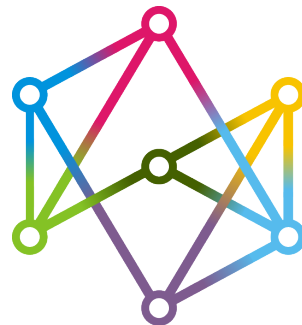


Computationally Efficient and Robust Belief Space Planning in High-Dimensional State Spaces

Vadim Indelman



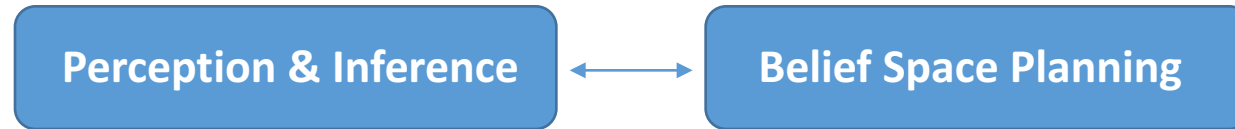
TECHNION
Israel Institute
of Technology



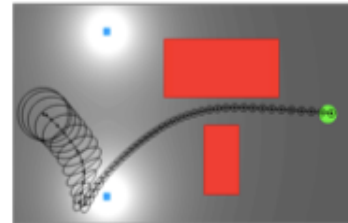
ANPL
Autonomous Navigation and
Perception Lab

Introduction

- Belief space planning (BSP) – determine optimal actions (policy) over the belief space with respect to a given objective, e.g. minimize state uncertainty
- A fundamental problem in robotics and AI
- Tight coupling with perception/inference



- Related problems: (multi-robot) informative planning/sensing, sensor deployment, active SLAM, autonomous navigation, graph sparsification etc.

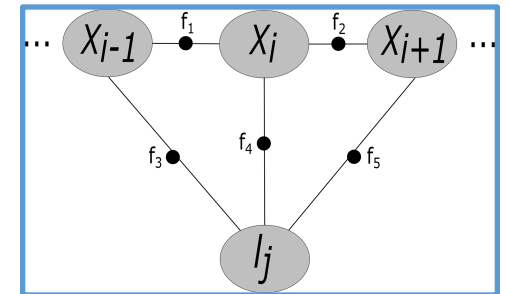


Introduction – Posterior Belief

- State vector $X_k \in \mathbb{R}^n$ at time t_k
- Posterior joint pdf can be represented by a factor graph
- Factors $F_i = \{f_i^1, \dots, f_i^{n_i}\}$ for $0 \leq t_i \leq t_k$

$$p(X_k | H_k) \propto \prod_{i=0}^k \prod_{j=1}^{n_i} f_i^j(X_i^j) \quad H_k \doteq \{u_{0:k-1}, z_{0:k}\}$$

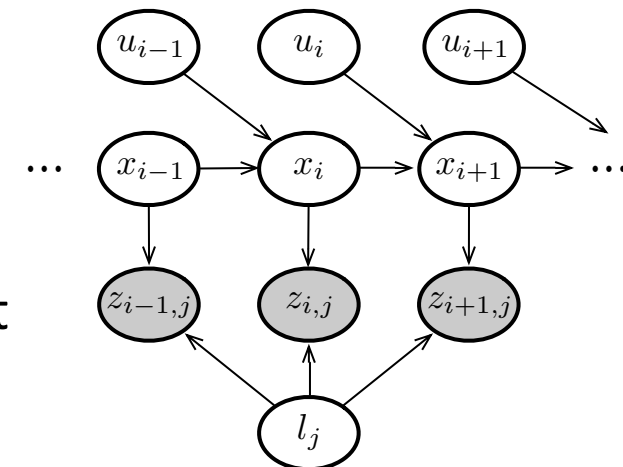
history



- Maximum a Posteriori (MAP) inference:

$$\underbrace{b[X_k]}_{\text{belief}} \doteq p(X_k | H_k) = N(X_k^*, \Sigma_k) = N^{-1}(\underbrace{\eta_k^*}_{\text{states/poses}}, \underbrace{\Lambda_k}_{\text{controls}})$$

- Usually (square root) information form is used, admits comp. efficient

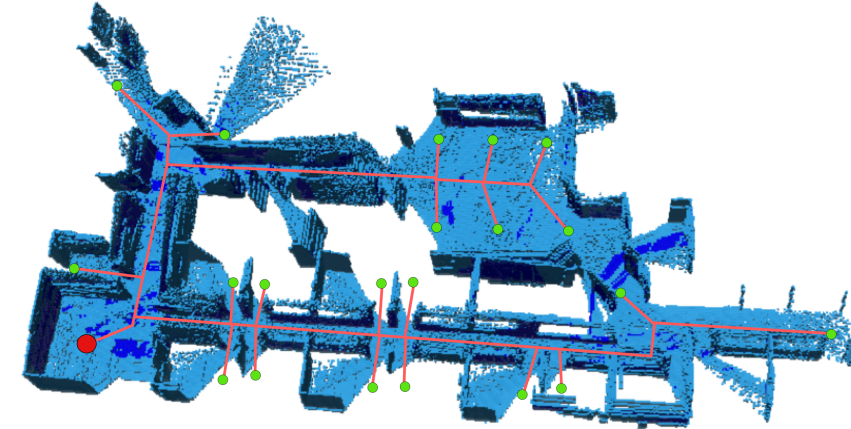


Belief Space Planning (BSP)

- Consider a set of candidate actions $\mathcal{A} \doteq \{a_1, a_2, \dots, a_N\}$
- For each (non-myopic) action $u_{k:k+L-1} \doteq a_i \in \mathcal{A}$
 - Belief at the l -th look-ahead step

$$b[X_{k+l}] \doteq p(X_k | \underbrace{u_{0:k-1}, z_{0:k}}_{\text{history } H_k}, \underbrace{u_{k:k+l-1}, z_{k+1:k+l}}_{\text{future actions \& observations}})$$

history H_k future actions & observations



- Given new factors and variables (if any), can be expressed as: $b[X_{k+l}] \propto b[X_k] \prod_{i=k+1}^{k+l} \prod_{j=1}^{n_i} f_i^j(X_i^j)$
- Objective function (e.g. **entropy**): $J(u_{k:k+L-1}) \doteq \mathbb{E} \left\{ \sum_{l=1}^L c_l(b[X_{k+l}], u_{k+l}) \right\}$

Multi-robot BSP

- Each robot r has its own discrete set of candidate actions \mathcal{A}^r
- Multi-robot joint belief for a specific candidate action permutation $P \doteq \{P^r, P^{r'}, \dots\}$

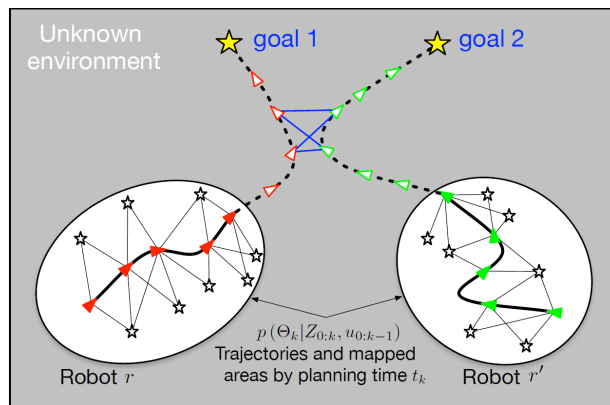
$$b[P] = p(X_k | Z_{0:k}, U_{0:k-1}) \prod_{r=1}^R \left[\prod_{l=1}^{L(P^r)} p(x_{v_l}^r | x_{v_{l-1}}^r, u_{v_{l-1}}^r) \cdot p(Z_{v_l}^r | X_{k+l}^r) \prod_{\{i,j\}} p(z_{i,j}^{r,r'} | x_{v_i}^r, x_{v_j}^{r'}) \right]$$

Current joint belief

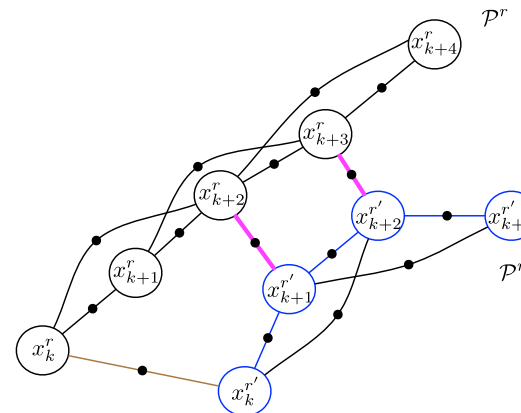
Local information

Multi-robot observations

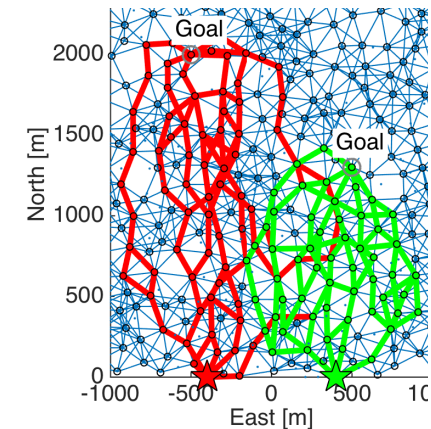
Multi robot future observations/constraints



Corresponding factor graph



Candidate actions



Challenges Include

- Calc. a globally optimal solution involves evaluating $J(\cdot)$ for **all** action permutations
 - **Comp. intractable** ($|\mathcal{A}|^R$)
- Belief is over a high-dimensional state – **comp. expensive** to evaluate **each** cost $c_l(b[X_{k+l}], u_{k+l})$
 - Involves propagating posterior belief for each action
 - Calc. of entropy is $O(N^3)$
 - Expensive also for **focused** case (reduce uncertainty of only some variables)
- Requires correct data association (e.g. loop closures, multi-robot constraints)
 - **Challenging** in presence of **ambiguity**

Agenda

Belief space planning in high-dimensional state spaces:

1. Computationally efficient information-theoretic BSP by re-using calculations and avoiding explicit belief propagation
2. Action consistent and bounded BSP problem representations:
 - Topological perspective (t-BSP)
 - Sparsification perspective (s-BSP)
3. Active perception in ambiguous environments – data association aware BSP

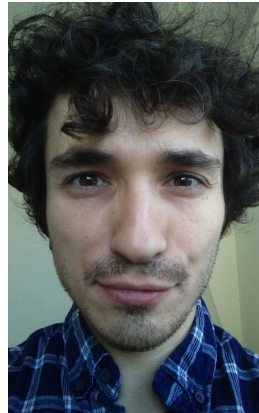
Agenda

Belief space planning in high-dimensional state spaces:

1. **Computationally efficient information-theoretic BSP by re-using calculations and avoiding explicit belief propagation**
2. Action consistent and bounded BSP problem representations:
 - Topological perspective (t-BSP)
 - Sparsification perspective (s-BSP)
3. Active perception in ambiguous environments – data association aware BSP

BSP via factor graphs, the matrix determinant lemma, and re-use of calculation (rAMD)

[Kopitkov and Indelman, IJRR'17]



