

# Vision-based Dynamic Target Trajectory and Ego-motion Estimation Using Incremental Light Bundle Adjustment

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# Overview

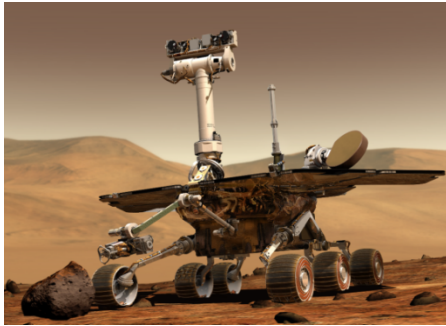
- Motivations
- Problem Formulation
- iLBA and Dynamic Target Tracking
- Optimization method
- Experiments Results
- Conclusions



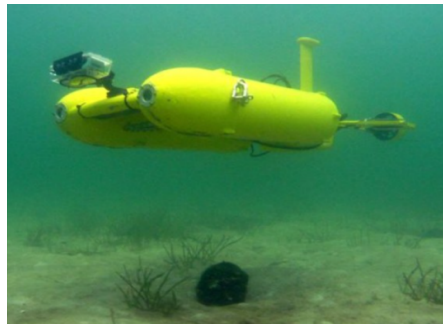
# → Why Motion Estimation ?

## - Autonomous Navigation

Space Exploration



Sub-marine exploration



Autonomous Driving



Indoor Operation



## - Others



Virtual/Augmented Reality

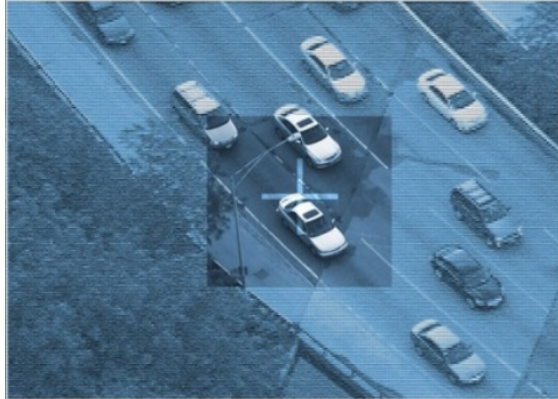


Pointing Devices



# → Why Target Tracking ?

Surveillance



Military



Robot – Human interaction



# Scenario



- Unknown environment
- No prior information about platform's trajectory
- No prior information about target's trajectory although motion model is assumed

- Interested in on-line operation
- No Global Positioning System

→ Use of onboard sensors :  
Monocular Camera

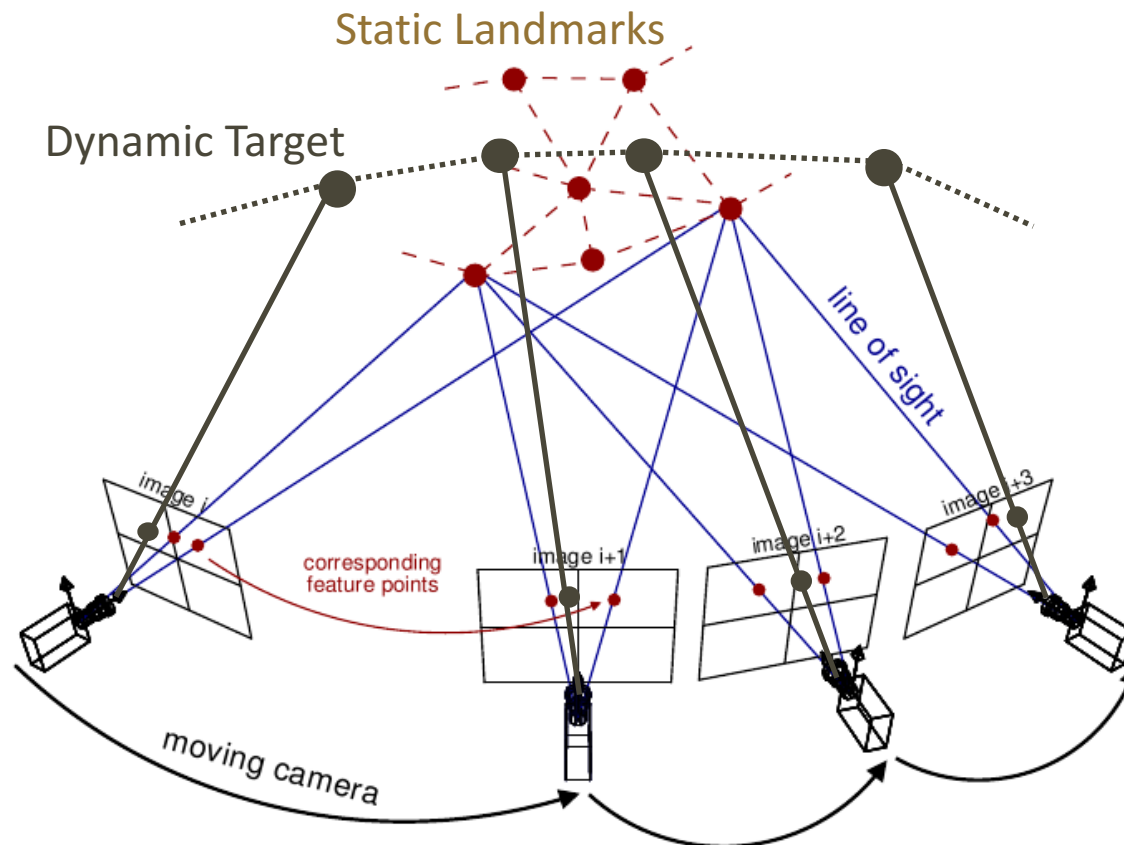


# Scenario



Efficiently and simultaneously estimate ego-motion and target trajectory

# BLAM + Detection from a (BAT) tracking of Moving Objects (DATMO) Mapping (SLAM)



## Related Work

- Target Tracking (or DATMO) : [Y. Bar-Shalom, 1988], [M. Breitenstein, 2009]
  - Assume **known/highly predictable sensor location**
  
- Combined SLAM and DATMO : [J. S. Ortega , 2007], [C. Wang, 2004], [T. D. Vu, 2009]
  - Different techniques : EKF, PF, ...
  - All involve optimization over the camera's state, the target's state **and the observed 3D structure !**
  
- "Structure-Less" BA : [Steffen et al., 2010], [Indelman, 2012]
  - All perform **batch optimization**



[www.cs.cmu.edu]

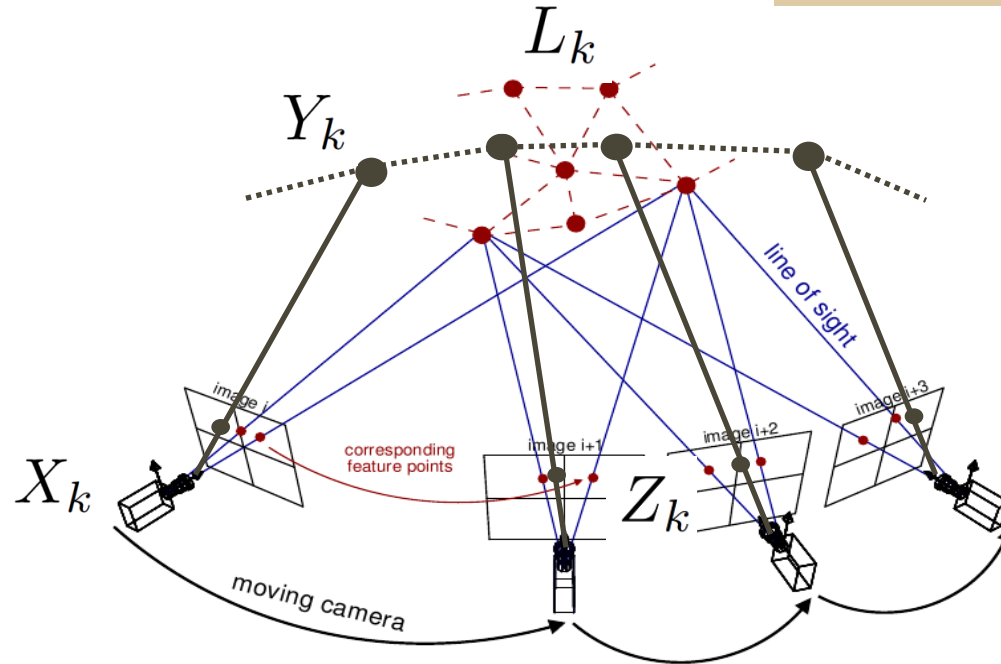


## Contributions

- Present Ego-motion estimation and Target tracking as a **combined optimization problem**
- **Integrate target tracking into efficient “structure-less” BA framework :**  
Use Incremental Light Bundle Adjustment (iLBA [Indelman et al., 2015]) to :
  - Improve computational efficiency compared to BA
  - Incremental optimization : Re-use calculations from previous steps
- Show results from **simulations** and **real-imagery experiments** performed at ANPL



## Notations



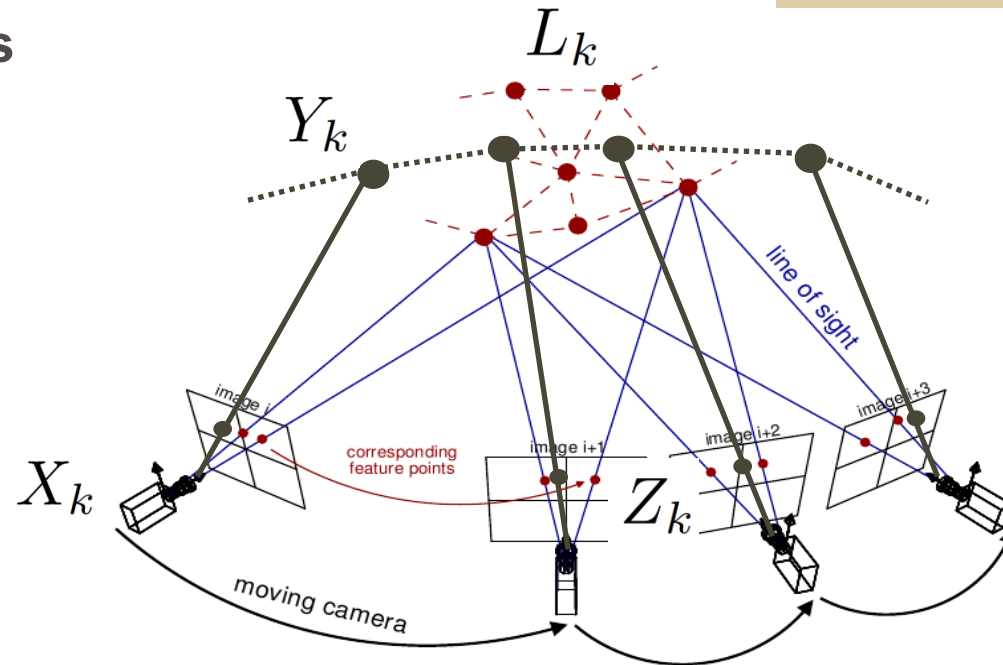
$X_k \doteq \{ x_1 \quad \dots \quad x_k \}$  where  $x_k$  is the 6DOF camera pose at time  $k$