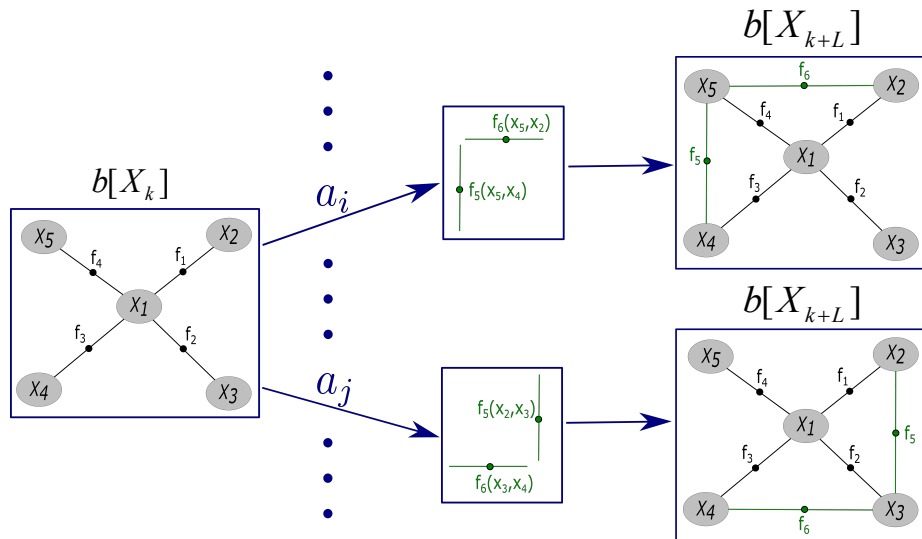
Objective & Key Idea

[Kopitkov and Indelman, IJRR'17]

- Consider an information-theoretic cost (e.g. entropy, info. gain)
- Existing approaches:
 - Propagate posterior belief for **each** action $a \in \mathcal{A}$
 - Calculate determinants of large matrices, $O(N^3)$
(reduced complexity in presence of sparsity)

$$J_H(a) = \dim.const - \frac{1}{2} \ln |\Lambda_{k+L}|$$

$$J_{IG}(a) = \frac{1}{2} \ln \frac{|\Lambda_{k+L}|}{|\Lambda_k|}$$



- Our objective: want to avoid explicitly calculating $|\Lambda_{k+L}|$
- Key ideas: (i) use (augmented) matrix determinant lemma
(ii) re-use calculations between cand. actions

Posterior Information Matrix

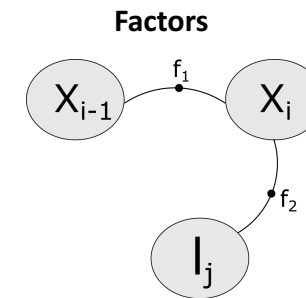
[Kopitkov and Indelman, IJRR'17]

Our objective: want to avoid explicitly calculating $|\Lambda_{k+L}|$

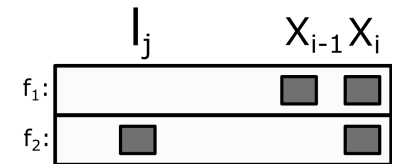
- For each action $a \in \mathcal{A}$:

Posterior belief
$$b[X_{k+L}] \propto b[X_k] \prod_{l=k+1}^{k+L} \prod_{j=1}^{n_l} f_l^j(X_l^j)$$

Posterior info. matrix
$$\Lambda_{k+L} = \Lambda_k + A^T \cdot A$$



Appropriate rows in Action Jacobian



- A is a **sparse** Jacobian matrix of **new** factors, with dimension $m \times N$
- Typically number of **involved** variables is very small

Matrix Determinant Lemma (MDL)

[Kopitkov and Indelman, IJRR'17]

- We use MDL to reduce calculations:

$$|\Lambda_k + A^T \cdot A| = |\Lambda_k| \cdot |I_m + A \cdot \Sigma_k \cdot A^T| \quad \text{where } \Sigma_k \equiv \Lambda_k^{-1} \in \mathbb{R}^{n \times n} \quad A \in \mathbb{R}^{m \times n}$$

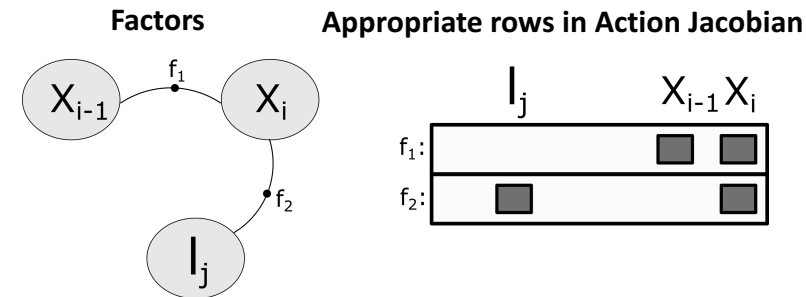
- Applying it to unfocused not-augmented BSP (see IJRR'17 paper):

$$J_{IG}(a) = \frac{1}{2} \ln |I_m + I_A \cdot \Sigma_k^{M, IX} \cdot (I_A)^T| \quad (\text{Instead of } |\Lambda_k + A^T \cdot A|)$$

where:

I_A is a partition of A with all non-zero columns

$\Sigma_k^{M, IX}$ is a prior marginal covariance of *involved* variables IX



Matrix Determinant Lemma (MDL)

[Kopitkov and Indelman, IJRR'17]

$$J_{IG}(a) = \frac{1}{2} \ln \left| I_m + I_A \cdot \sum_k^{M, I_X} \cdot (I_A)^T \right|$$

- We can **avoid** posterior propagation and calc. of determinants of **large** matrices
- Calculation of action impact **does not** depend on N , **given** \sum_k^{M, I_X}
- Calculation complexity depends on m and $\dim(I_X)$, typically very cheap
- We propose **re-use of calculation**:
 - Only **few entries** from the prior covariance are actually required!
 - Different candidate actions often **share** many **involved** variables I_X
 - Combine variables involved in all candidate actions into set $X_{All} \subseteq X_k$
 - Perform one-time calculation of $\sum_k^{M, X_{All}}$ (depends on N)
 - Calculate $J_{IG}(a)$ for each action, using $\sum_k^{M, X_{All}}$

Extensions

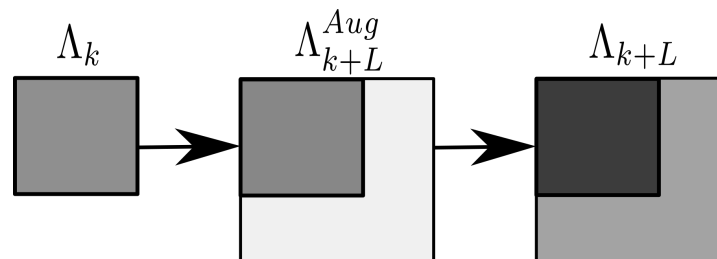
[Kopitkov and Indelman, IJRR'17]

- Approach has been extended to support other BSP problem types (see **IJRR'17 paper**)

BSP problem type	Non-Augmented	Augmented
Unfocused	✓	✓
Focused (reduce uncertainty only of some states)	✓	✓

- **Focused:**
$$J_{IG}^F(a) = \frac{1}{2} \ln \left| I_m + I_A \cdot \Sigma_k^{M, I_X} \cdot (I_A)^T \right| - \frac{1}{2} \ln \left| I_m + I_{A^U} \cdot \Sigma_k^{I_X^U | F} \cdot (I_{A^U})^T \right|$$

- For **augmented** case (e.g. active SLAM), Matrix Determinant Lemma (MDL) **cannot** be used!
 - We extended MDL to an augmented case (AMDL), details in the paper

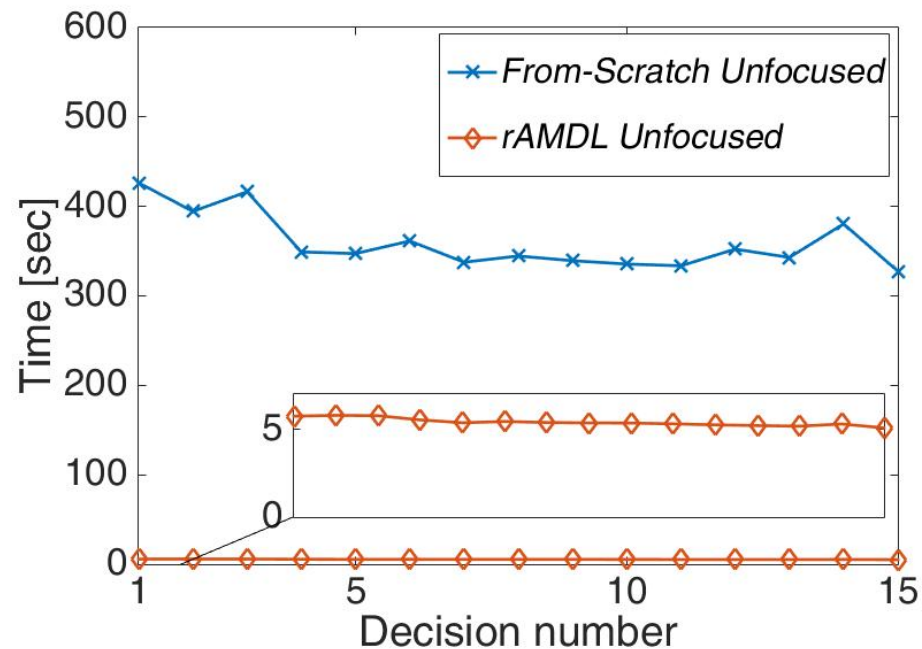


$$\Lambda_{k+L} = \Lambda_{k+L}^{Aug} + A^T \cdot A$$

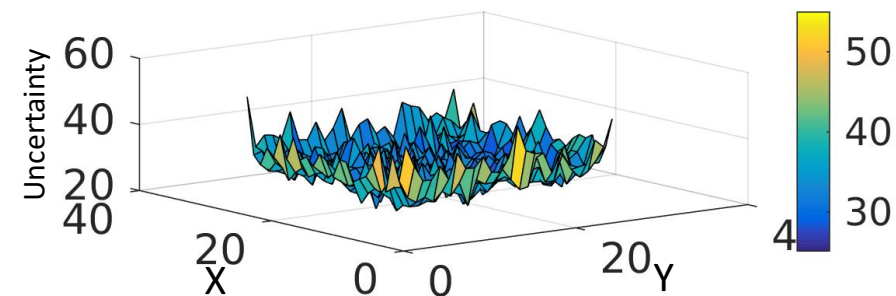
Results – Unfocused, Non-Augmented - Sensor Deployment

[Kopitkov and Indelman, IJRR'17]

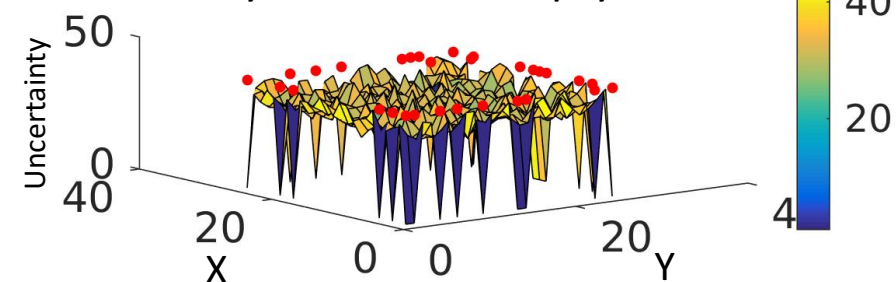
- Significant time reduction in *Unfocused* case



Uncertainty field (**dense** prior information matrix)



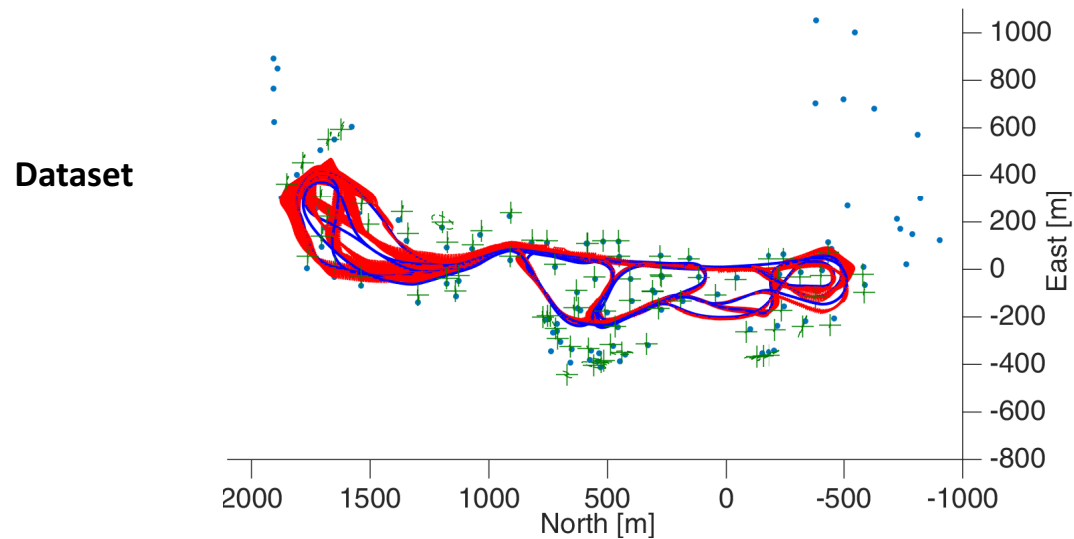
Uncertainty field after sensors' deployment



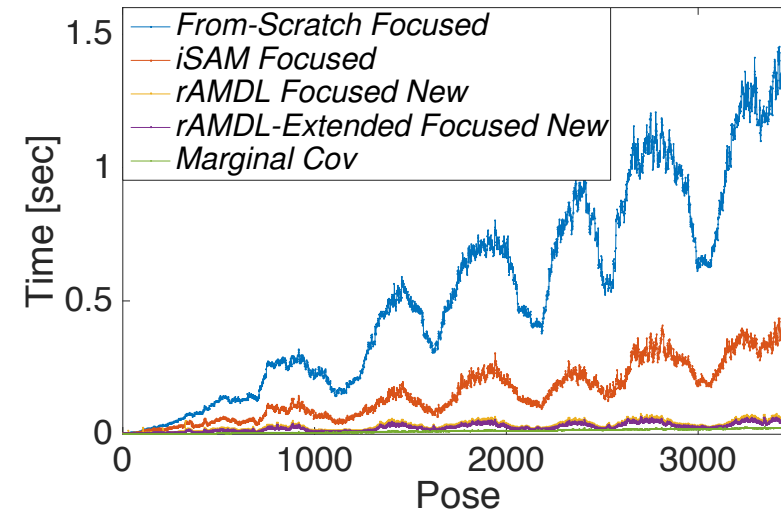
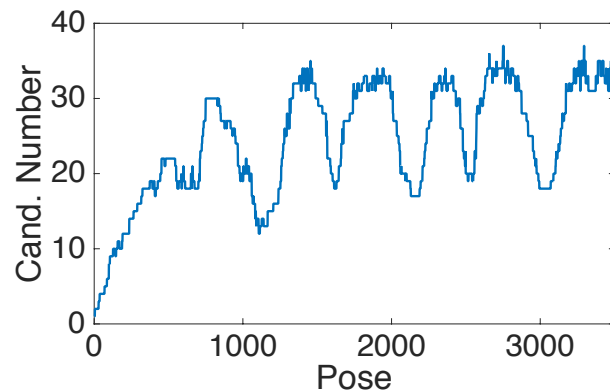
Results – Focused, Augmented – SLAM – Viktoria Park

[Kopitkov and Indelman, IJRR'17]

- Significant time reduction in *Focused* case – focus on **last robot pose**

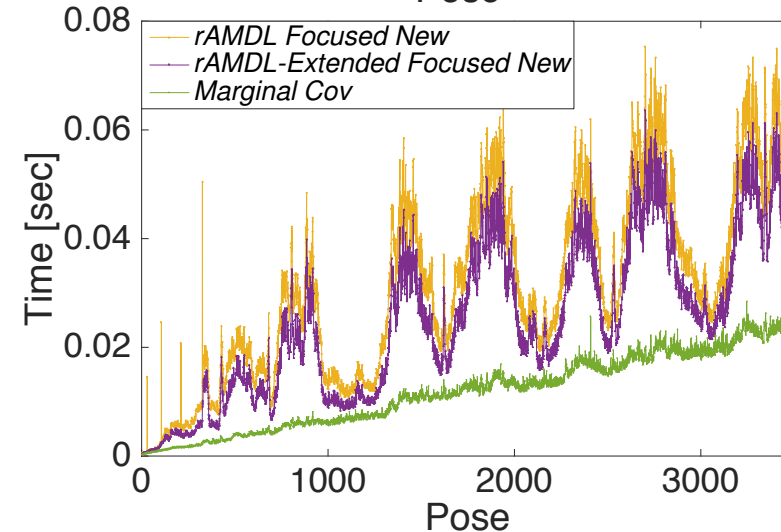


Number of candidate actions



Running time of planning

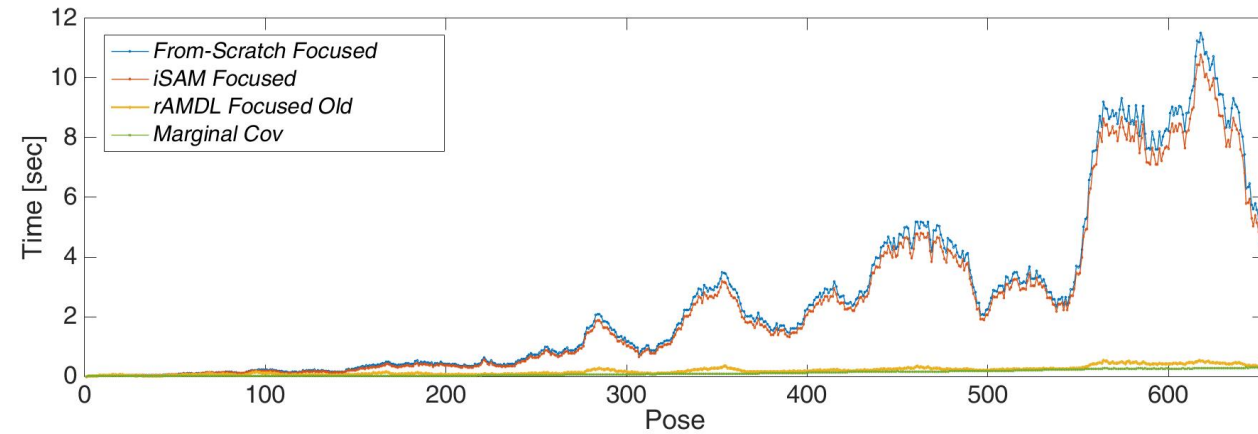
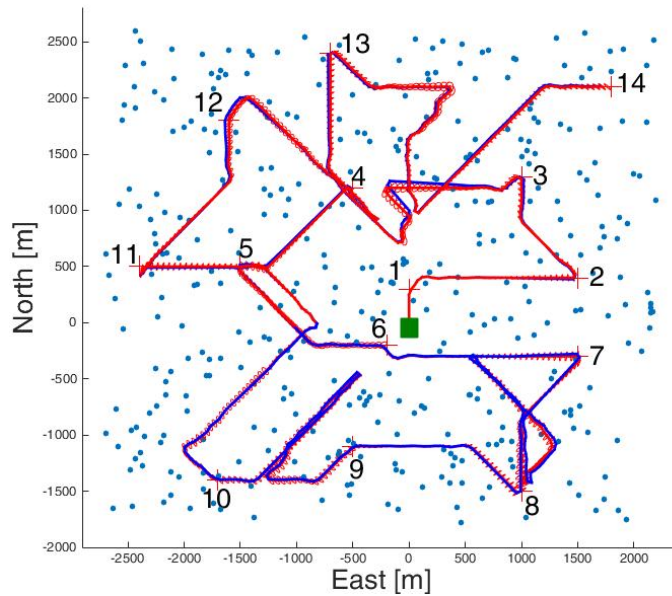
Zoom-in



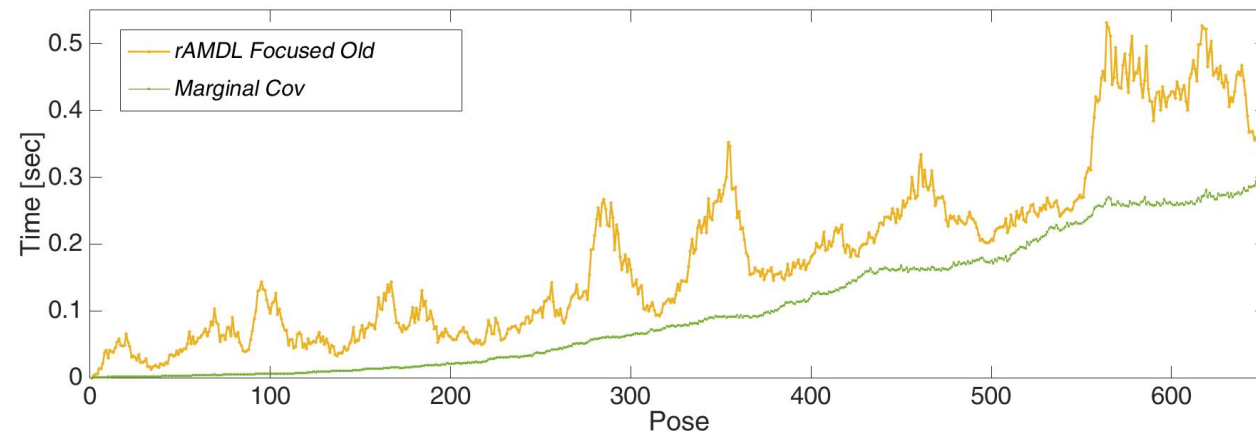
Results – Focused, Augmented – SLAM – Simulation

[Kopitkov and Indelman, IJRR'17]

- Significant time reduction in *Focused* case – focus on **mapped landmarks**



Zoom-in



rAMD L - Summary

[Kopitkov and Indelman, IJRR'17]

- We address all 4 BSP problem types:

BSP cases	Non-Augmented	Augmented
Unfocused	✓	✓
Focused	✓	✓

- **No need** for posterior belief propagation
- **An exact** solution
- **Avoid** calculating determinants of large matrices
- Calculation **Re-use**
- Per-action evaluation **does not depend on state dimension**, given marginal prior covariances
- Still requires a **one-time** recovery of marginal covariances of involved variables

Agenda

Belief space planning in high-dimensional state spaces:

1. Computationally efficient information-theoretic BSP by re-using calculations and avoiding explicit belief propagation
2. Action consistent and bounded BSP problem representations:
 - Topological perspective (t-BSP)
 - Sparsification perspective (s-BSP)
3. Active perception in ambiguous environments – data association aware BSP

Action Consistent & Bounded Approximations

[Elimelech and Indelman, ICRA'17, IROS'17, ISRR'17]



Action Consistent & Bounded Approximations

[Elimelech and Indelman, ISRR'17]

- **Paradigm:** generate and solve a simplified decision making problem, b_s, J_s , which has a minimal impact on the best-action selection
- **Key observations:**
 - In decision making, only need to sort actions from best to worst
 - Changing reward values w/o changing order of actions does not change action selection