$Y_k \doteq \{ y_1 \quad \dots \quad y_k \}$  where  $y_k = \begin{bmatrix} y_k^T \\ \dot{y}_k^T \end{bmatrix}$

$L_k \doteq \{ l_1 \quad \dots \quad l_n \}$

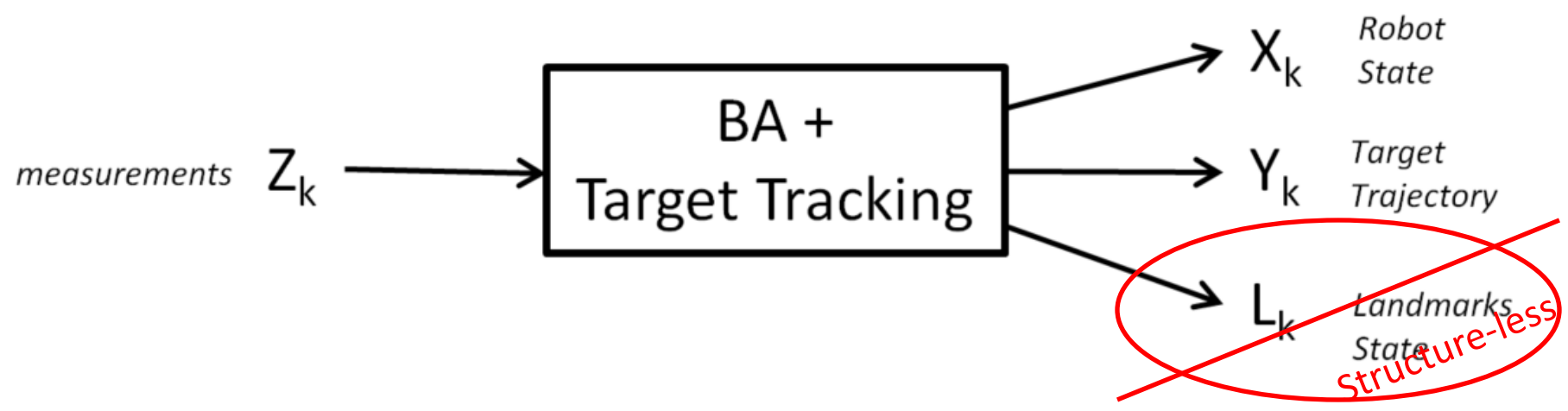
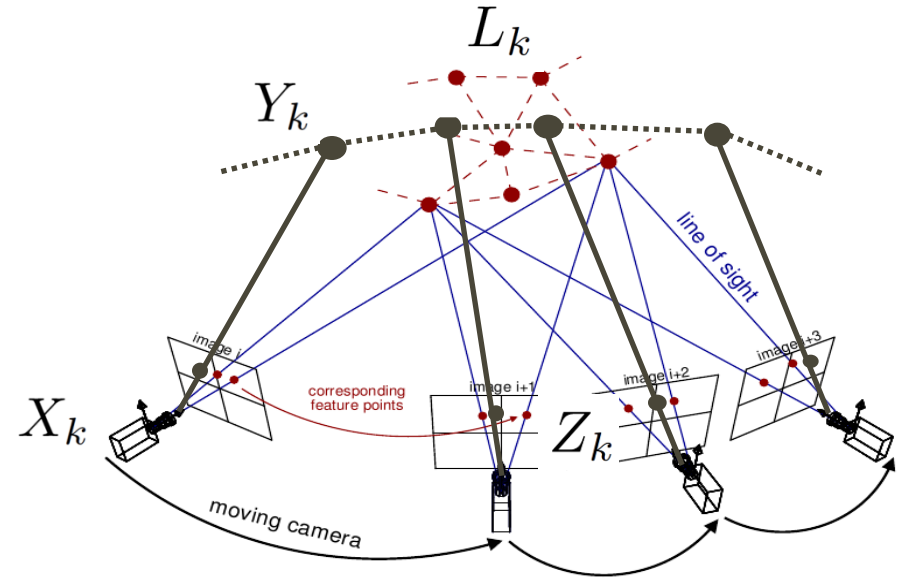
$Z_k \doteq \{ z_1 \quad \dots \quad z_k \}$  where  $z_k = [z_k^1, \dots, z_k^i]$

## Assumptions

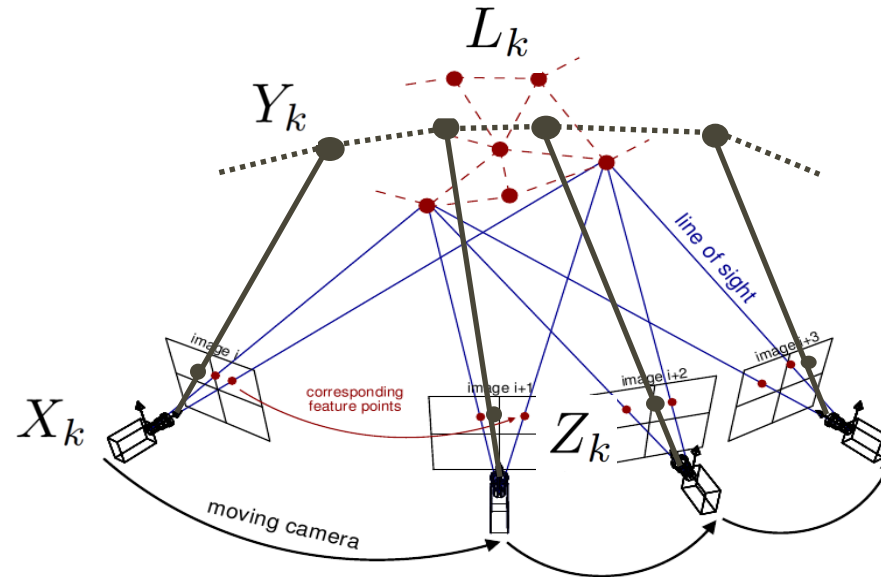


- Known image correspondences for landmarks & target
- White-Gaussian Noises
- Markov process : Models depend only on the current state and previous state
- Target is represented by a single landmark
- Prior information on first camera pose and target state

# Problem Formulation : BA and Target Tracking



## Problem Formulation : BA and Target Tracking



- Joint probability distribution function (pdf)

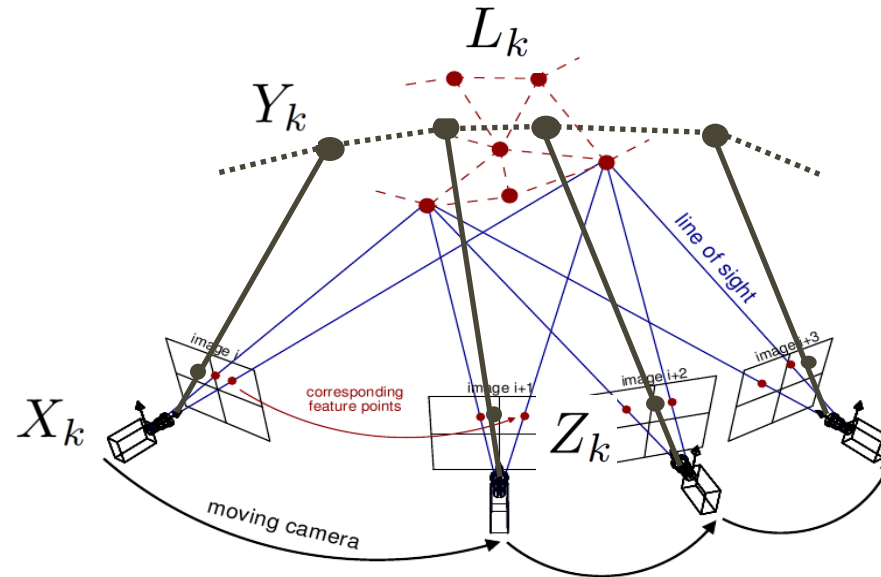
$$P(X_k, Y_k, L_k | Z_k) \propto \underbrace{\text{priors}}_{\text{Prior Information}} \prod_{i=1}^k \left( \underbrace{p(y_i | y_{i-1})}_{\text{Motion Model}} \underbrace{p(z_i^T | x_i, y_i)}_{\text{Measurement Model}} \prod_{j \in \mathcal{M}_i} \underbrace{p(z_i^j | x_i, l_j)}_{\text{Measurement Model}} \right)$$