- **Action-consistent** representation b_s, J_s :

$$\forall a, a' \in \mathcal{A}: J(b, a) < J(b, a') \iff J_s(b_s, a) < J_s(b_s, a')$$

$$J(b, a) = J(b, a') \iff J_s(b_s, a) = J_s(b_s, a')$$

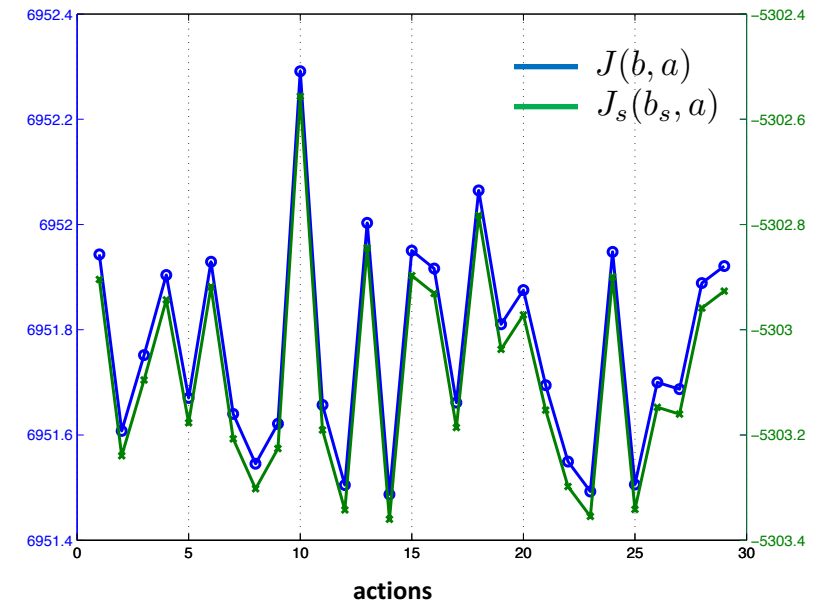


Image from Indelman RA-L'16 21

Action Consistent & Bounded Approximations

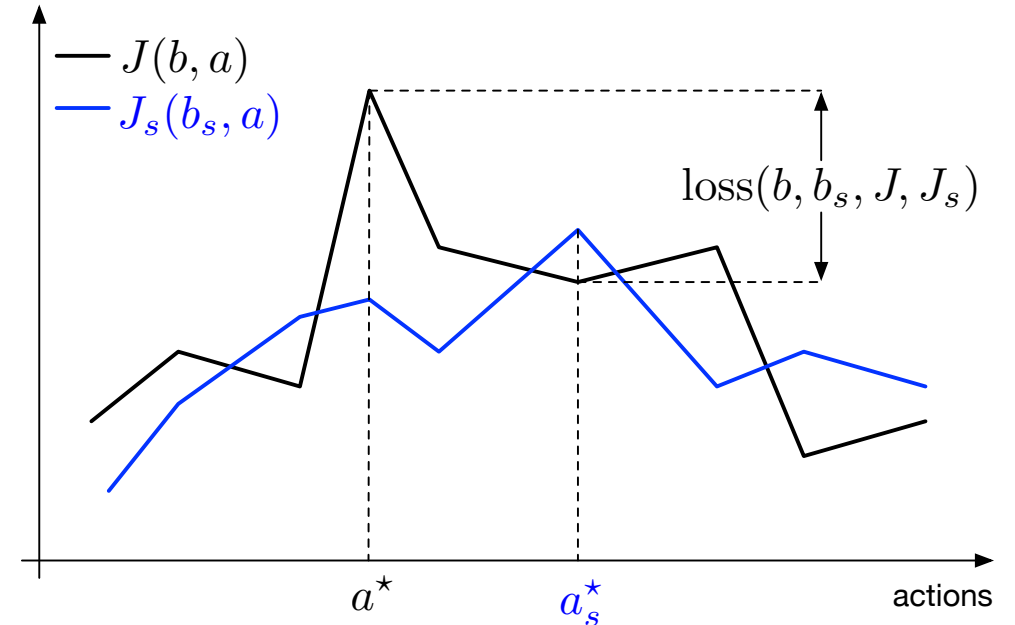
[Elimelech and Indelman, ISRR'17]

- Action consistency cannot be always guaranteed
- Sacrifice in performance - definition:

$$\text{loss}(b, b_s, J, J_s) \doteq J(b, a^*) - J(b, a_s^*)$$

with $a^* \doteq \operatorname{argmax}_{a \in \mathcal{A}} J(b, a)$

$$a_s^* \doteq \operatorname{argmax}_{a \in \mathcal{A}} J_s(b_s, a)$$

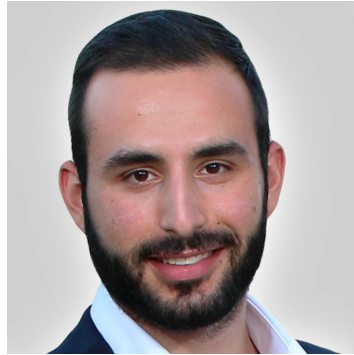


- Often possible to settle for a sub-optimal action, in order to reduce the solution complexity
- Need tight bounds on $\text{loss}(b, b_s, J, J_s)$!

Perspectives

- **Belief sparsification for BSP (s-BSP)**

[Elimelech and Indelman, ICRA'17, IROS'17, ISRR'17]



- **Topological BSP (t-BSP)**

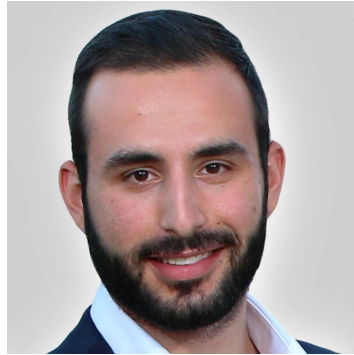
[Kitanov and Indelman, ICRA'18]



Perspectives

- **Belief sparsification for BSP (s-BSP)**

[Elimelech and Indelman, ICRA'17, IROS'17, ISRR'17]



- **Topological BSP (t-BSP)**

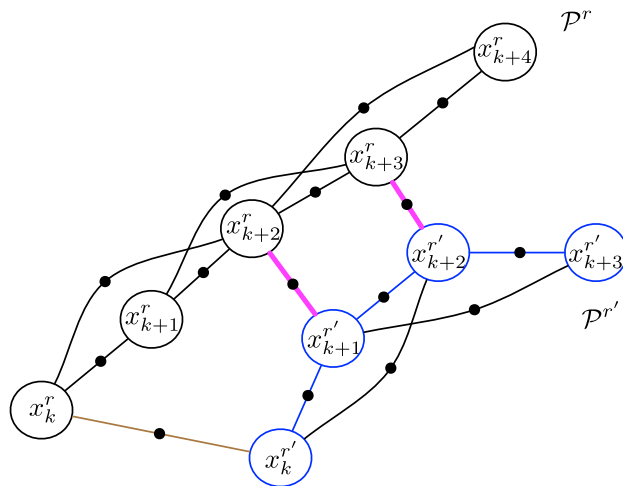
[Kitanov and Indelman, ICRA'18]



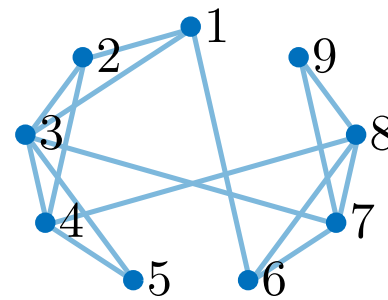
Topological Belief Space Planning (t-BSP)

[Kitanov and Indelman, ICRA'18]

- Topological properties of factor graphs dominantly determine estimation accuracy
[Khosoussi et al. IROS'14, IJRR'17]
- **Key idea:**
 - Design a metric of factor graph topology that is strongly correlated with entropy
 - Determine best action using that topological metric (instead of entropy)
 - **Does not require explicit inference, nor partial state covariance recovery**



Factor graph for a 2-robot scenario,
considering some specific candidate actions



Corresponding topology represented
by a graph $G(\Gamma, E)$



topological
metric $s(G)$

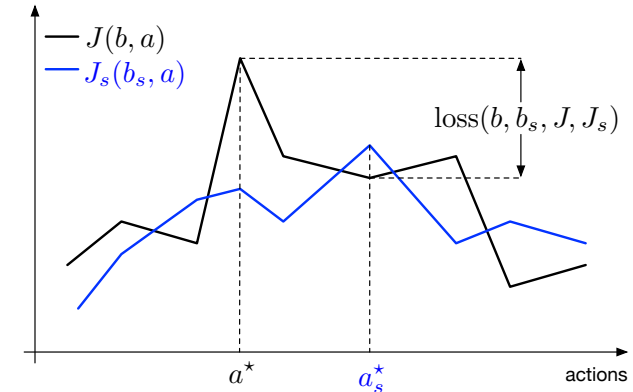
graph signature

Topological Belief Space Planning (t-BSP)

[Kitanov and Indelman, ICRA'18]

- Relation to action-consistent & bounded approximations framework:

- b_s - Factor graph topology
- J_s - Graph signature $s(G)$



- Two graph signatures currently considered in t-BSP:

- Von Neumann entropy of G (VN) which is further simplified with a function of graph node degrees d

$$s(G) = H_{VN}(G) = - \sum_{i=1}^{|\Gamma|} \frac{\hat{\lambda}_i}{|\Gamma|} \ln \frac{\hat{\lambda}_i}{|\Gamma|} \approx 1 - \frac{1}{|\Gamma|} - \frac{1}{|\Gamma|^2} \sum_{(i,j) \in E} \frac{1}{d(i)d(j)}$$

- Signature based on the number of spanning trees of G (ST)

Cheap to calculate, only a function of node degrees!

$$s(G) = \frac{3}{2} \tau(G) + \frac{|\Gamma|}{2} [\ln |\Omega_w| - \ln(2\pi e)]$$

Topological Belief Space Planning (t-BSP)

[Kitanov and Indelman, ICRA'18]

Metric Space

$$J(\mathcal{U}) = \frac{n}{2} \ln(2\pi e) + \frac{1}{2} \ln |\Sigma(X_{k+L})|$$

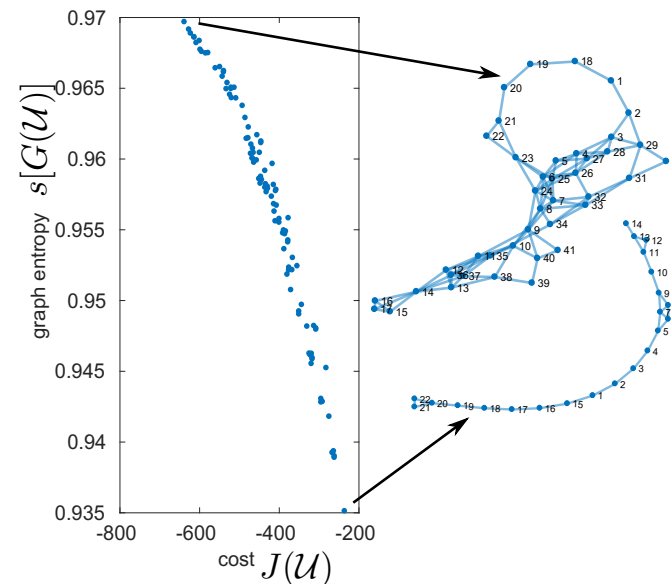
$$\mathcal{U}^* = \arg \min_{\mathcal{U}} J(\mathcal{U})$$