- Maximum a posteriori (MAP) estimate :

$$X_k^*, Y_k^*, L_k^* = \arg \max_{X_k, Y_k, L_k} P(X_k, Y_k, L_k | Z_k)$$



## Problem Formulation : BA and Target Tracking

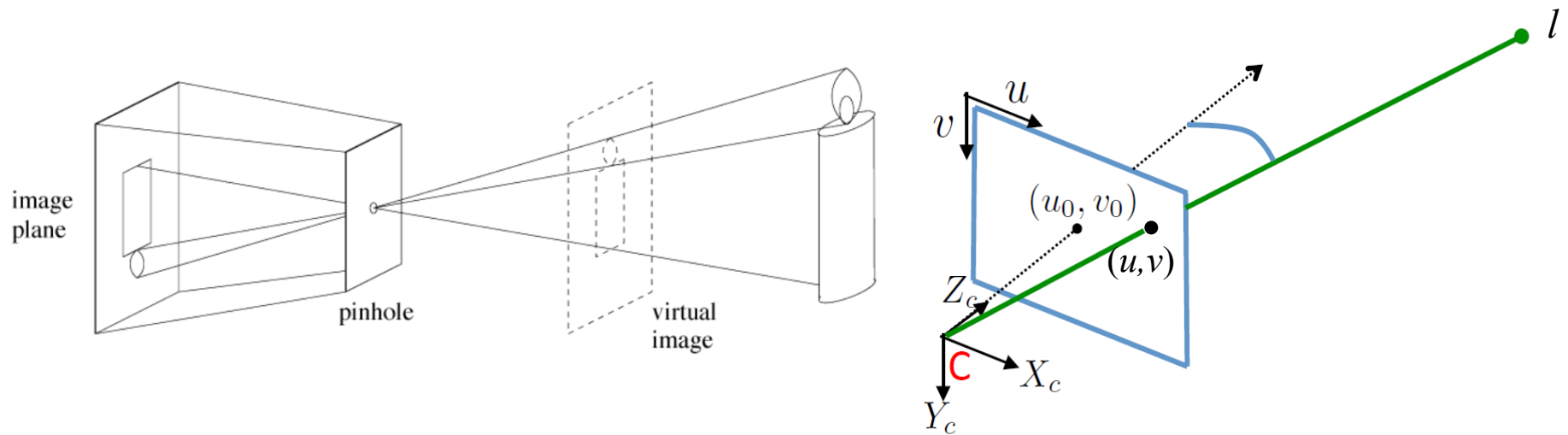


- Joint probability distribution function (pdf)

$$P(X_k, Y_k, L_k | Z_k) \propto \text{priors} \cdot \prod_{i=1}^k \left( p(y_i | y_{i-1}) p(z_i^T | x_i, y_i) \prod_{j \in \mathcal{M}_i} p(z_i^j | x_i, l_j) \right)$$

↑ Prior Information     
 ↑ Motion Model     
 ← Measurement Model     
 →

## Measurement Model : Pinhole Camera

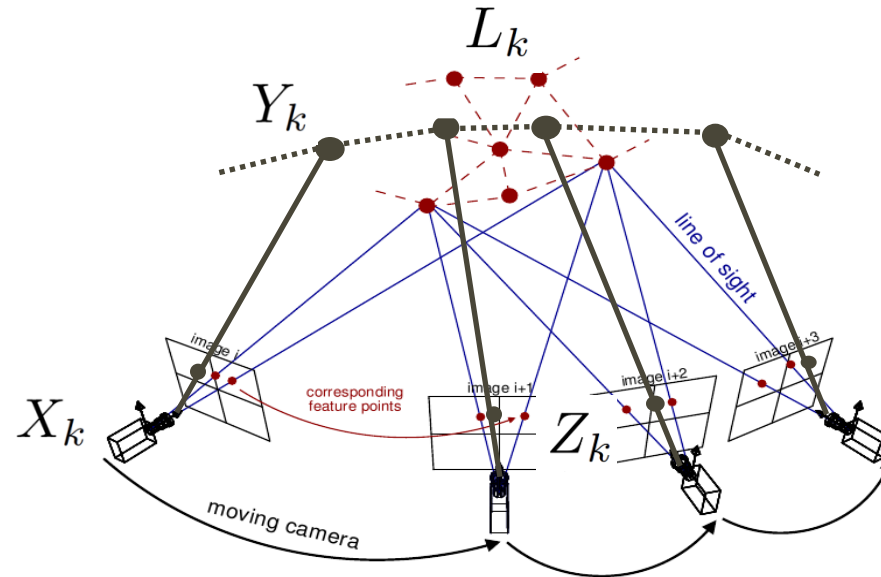


- Defining the Projection Operator :  $proj(x, l) \doteq K[R \quad t]l$  [R.I. Hartley, 2004]

- Observation Model :  $z = proj(x, l) + v$  where  $v \sim \mathcal{N}(0, \Sigma_v)$

- Measurement Likelihood :  $p(z|x, l) = \frac{1}{\sqrt{|2\pi\Sigma_v|}} \exp\left(-\frac{1}{2} \underbrace{\|z - proj(x, l)\|_{\Sigma_v}^2}_{\text{Re-projection error}}\right)$

## Problem Formulation : BA and Target Tracking



- Joint probability distribution function (pdf)

$$P(X_k, Y_k, L_k | Z_k) \propto \text{priors} \cdot \prod_{i=1}^k \left( p(y_i | y_{i-1}) p(z_i^T | x_i, y_i) \prod_{j \in \mathcal{M}_i} p(z_i^j | x_i, l_j) \right)$$

↑
↑
↑

Prior Information                      Motion Model                      Measurement Model

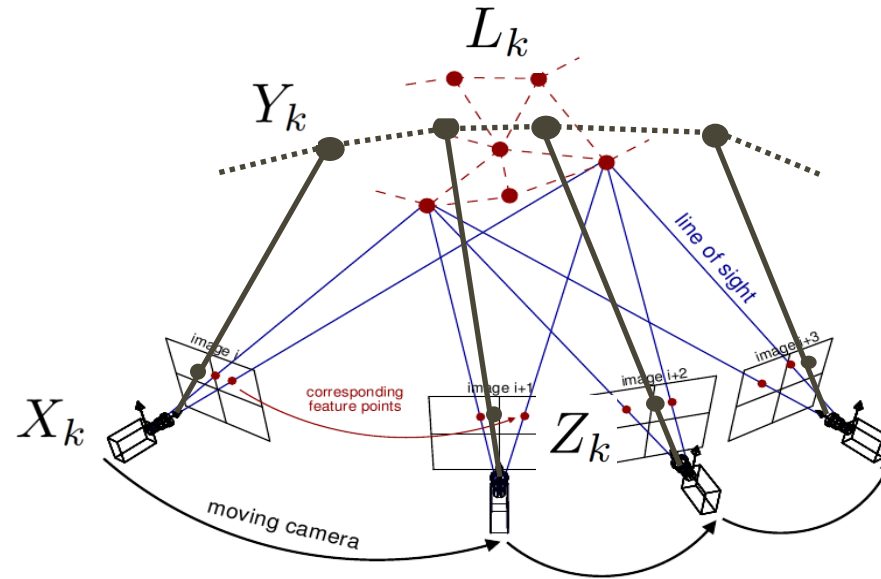
## Motion Model : Constant Velocity

- Target state : 
$$y_k = \begin{bmatrix} y_k^T \\ \dot{y}_k^T \end{bmatrix}$$
- State Propagation : 
$$y_{k+1} = \underbrace{\Phi_k}_{\text{Transition Matrix}} y_k + \underbrace{G_k}_{\text{Process noise Jacobian}} w_k \quad \text{where} \quad w_k \sim \mathcal{N}(0, \Sigma_w)$$
- Constant Velocity : 
$$\Phi_k = \begin{bmatrix} I_{3 \times 3} & \Delta t \\ 0 & I_{3 \times 3} \end{bmatrix} \in \mathbb{R}^{6 \times 6} \quad \text{and} \quad G_k = \begin{bmatrix} 0 \\ I_{3 \times 3} \end{bmatrix} \in \mathbb{R}^{6 \times 3}$$
- Probabilistic Representation : 
$$p(y_{k+1} | y_k) \doteq \frac{1}{\sqrt{|2\pi \Sigma_{mm}|}} \exp \left( -\frac{1}{2} \|y_{k+1} - \Phi_k y_k\|_{\Sigma_{mm}}^2 \right)$$
  

$$\Sigma_{mm} \doteq G^T \Sigma_w G$$



## Problem Formulation : BA and Target Tracking



- Joint probability distribution function (pdf)

$$P(X_k, Y_k, L_k | Z_k) \propto \text{priors} \cdot \prod_{i=1}^k \left( \underbrace{p(y_i | y_{i-1})}_{\text{camera motion}} \underbrace{p(z_i^T | x_i, y_i)}_{\text{feature observation}} \prod_{j \in \mathcal{M}_i} \underbrace{p(z_i^j | x_i, l_j)}_{\text{target observation}} \right)$$

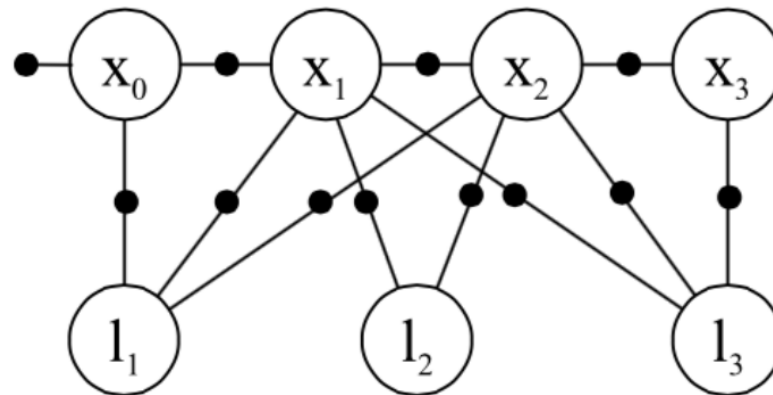
$$\propto \exp\left(-\frac{1}{2} \|y_i - \Phi_i y_{i-1}\|_{\Sigma_{mm}}^2\right) \propto \exp\left(-\frac{1}{2} \|z_i^T - \text{proj}(x_i, y_i)\|_{\Sigma_v}^2\right) \propto \exp\left(-\frac{1}{2} \|z_i^j - \text{proj}(x_i, l_j)\|_{\Sigma_v}^2\right)$$