Topological space

$$s(G) = H_{VN}(G) \approx 1 - \frac{1}{|\Gamma|} - \frac{1}{|\Gamma|^2} \sum_{(i,j) \in E} \frac{1}{d(i)d(j)}$$

$$\hat{\mathcal{U}}^* = \arg \max_{\mathcal{U}} s[G(\mathcal{U})]$$

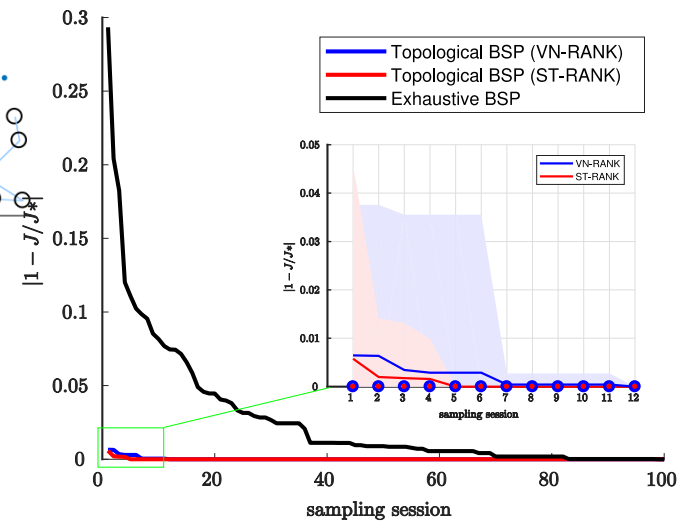
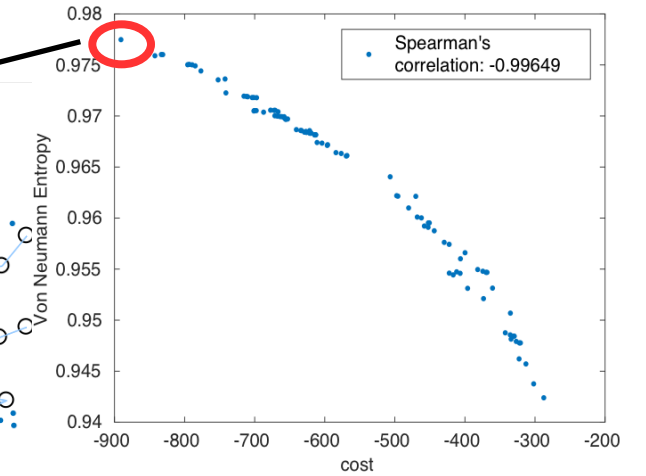
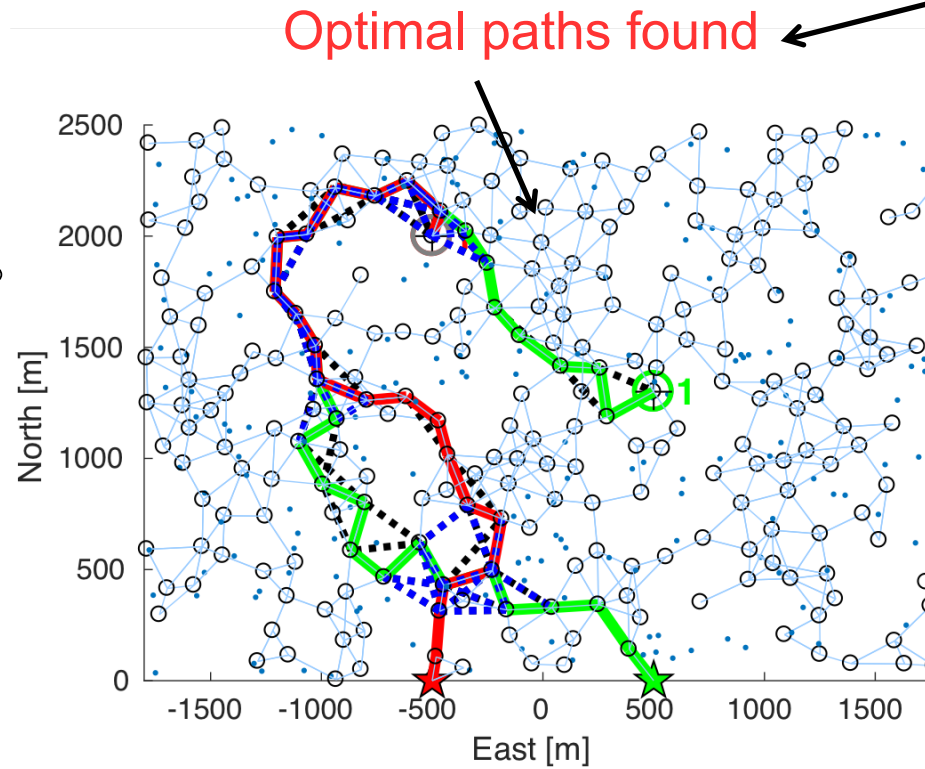
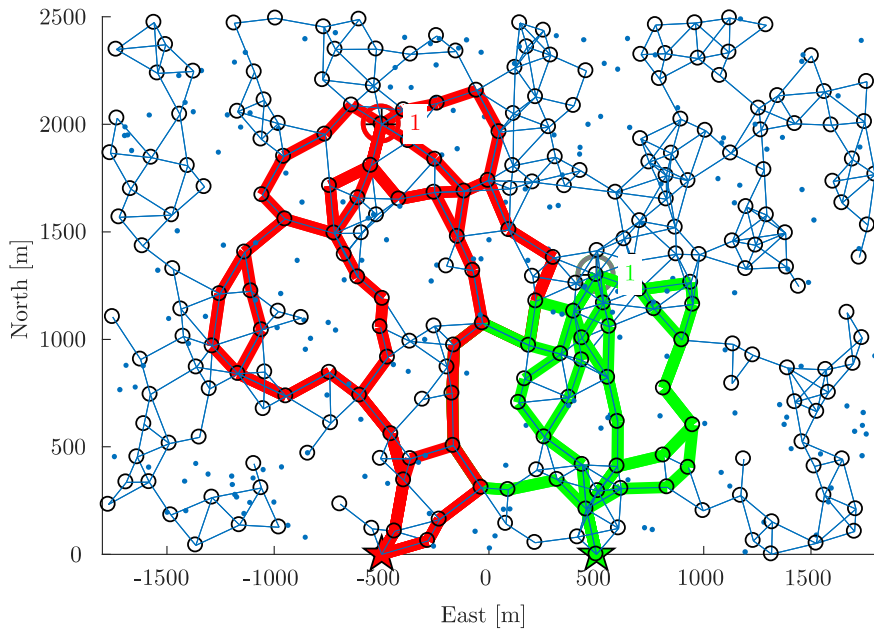
Topological and info-theoretic metrics are strongly correlated!



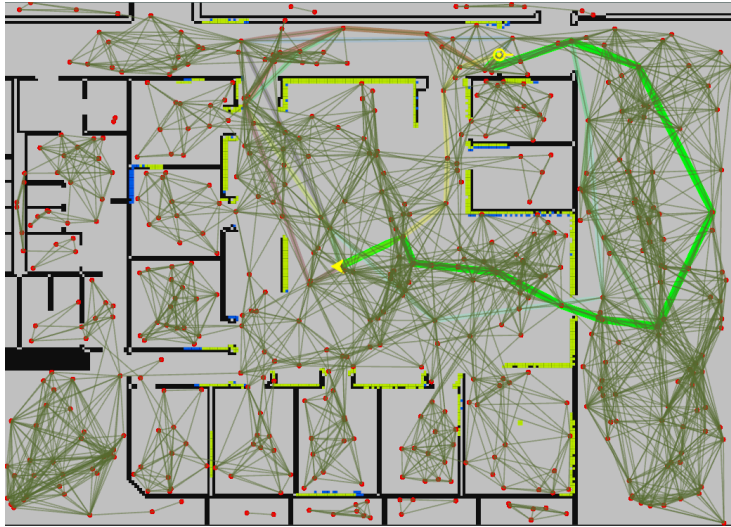
t-BSP: Application to Multi-Robot BSP

[Kitanov and Indelman, ICRA'18]

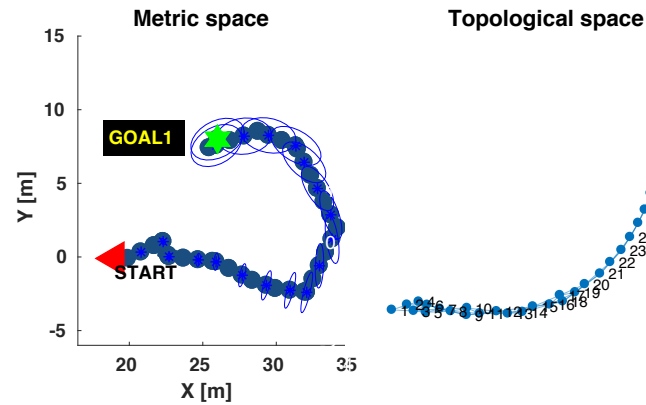
Candidate paths of two robots (red and green) generated on top of PRM in a single planning session:



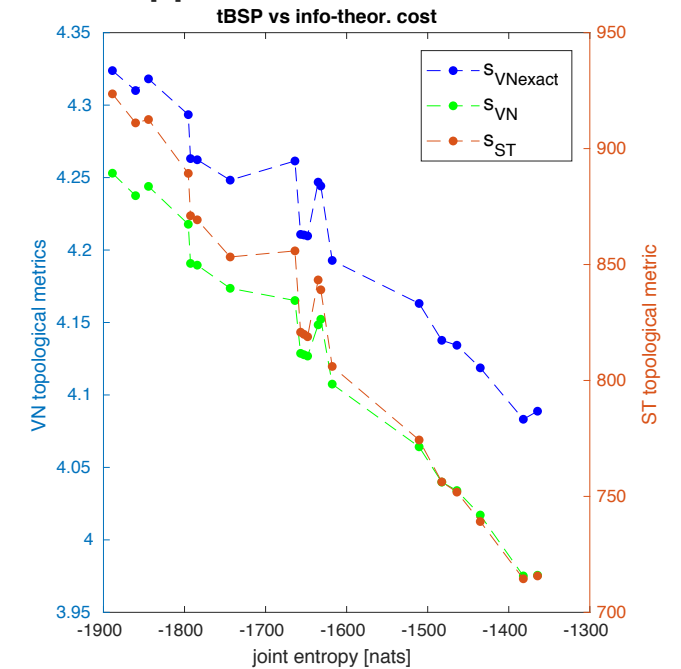
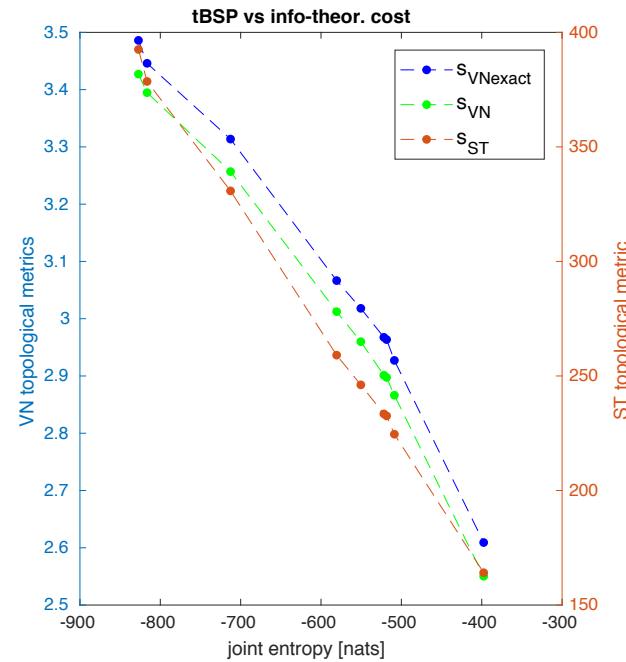
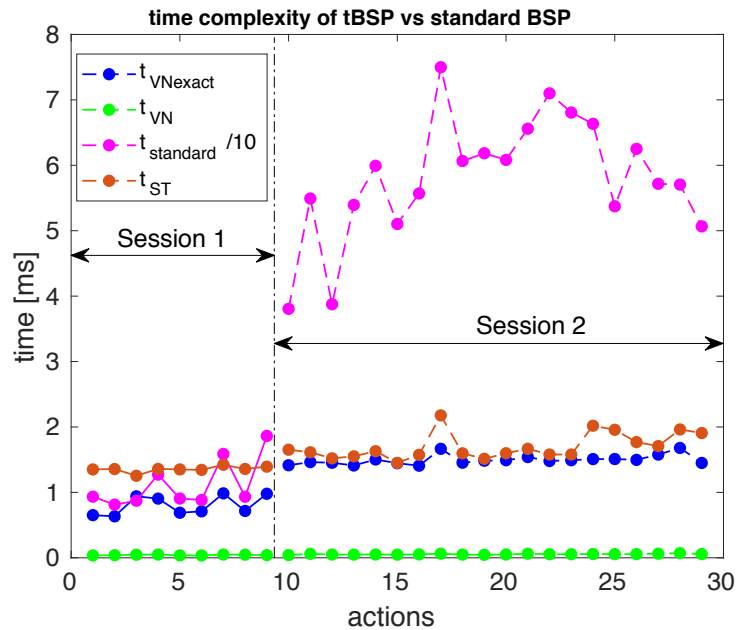
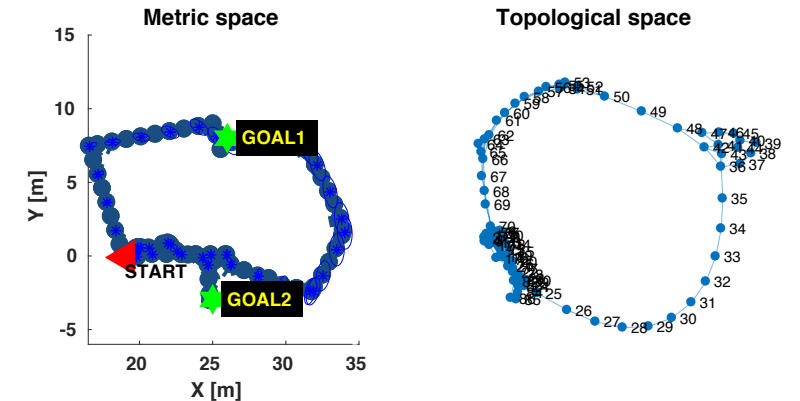
t-BSP: Gazebo Initial Results