- Pdf can also be represented by graphical models : **Factor Graph**



## Factor Graph

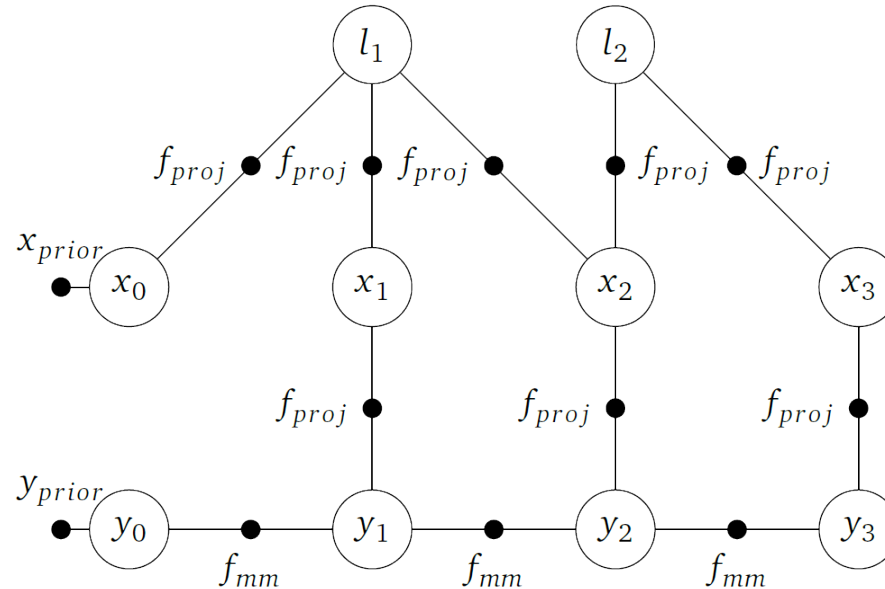
Describes a factorization of a joint pdf in terms of process and measurement models



- Vertices represent the variables
- Nodes represent constrains between variables, also known as **factors**

→ Allows for computationally efficient probabilistic inference

# Factor Graph : BA and Target Tracking



- Joint pdf :

$$P(X_k, Y_k, L_k | Z_k) \propto p(x_0) p(y_0) \prod_{i=1}^k \left( \underbrace{f_{mm}(y_{i+1}, y_i)}_{\text{Motion Model}} \underbrace{f_{proj}(x_i, y_i)}_{\text{Projection}} \prod_{j \in \mathcal{M}_i} \underbrace{f_{proj}(x_i, l_j)}_{\text{Projection}} \right)$$

$\doteq \exp\left(-\frac{1}{2} \|y_i - \Phi_i y_{i-1}\|_{\Sigma_{mm}}^2\right)$

$\doteq \exp\left(-\frac{1}{2} \|z_i^T - proj(x_i, y_i)\|_{\Sigma_v}^2\right)$

$\doteq \exp\left(-\frac{1}{2} \|z_i^j - proj(x_i, l_j)\|_{\Sigma_v}^2\right)$

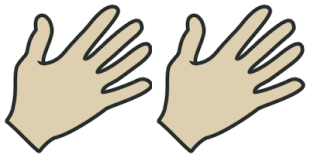
- Maximum a posteriori (MAP) estimate : Involves reconstruction of the 3D structure
- $$X_k, Y_k, L_k = \arg \max_{X_k, Y_k, L_k} P(X_k, Y_k, L_k | Z_k)$$



## Computational Efficiency



- For BA and target tracking and with N frames observing M landmarks :
  - **12N + 3M** elements to optimize (**6N – Camera, 6N – Target, 3M - Landmarks**)
- Performed Incrementally as new surrounding features are observed
  - Increases the computational complexity of the problem



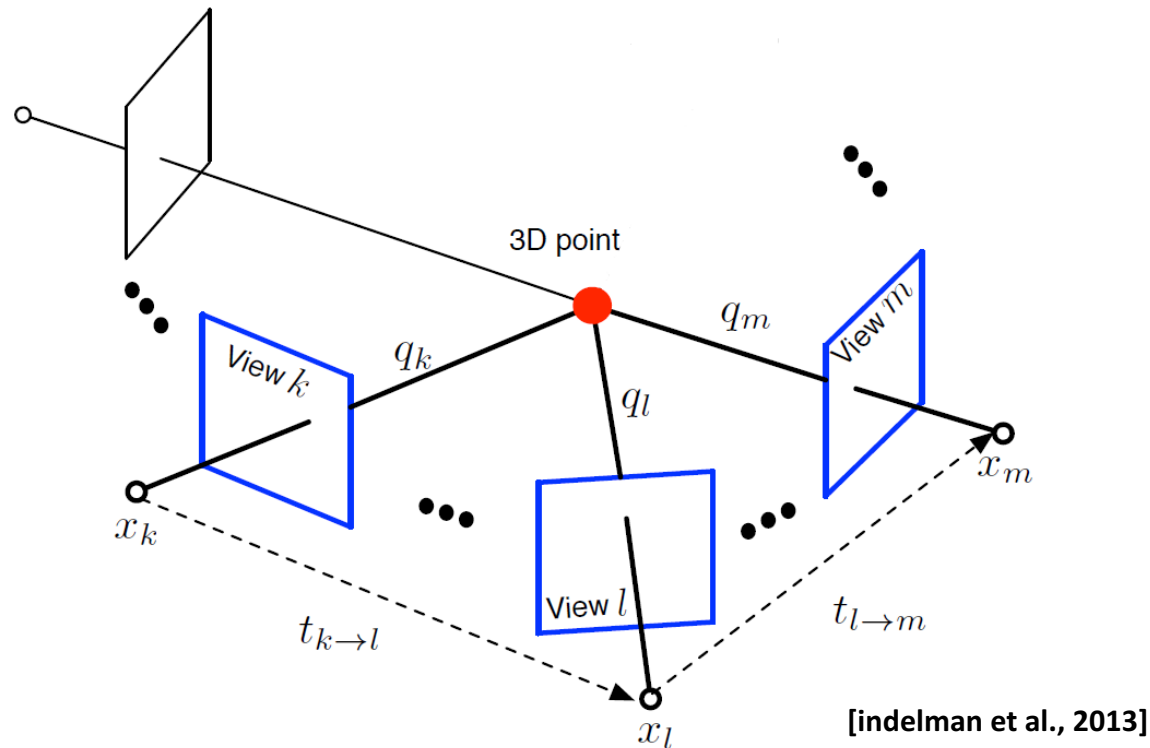
- On-line 3D structure reconstruction is of no interest :  $P(X_k, Y_k | Z_k)$  : **12N** elements

$$P(X_k, Y_k, L_k | Z_k) \xrightarrow{\text{Marginalization}} P(X_k, Y_k | Z_k) = \int P(X_k, Y_k, L_k | Z_k) dL_k$$

Computationally Expensive Process !

→ **Light Bundle Adjustment (LBA)**

## Incremental Light Bundle Adjustment (iLBA) – [Indelman et al., 2015]



- Intuition : 3 frames from which the same landmark is observed are related by geometrical constraints : **Multiview constraints**
- Allows to algebraically eliminate the landmarks from the optimization

→ Less Variables involved! (No need to calculate full BA first)



## Incremental Light Bundle Adjustment (iLBA) – [Indelman et al., 2015]

$$g_{2v}(x_k, x_l, z_k, z_l) = q_k \cdot (t_{k \rightarrow l} \times q_l)$$

$$g_{2v}(x_l, x_m, z_l, z_m) = q_l \cdot (t_{l \rightarrow m} \times q_m)$$

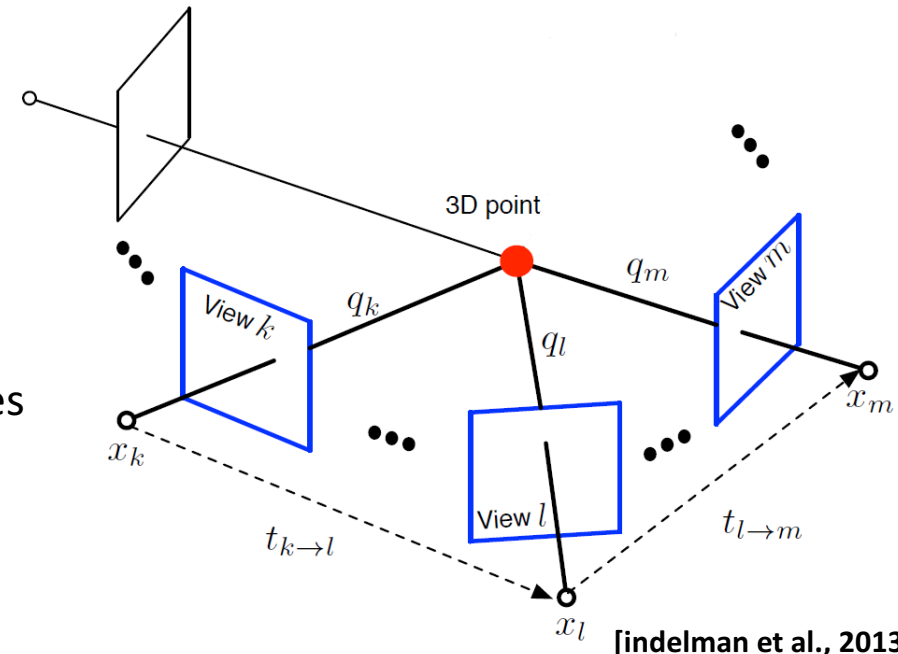
$$g_{3v}(x_k, x_l, x_m, z_k, z_l, z_m) =$$

$$(q_l \times q_k) \cdot (q_m \times t_{l \rightarrow m}) - (q_k \times t_{k \rightarrow l}) \cdot (q_m \times q_l)$$

2 view constraints

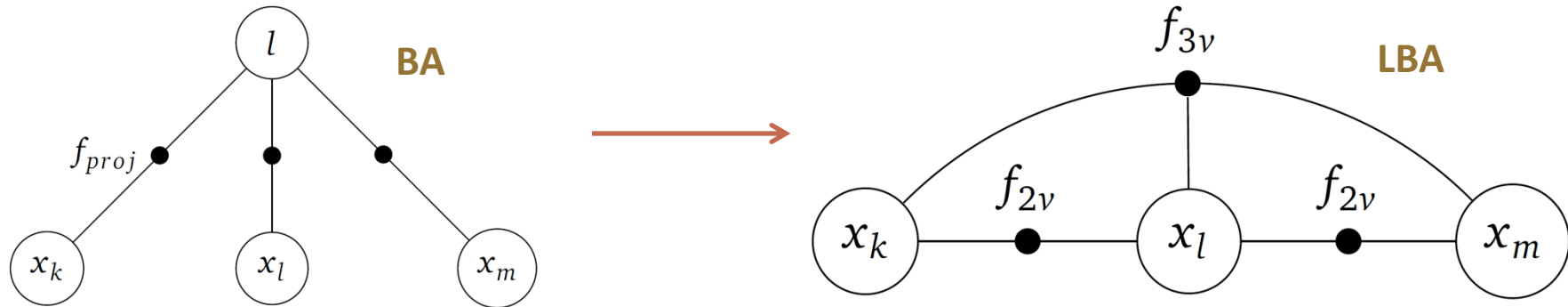
3 view constraint

- **2 view constraints** : epipolar geometry
- **3 view constraints** : relates between the scales of  $t_{l \rightarrow m}$  and  $t_{k \rightarrow l}$





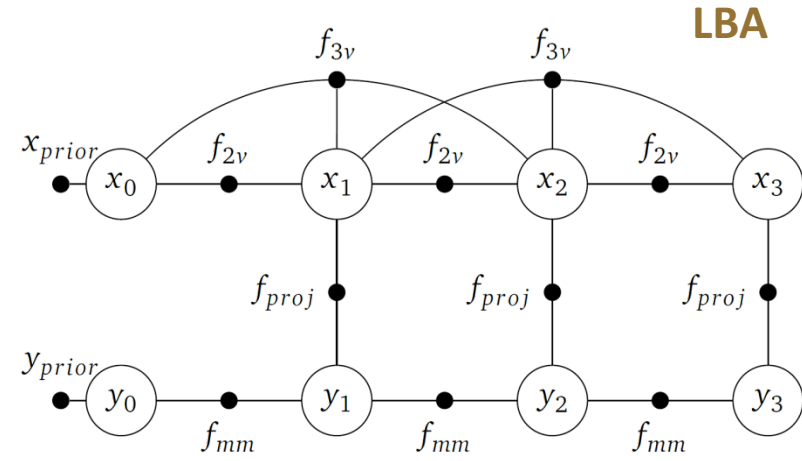
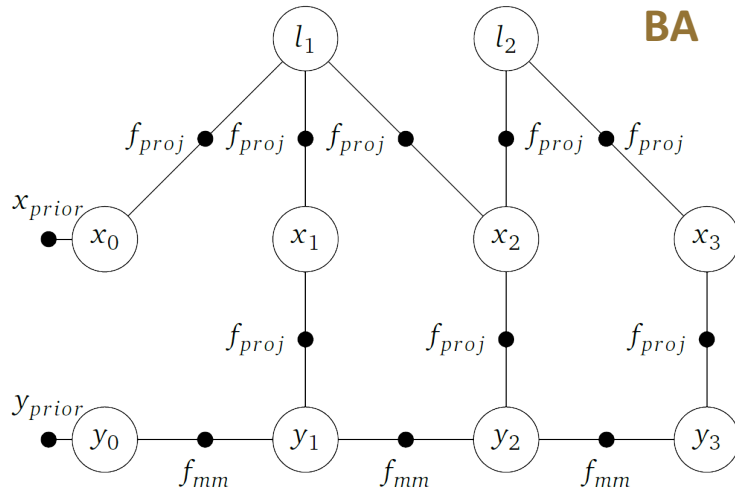
# Incremental Light Bundle Adjustment (iLBA) – [Indelman et al., 2015]



$$f_{2v}(x_k, x_l) \doteq \exp\left(-\frac{1}{2} \|g_{2v}(x_k, x_l, z_k, z_l)\|_{\Sigma_{2v}}^2\right) \longrightarrow \text{2 view factor}$$

$$f_{3v}(x_k, x_l, x_m) \doteq \exp\left(-\frac{1}{2} \|g_{3v}(x_k, x_l, x_m, z_k, z_l, z_m)\|_{\Sigma_{3v}}^2\right) \longrightarrow \text{3 view factor}$$

## ... With target tracking



$$P(X_k, Y_k, L_k | Z_k) \propto \text{priors}$$

$$\prod_{i=1}^k \left( f_{mm}(y_i, y_{i-1}) f_{proj}(x_i, y_i) \prod_{j \in \mathcal{M}_i} f_{proj}(x_i, l_j) \right)$$

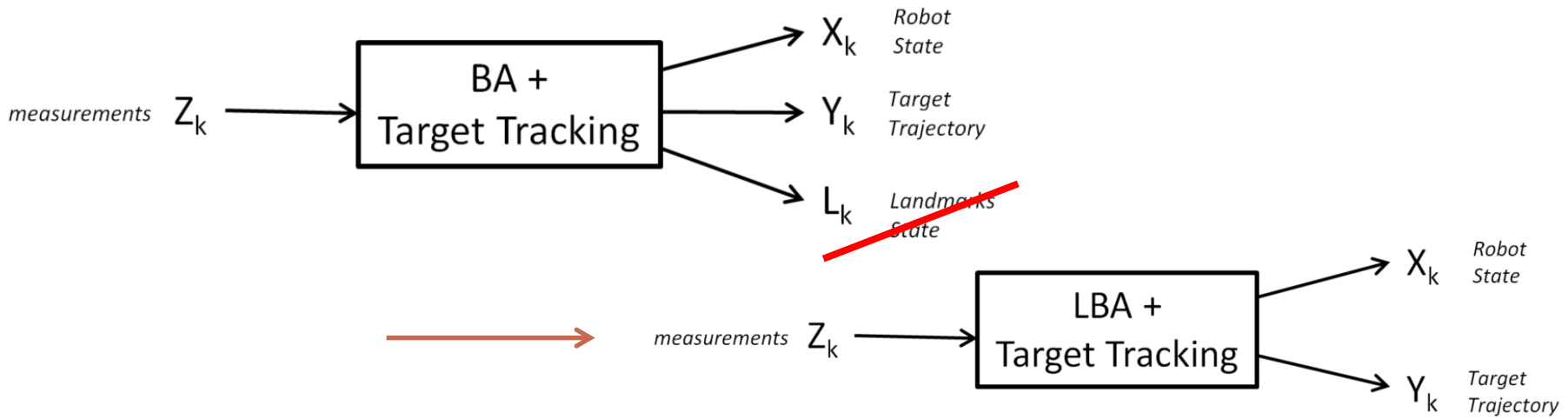


$$P(X_k, Y_k | Z_k) \propto p(x_0) p(y_0) \prod_{i=1}^k \left( f_{mm}(y_i, y_{i-1}) f_{proj}(x_i, y_i) \prod_{j=1}^N f_{2v/3v}(X_j) \right)$$



**The target is the only re-constructed 3D point !**

## Up till now



- Less variables to optimize :  $N$  frames  $M$  landmarks  $12N + 3M \longrightarrow 12N$
- We solve :  $X_k^*, Y_k^*, L_k^* = \arg \max_{X_k, Y_k, L_k} p(X_k, Y_k, L_k | Z_k) \longrightarrow X_k^*, Y_k^* = \arg \max_{X_k, Y_k} p(X_k, Y_k | Z_k)$
- How ?

## LBA and Target Tracking

- Recall :  $P(X_k, Y_k | Z_k) \propto p(x_0) p(y_0) \prod_{i=1}^k \left( \underbrace{f_{mm}(y_i, y_{i-1})}_{\text{Motion model}} \underbrace{f_{proj}(x_i, y_i)}_{\text{Observation model}} \prod_{j=1}^N \underbrace{f_{2v/3v}(X_j)}_{\text{2v/3v constraints (LBA)}} \right)$

$$\doteq \exp\left(-\frac{1}{2} \|y_i - \Phi_i y_{i-1}\|_{\Sigma_{mm}}^2\right) \quad \doteq \exp\left(-\frac{1}{2} \|z_i^T - proj(x_i, y_i)\|_{\Sigma_v}^2\right) \quad \doteq \exp\left(-\frac{1}{2} \|h_j(X_j, Z_j)\|_{\Sigma_j}^2\right)$$

**Motion model**
**Observation model**
**2v/3v constraints (LBA)**

- Find the MAP :  $X_k^*, Y_k^* = \arg \max_{X_k, Y_k} p(X_k, Y_k | Z_k)$



**Log is monotonic (same max/min)**

- Equivalent to minimizing :

$$J(X_k, Y_k) = \|x_0 - \hat{x}_0\|_{\Sigma_x}^2 + \|y_0 - \hat{y}_0\|_{\Sigma_y}^2 + \sum_{i=1}^k \left( \|y_{i+1} - \Phi_i y_i\|_{\Sigma_{mm}}^2 + \|z_i^T - proj(x_i, y_i)\|_{\Sigma_v}^2 + \sum_j^{N_h} \|h_j(X_j, Z_j)\|_{\Sigma_j}^2 \right)$$

- Approaches : Gauss-Newton, Levenberg-Marquardt, ...



# Optimization

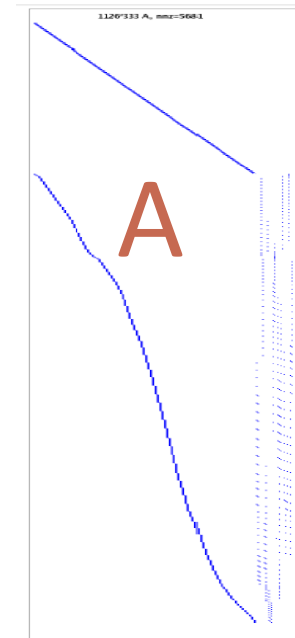
Pictures from [Dellaert et al., 2006]

## Recipe for Gauss-Newton :

- Linearize cost function
- Re-arrange RHS such that  $J(\bar{\Theta} + \Delta\Theta) \approx \|A\Delta\Theta - b\|^2$
- Solve for  $\Delta\Theta$
- Update linearization point  $\bar{\Theta} + \Delta\Theta \rightarrow \bar{\Theta}$
- Repeat until convergence

Note :

- $A$  contains the Jacobians of all the measurements with respect to the variables -  $A$  is large !
- $A$  is sparse !