1st planning session (exploration)



2nd planning session (exploitation, loop closures)



Perspectives

- **Belief sparsification for BSP (s-BSP)**

[Elimelech and Indelman, ICRA'17, IROS'17, ISRR'17]



- **Topological BSP (t-BSP)**

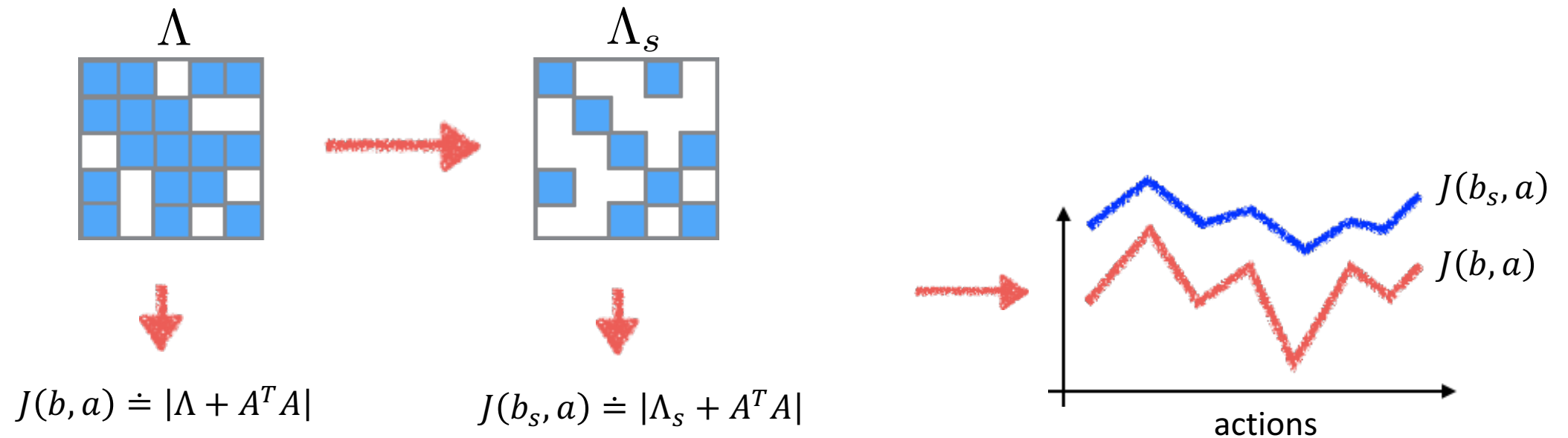
[Kitanov and Indelman, ICRA'18]



Belief sparsification for BSP (s-BSP): Key Idea

[Elimelech and Indelman, ICRA'17, IROS'17, ISRR'17]

- Find an appropriate **sparsified** information space (more generally, belief)
- Perform decision making over that, rather the original, information space

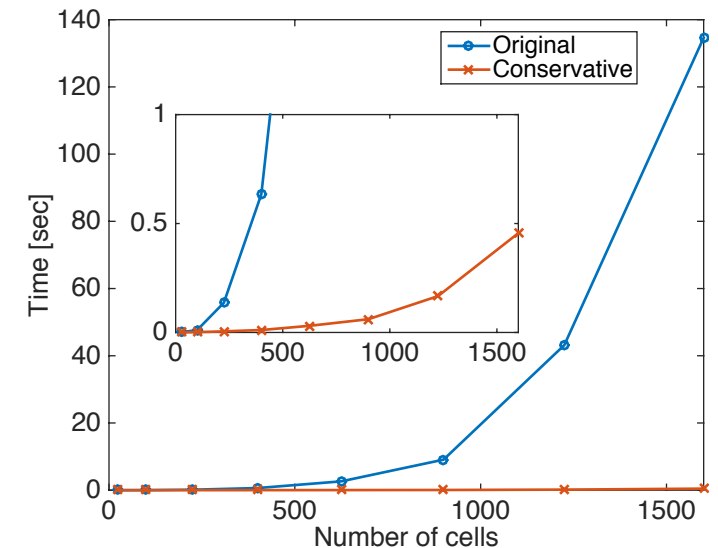
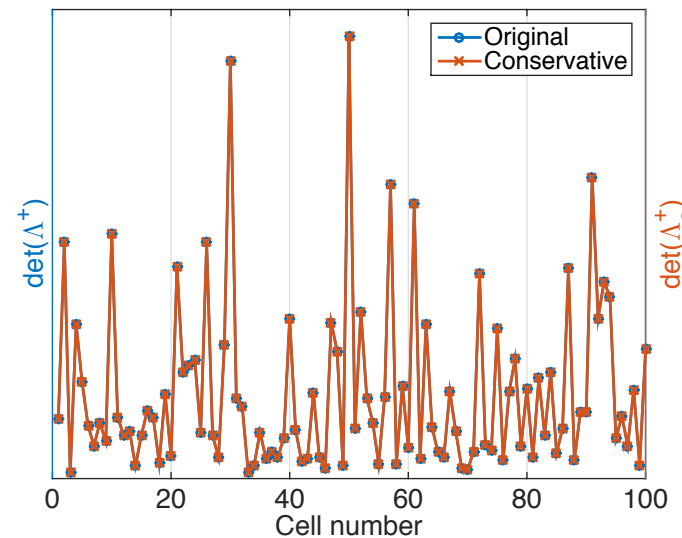
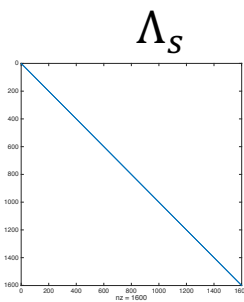
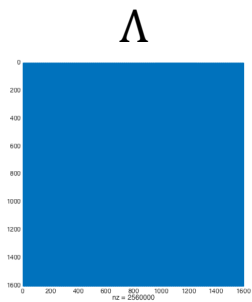
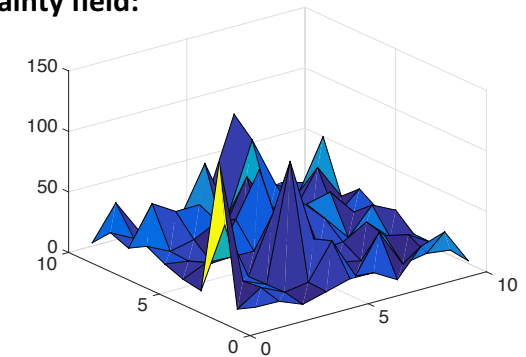


- Do we get the same performance (decisions), i.e. is it action consistent?
- If not, can we bound the loss?

s-BSP: Initial Results – Sensor Deployment

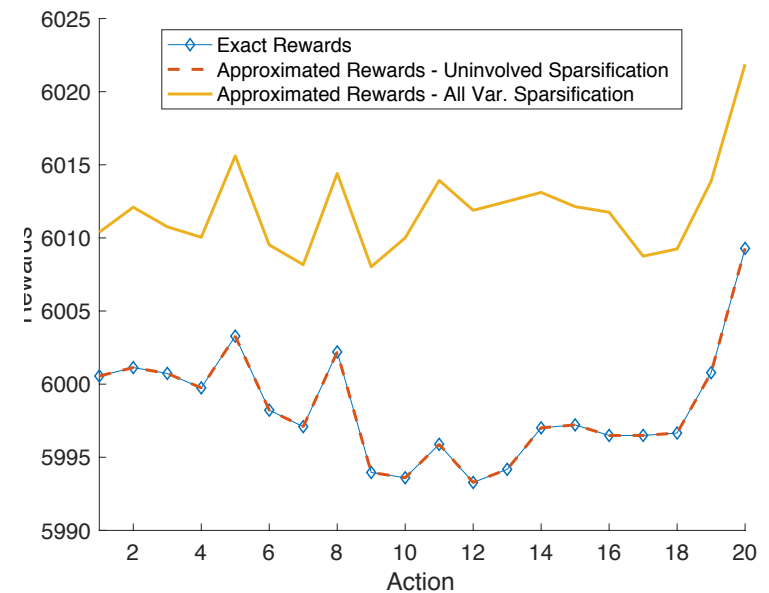
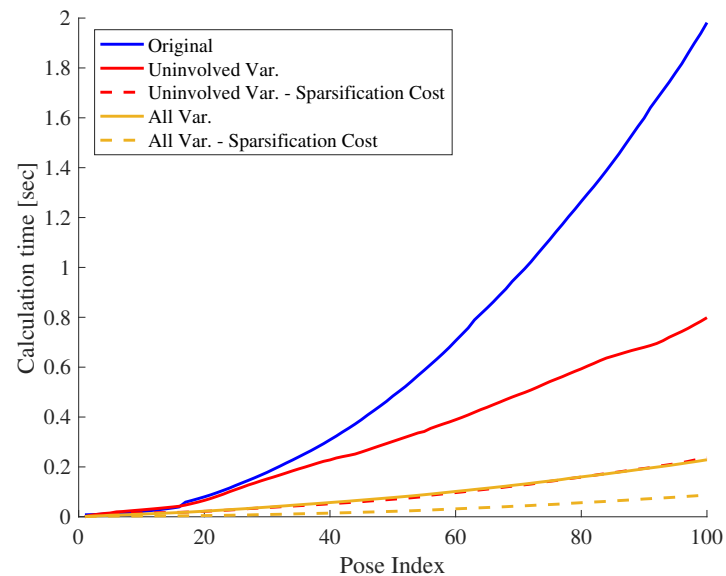
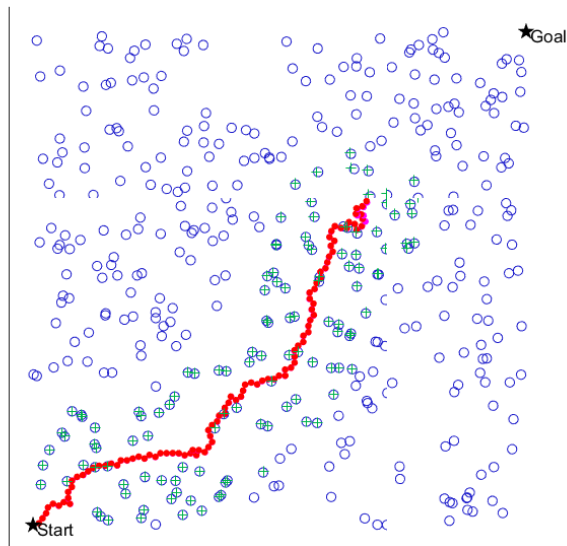
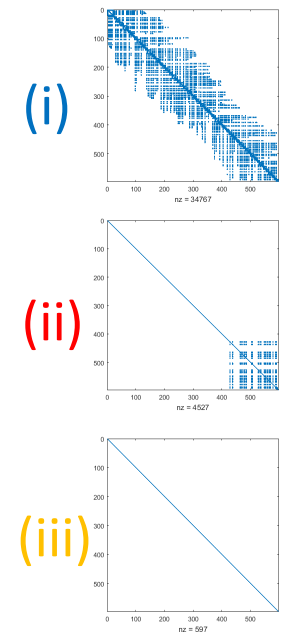
- **Objective:** deploy k sensors in an $N \times N$ area
- **Motivating example:** **extreme** sparsification – drop **all** off-diagonal terms
- **Action consistency is guaranteed** [Indelman RA-L'16] for a restricted problem setting (myopic, single-row measurement Jacobians)

Prior uncertainty field:



s-BSP: Initial Results - Active SLAM

- Active full SLAM scenario – navigation to a goal in an unknown environment
- LIDAR sensor – range-bearing observations of surrounding landmarks
- Primitive actions – star-pattern search of best progression angle (20 actions)
- Results considering 3 sparsity levels:
 - (i) original, (ii) sparsification of uninvolved variables, (iii) sparsification of all variables (diag. info. matrix)



Agenda

Belief space planning in high-dimensional state spaces:

1. Computationally efficient information-theoretic BSP by re-using calculations and avoiding explicit belief propagation
2. Action consistent and bounded BSP problem representations:
 - Topological perspective (t-BSP)
 - Sparsification perspective (s-BSP)
3. Active perception in ambiguous environments – data association aware BSP

[Pathak, Thomas, Indelman, IJRR'18]



Active Robust Perception

[Pathak, Thomas, Indelman, IJRR'18]

- What happens if the environment is **ambiguous, perceptually aliased**?
- BSP approaches typically assume data association is **given** and **perfect**! We **relax** this assumption
- Our **Data Association Aware BSP (DA-BSP)** algorithm considers both
 - **Ambiguous data association** (DA) due to perceptual aliasing, and
 - **Localization uncertainty** due to stochastic control and imperfect sensing
- Approach can be used for **active disambiguation** (for example)



Angeli et al., TRO'08

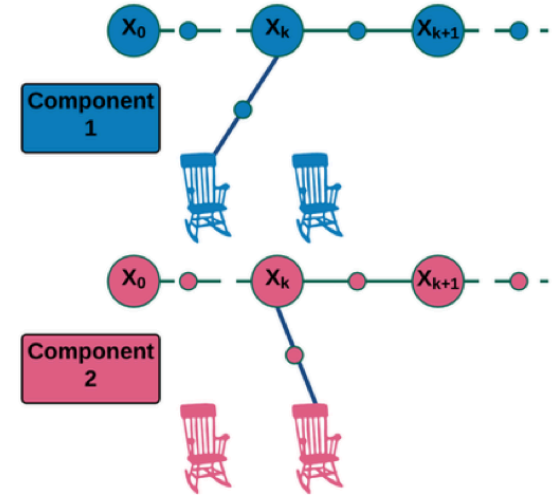
2nd Workshop on Multi-Robot Perception-Driven Control and Planning, IROS'18

Approach Overview

[Pathak, Thomas, Indelman, IJRR'18]

- Belief is represented by a Gaussian Mixture Model (GMM)

$$b[X_k] = \mathbb{P}(X_k | \mathcal{H}_k) = \sum_{j=1}^{M_k} \underbrace{\xi_k^j}_{\text{Weight}} \underbrace{\mathbb{P}(X_k | \mathcal{H}_k, \gamma = j)}_{\text{Conditional Gaussian, represented by a factor graph}}$$



Pathak et al., IJRR'18

- **Main idea:** Reason how a GMM belief will evolve for different candidate actions
- Number of modes can go down, and go up (!)

Approach Overview

[Pathak, Thomas, Indelman, IJRR'18]

$$J(u_k) \doteq \mathbb{E} \{c(b[X_{k+1}])\} \equiv \int_{z_{k+1}} \boxed{p(z_{k+1} | \mathcal{H}_{k+1}^-)} c \left(\boxed{p(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1})} \right)$$

- Marginalize over possible data associations
- Maintain & track data association hypotheses

- Likelihood of a specific z_{k+1} to be captured

$$\underline{p(z_{k+1} | \mathcal{H}_{k+1}^-)} \equiv \sum_j \int_x p(z_{k+1}, x, A_j | \mathcal{H}_{k+1}^-) \doteq \sum_j w_j$$

- Posterior *given a specific observation* z_{k+1}

$$\begin{aligned} \underline{b[X_{k+1}]} &= \sum_j p(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1}, A_j) p(A_j | \mathcal{H}_{k+1}^-, z_{k+1}) \\ &= \sum_j \tilde{w}_j b[X_{k+1}^{j+}] \quad \tilde{w}_j = \eta w_j \end{aligned}$$

Approach Overview

[Pathak, Thomas, Indelman, IJRR'18]

$$J(u_k) \doteq \int_{z_{k+1}} p(z_{k+1} | \mathcal{H}_{k+1}^-) c \left(\boxed{p(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1})} \right)$$

- Posterior belief

$$b[X_{k+1}] = \sum_j^{\{A_N\}} p(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1}, A_j) p(A_j | \mathcal{H}_{k+1}^-, z_{k+1})$$

- In other words
 - Observation is given, hence, **must** capture **one** (unknown) scene
 - Which one? Consider all possible scenes

Approach Overview

[Pathak, Thomas, Indelman, IJRR'18]

$$J(u_k) \doteq \int_{z_{k+1}} \boxed{p(z_{k+1} | \mathcal{H}_{k+1}^-)} c \left(p(X_{k+1} | \mathcal{H}_{k+1}^-, z_{k+1}) \right)$$

- Likelihood of a specific z_{k+1} to be captured
- Marginalize over all scenes A_j and viewpoints x_{k+1}

$$p(z_{k+1} | \mathcal{H}_{k+1}^-) \equiv \sum_j \int_x p(z_{k+1}, x, A_j | \mathcal{H}_{k+1}^-) \doteq \sum_j w_j$$

Perceptual Aliasing Aspects

[Pathak, Thomas, Indelman, IJRR'18]

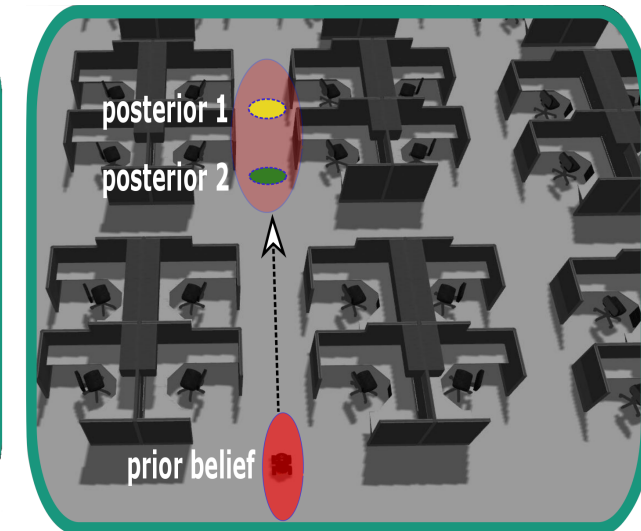
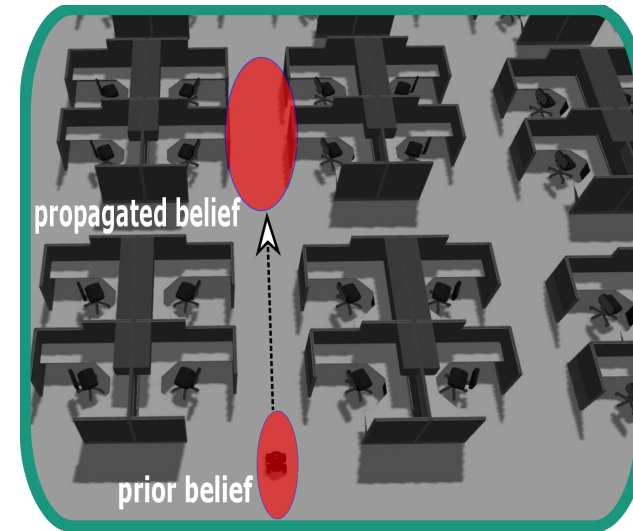
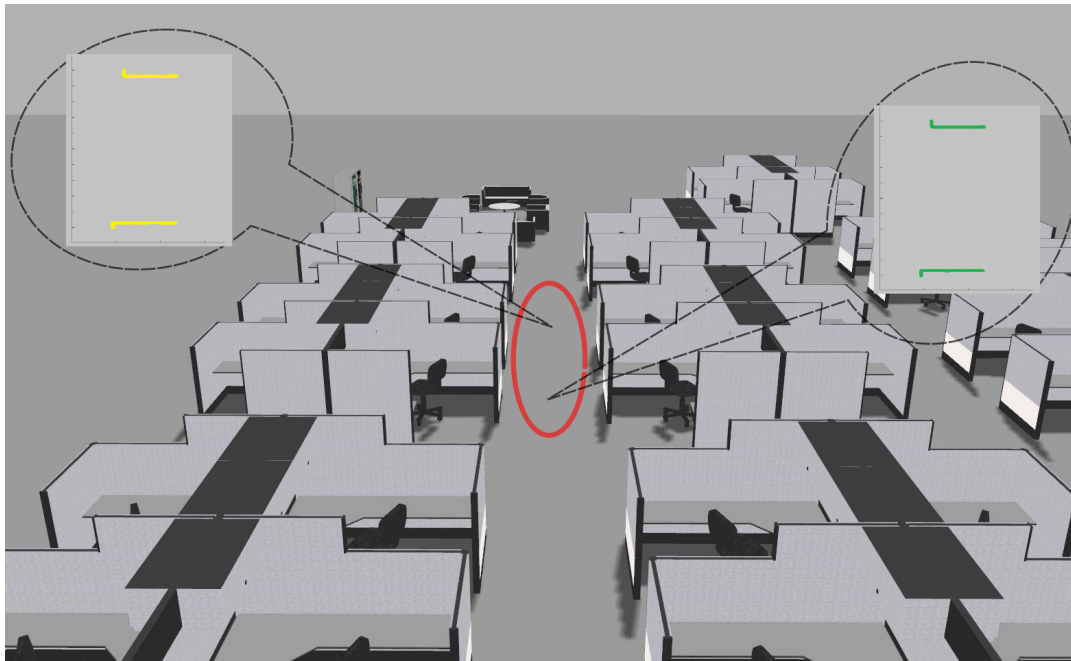
$$J(u_k) \doteq \int_{z_{k+1}} \underbrace{\left(\sum_j w_j\right)}_{\text{blue}} c\left(\underbrace{\sum_j \tilde{w}_j b[X_{k+1}^{j+}]}_{\text{red}}\right)$$