Jacobian matrix



# Optimization

Pictures from [Dellaert et al., 2006]

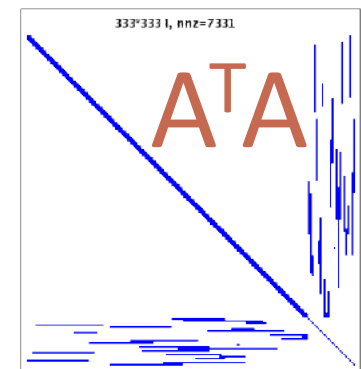
## Recipe for Gauss-Newton :

- Linearize cost function
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- Update linearization point
- Repeat until convergence

Need to solve  $A\Delta\Theta = b$   $\longrightarrow$   $A^T A\Delta\Theta = A^T b$

$$\Delta\Theta = (A^T A)^{-1} A^T b$$

Expensive process !



Information matrix

## Optimization

- Two issues :
  1. Naïve approach is **expensive**
  2. The **Entire process needs to be performed from scratch** each time a new variable/measurement is added to the problem
- Recently Developed Techniques :
  - Square Root SAM (Dellaert et al., 2006)
  - Incremental SAM - iSAM (Kaess et al., 2008)
  - iSAM2 (Kaess et al., 2012)
  1. Exploits **sparsity** of the involved matrices to **simplify  $\Delta\Theta$  recovery**
  2. Uses **graphical models** to perform **Incremental optimization** : Calculations from previous steps can be reused

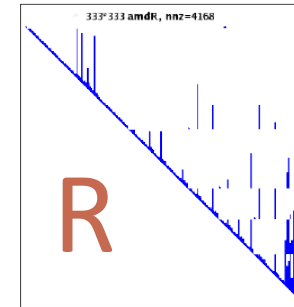


## Incremental Smoothing and Mapping (iSAM)

### 1. Exploiting matrix sparsity

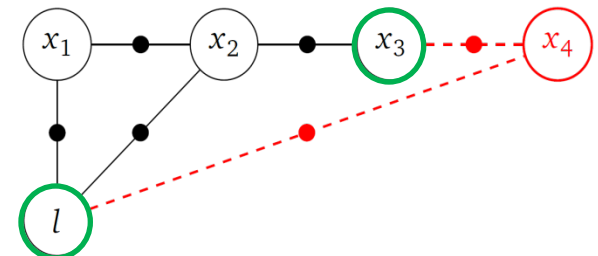
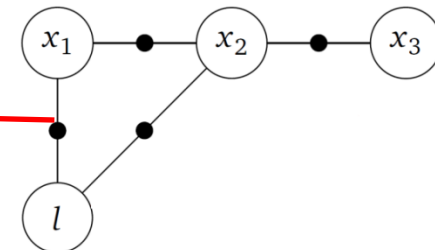
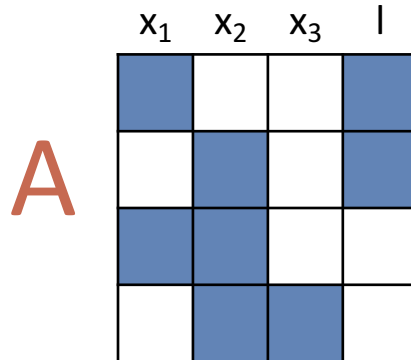
- Factorization : QR (A), Cholesky ( $A^T A$ )

$$\text{QR : } \longrightarrow R \Delta \Theta = d$$



Square-root matrix

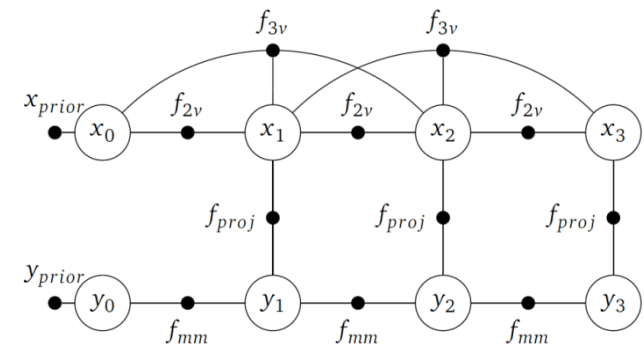
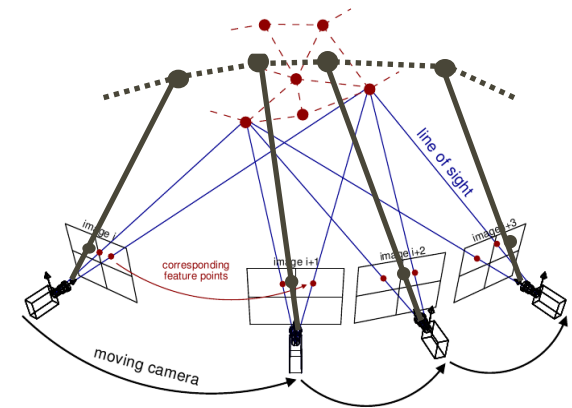
### 2. Using graphical models to allow for incremental optimization



- Identify which variables are **involved in new factors**
- R matrix can be **updated**, not recalculated

## Up till now

- **One big optimization process** including camera and target states
- **Integrated target tracking into iLBA framework** :
  - Involves less variables (structure-less)
  - Performs incremental inference over graphical models



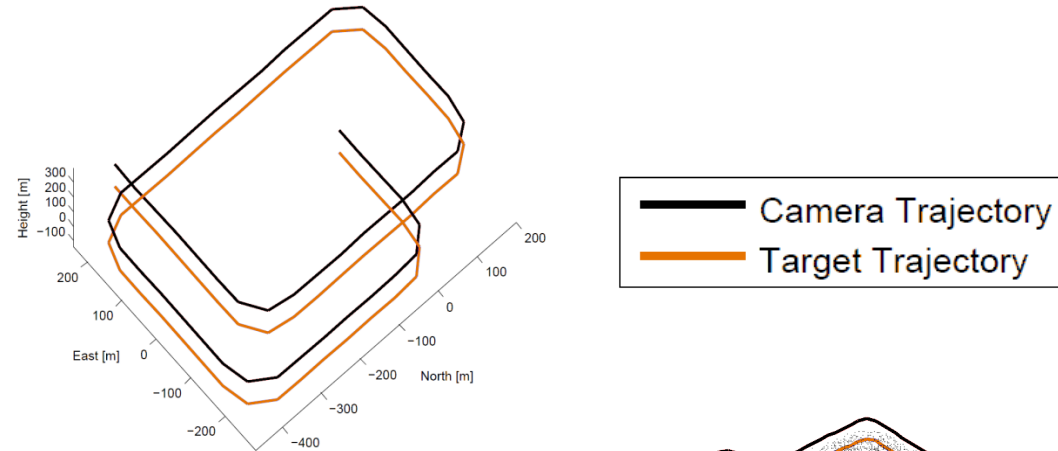
**Let's see some results !**

## Scenario

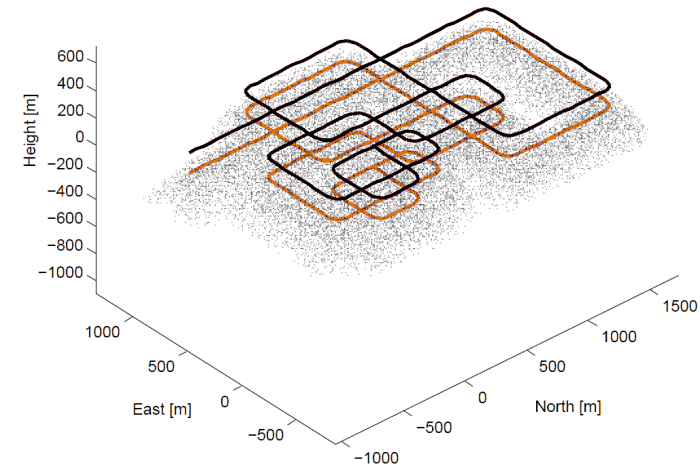


## Experiments and Simulations

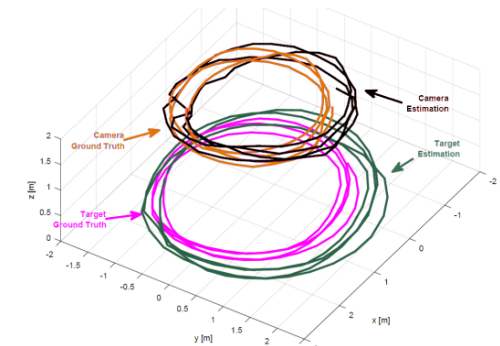
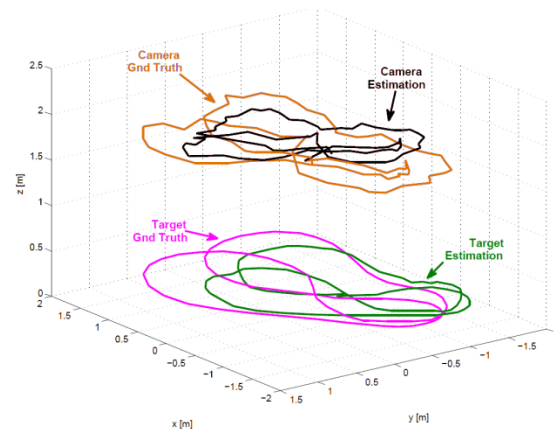
1. Statistical Simulation :  
- Short Scenario



2. Case Study :  
- Large Scale Scenario

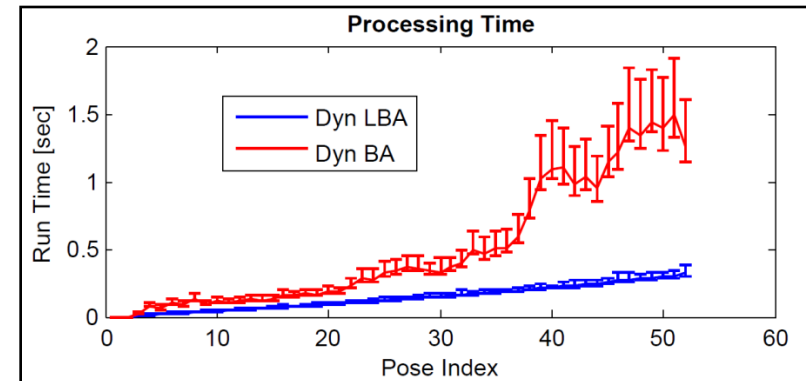
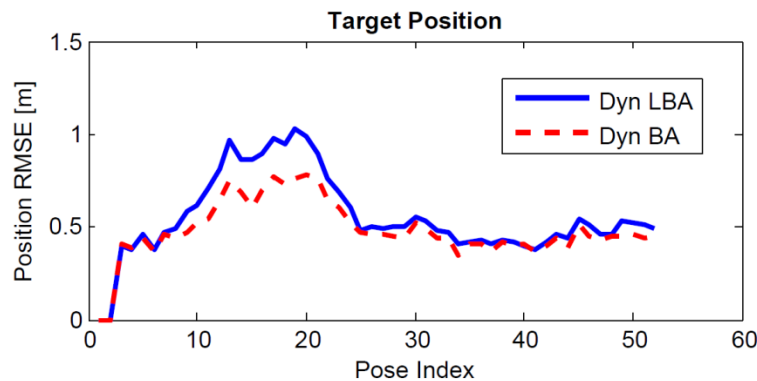
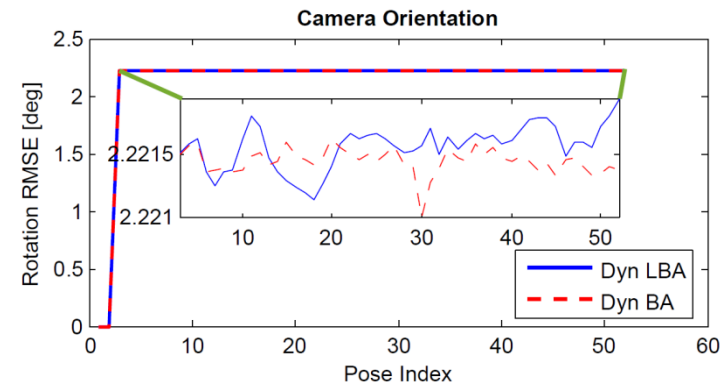
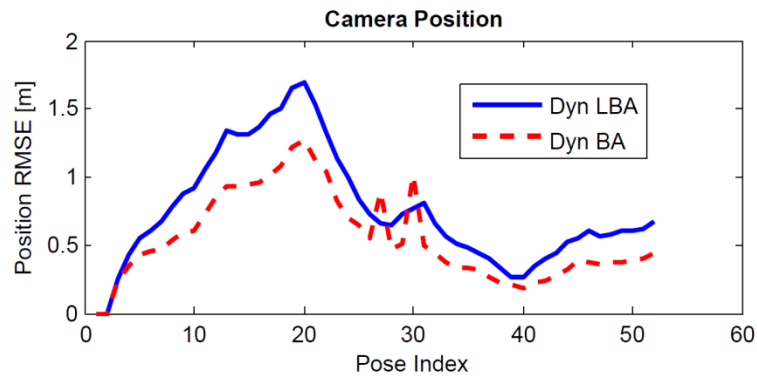
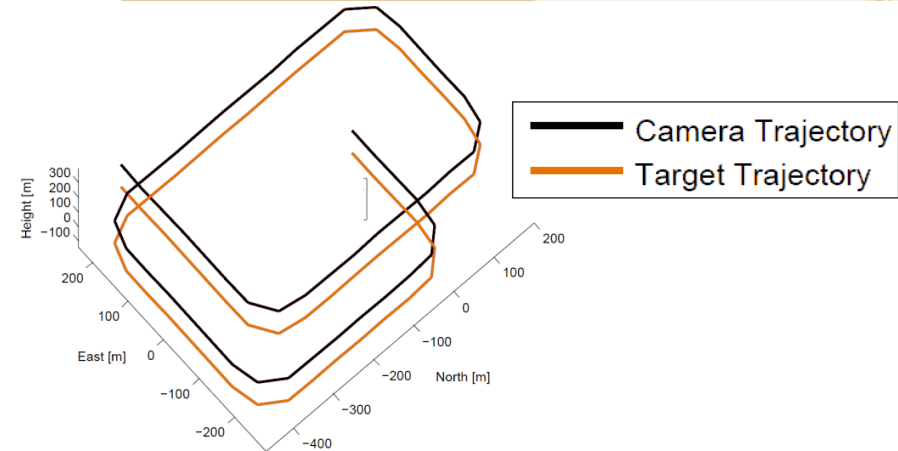


3. Real-Imagery Experiments



# 1. Statistical Simulation

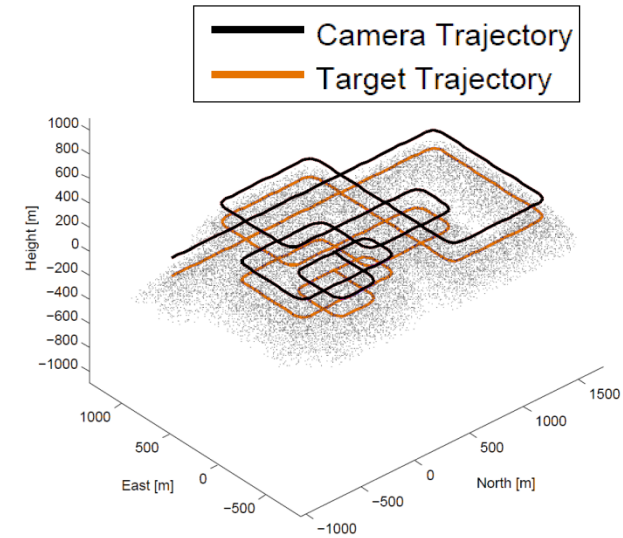
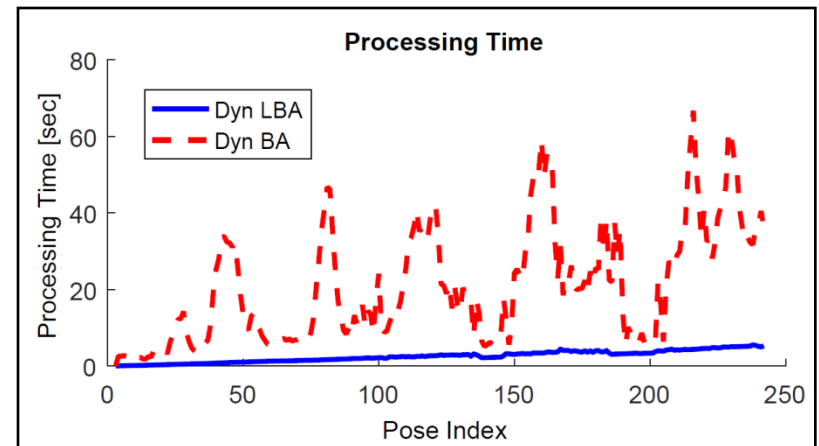
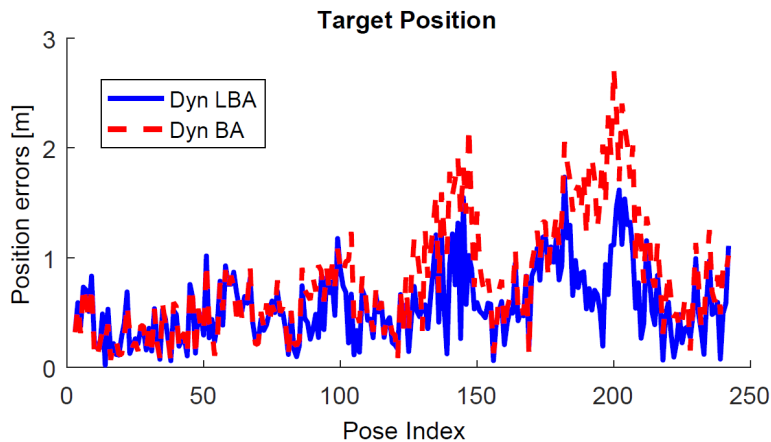
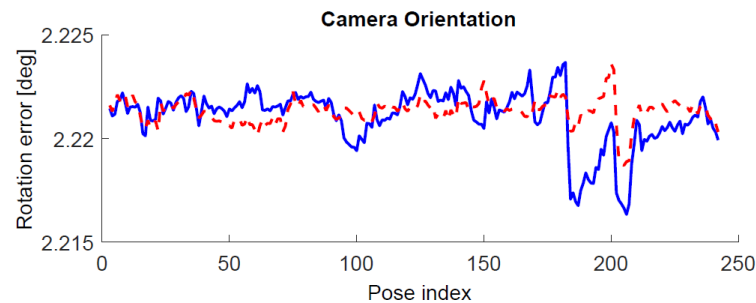
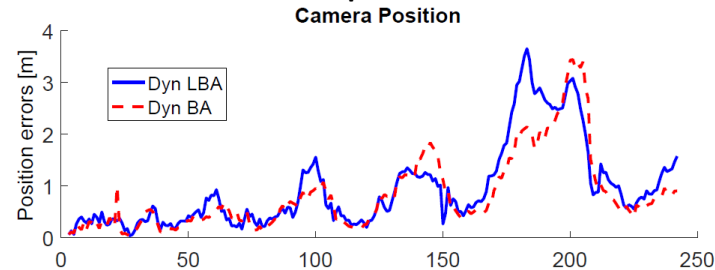
- 45 run Monte-Carlo study
- Short Scenario : 52 frames, 160 seconds
- 2 Loop Closures