- No perceptual aliasing:
 - Only **one** non-negligible weight \tilde{w}_j
 - Reduces to state of the art belief space planning
- With perceptual aliasing:
 - Multiple non-negligible weights \tilde{w}_j , correspond to aliased scenes (given z_{k+1})
 - Posterior **becomes a mixture of pdfs (GMM)**
 - In practice, hypotheses pruning/merging is performed (**see IJRR'18 paper**)
- Approach can be used for **active disambiguation** (between DA hypotheses)

Number of GMM Components Can Increase

[Pathak, Thomas, Indelman, IJRR'18]

- Gazebo simulation



Real Experiment with April Tags

[Pathak, Thomas, Indelman, IJRR'18]

- Octagonal world with a known map
- April Tags used to simulate aliasing environment and for localization

Stairs

Door

Elevator

Elevator

Legend:

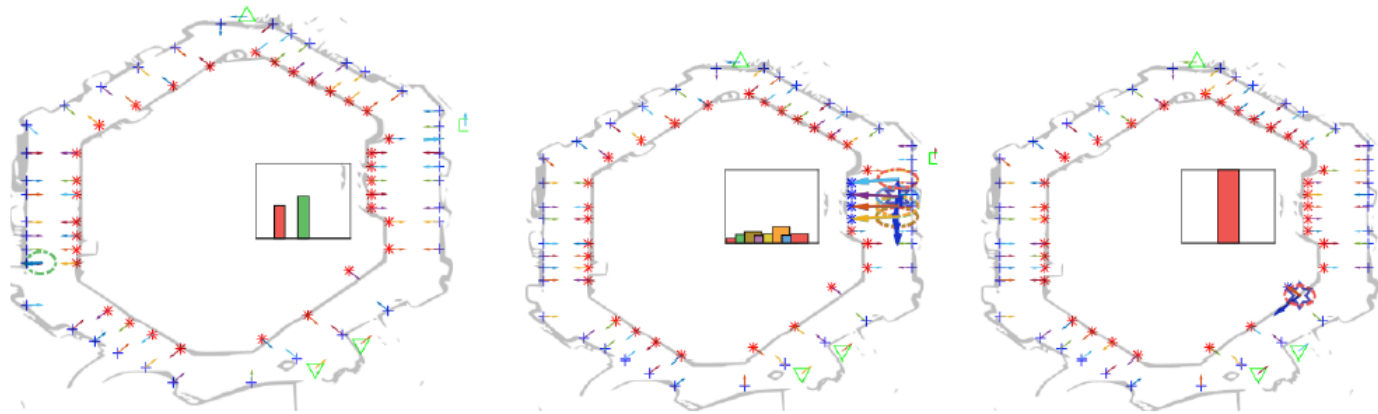
- ▽ Elevators
- △ Stairs
- + AprilTag-1
- * AprilTag-2
- Doors

No object detected

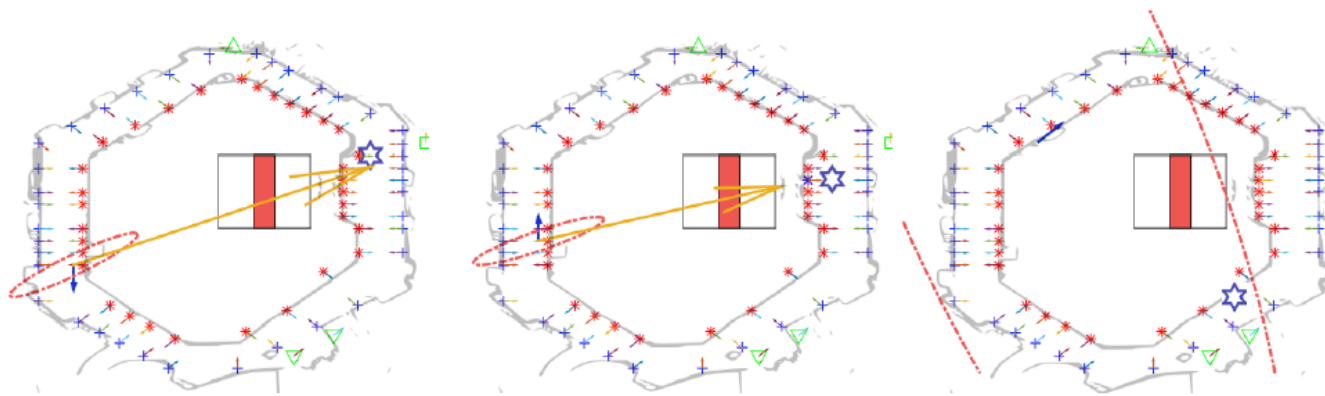
An object detected

Real Experiment with April Tags

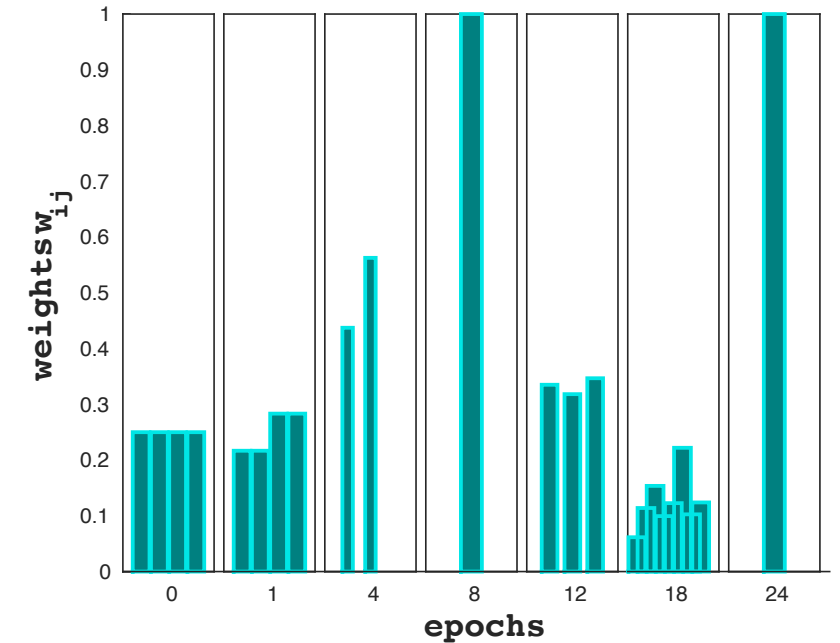
[Pathak, Thomas, Indelman, IJRR'18]



(a) DA-BSP for epochs {4,18,24}.



(b) BSP-uni for epochs {4,18,24}

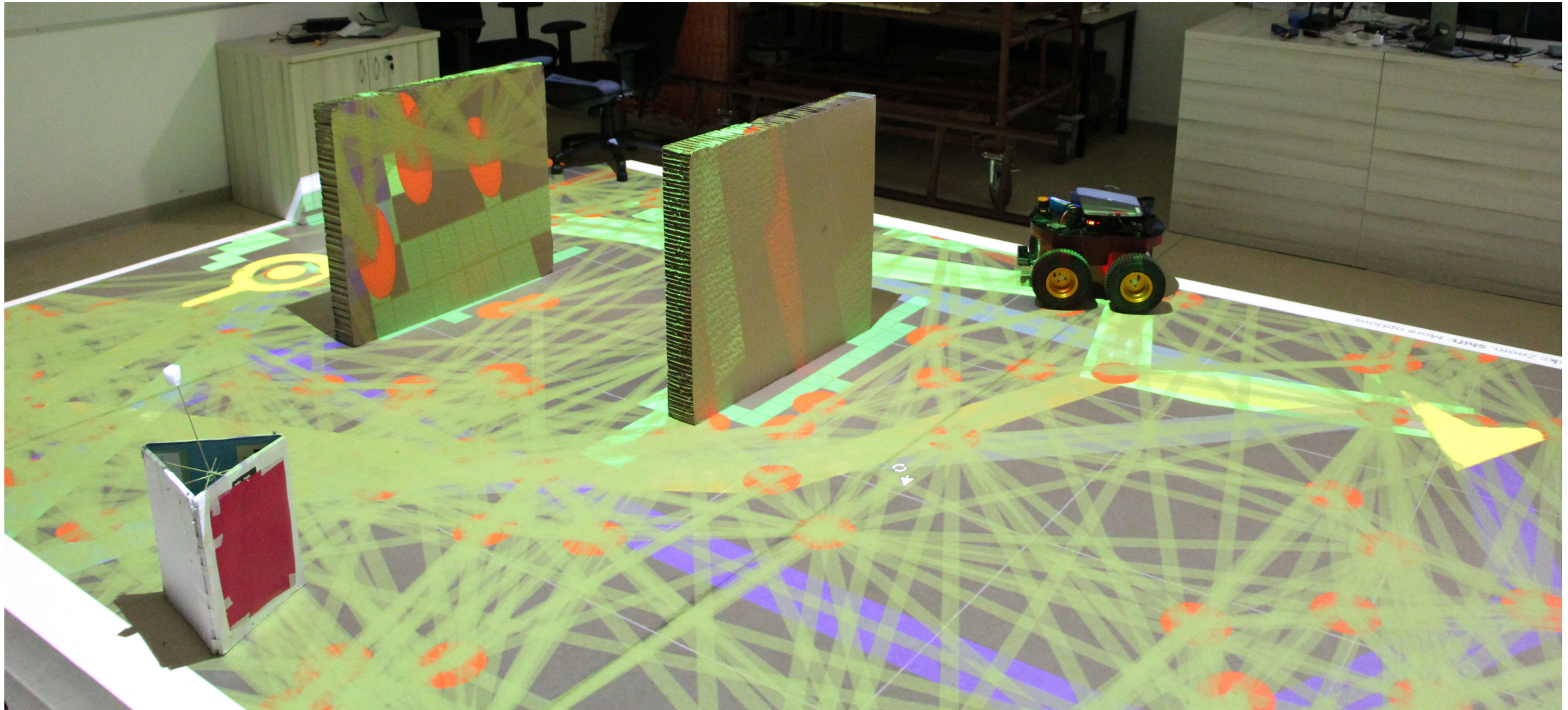


DA-BSP - Summary

[Pathak, Thomas, Indelman, IJRR'18]

- **Data association aware belief space planning (DA-BSP)**
 - Considers **data association** within BSP
 - Relaxes typical assumption in BSP that DA is **given** and **correct**
 - Approach in particular suitable to handle scenarios with **perceptual aliasing** and **localization uncertainty**
 - Unified framework for **robust active** and **passive perception**

Experiments at ANPL – In Process



Summary

Belief space planning (BSP) in high-dimensional state spaces:

- rAMD: Computationally efficient BSP in high dim. state spaces
- Action consistency & bounded approximations
 - s-BSP: belief sparsification for BSP
 - t-BSP: topological BSP
- Active perception in ambiguous environments: Data association aware BSP