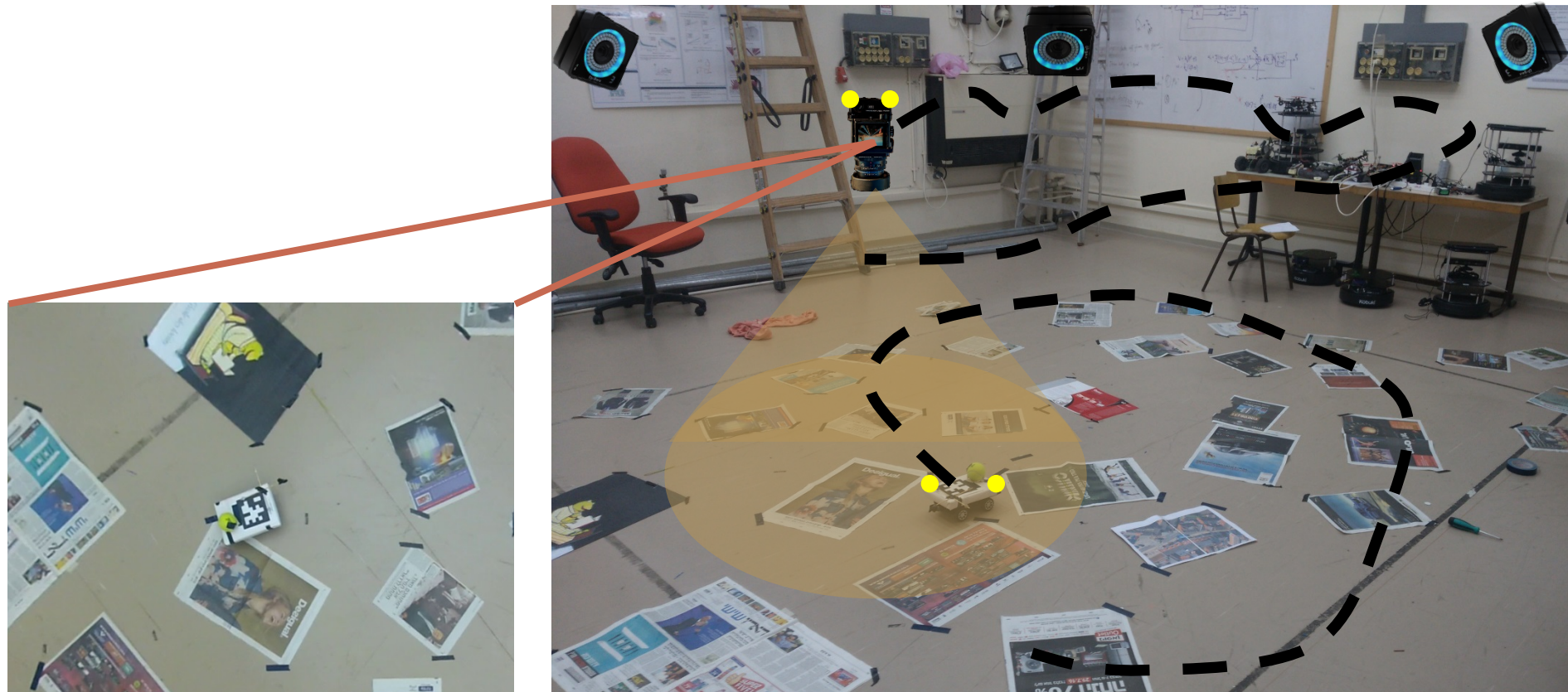
## 2. Large Scale Scenario Simulation

- Large scenario : 240 frames, 14.5 km
- ~25300 observed Landmarks, ~10 Loop Closures



### 3. Real-Imagery Experiments

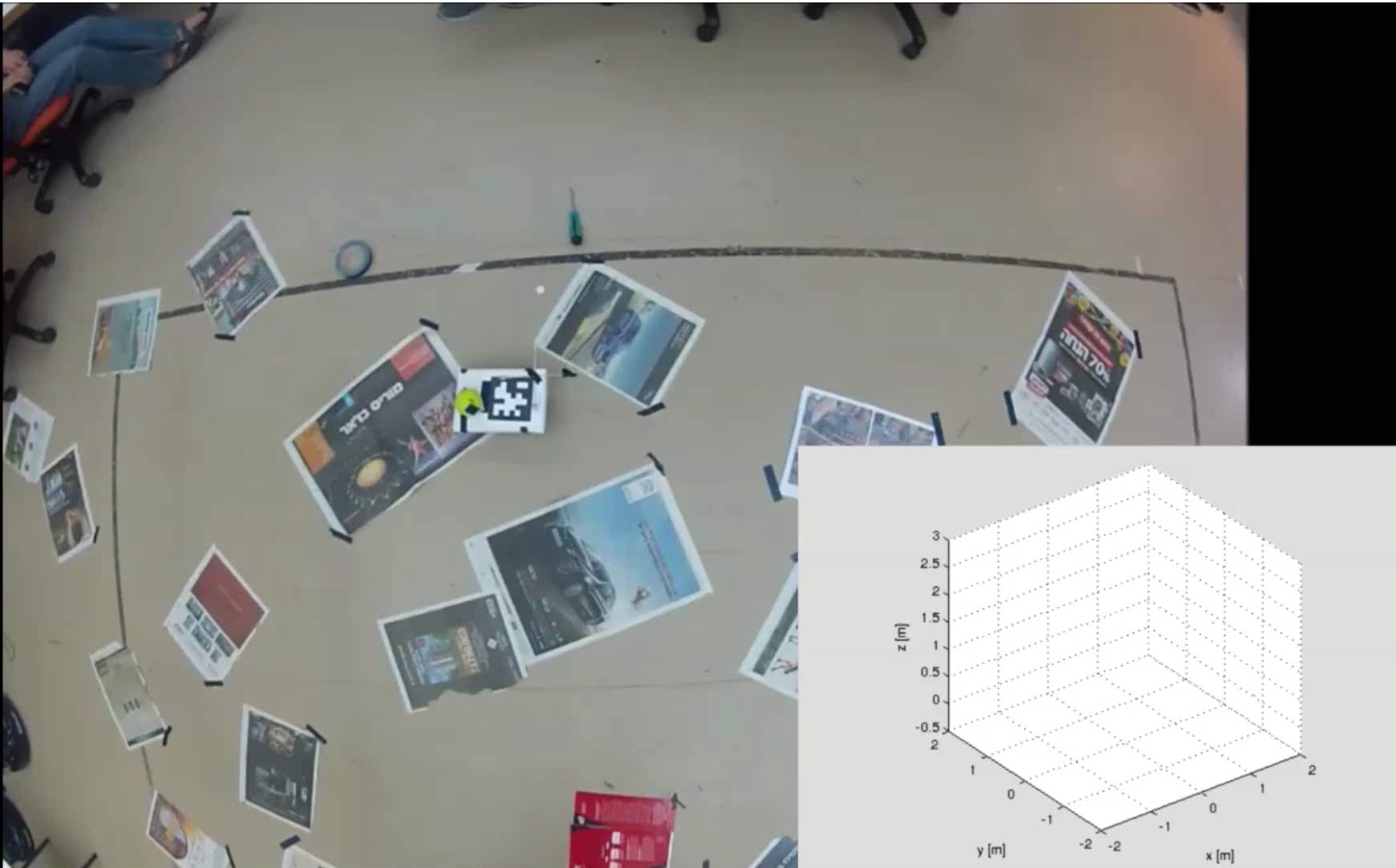
- Aerial Scenario : Downward facing camera observing a dynamic target on the ground
- Ground Truth from 6DoF optical tracking system



- Datasets publicly available at : <http://vindelman.net.technion.ac.il/software/>



### 3. Real-Imagery Experiments

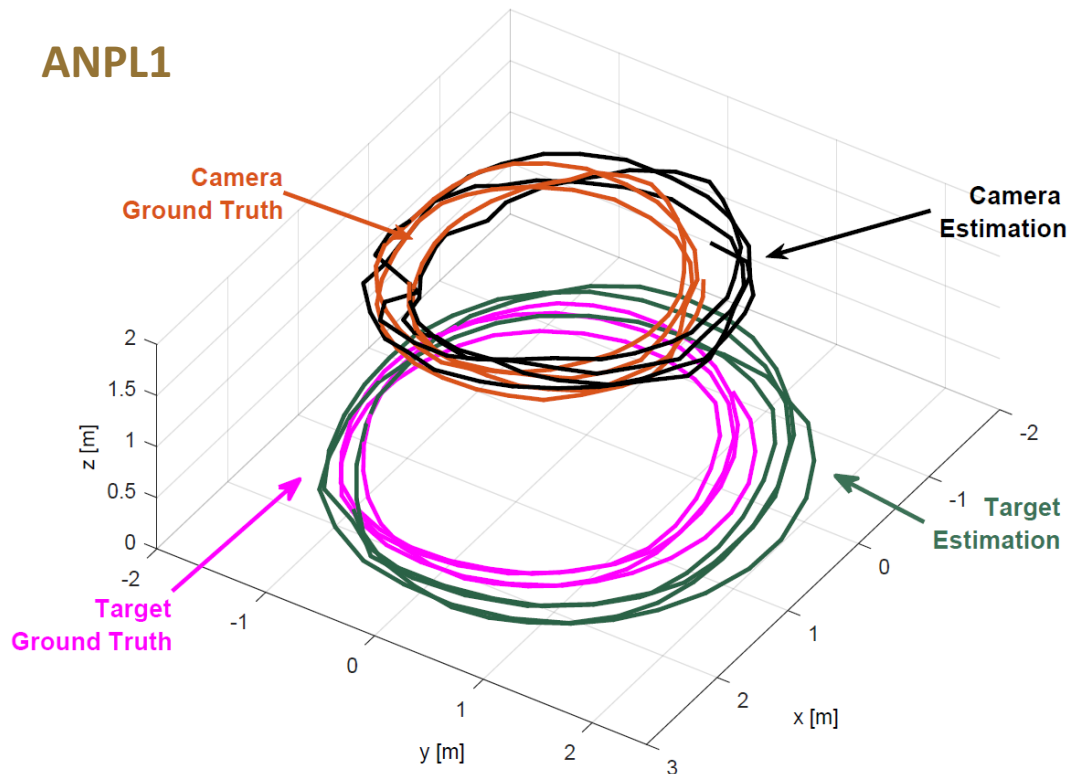


### 3. Real-Imagery Experiments

- 2 different datasets

	Camera Resolution [pix]	Frames	Duration [sec]	Landmarks	Observations
<i>ANPL1</i>	1280 × 960	80	40	2439	31333
<i>ANPL2</i>	1920 × 1080	73	117	3366	25631

#### ANPL1



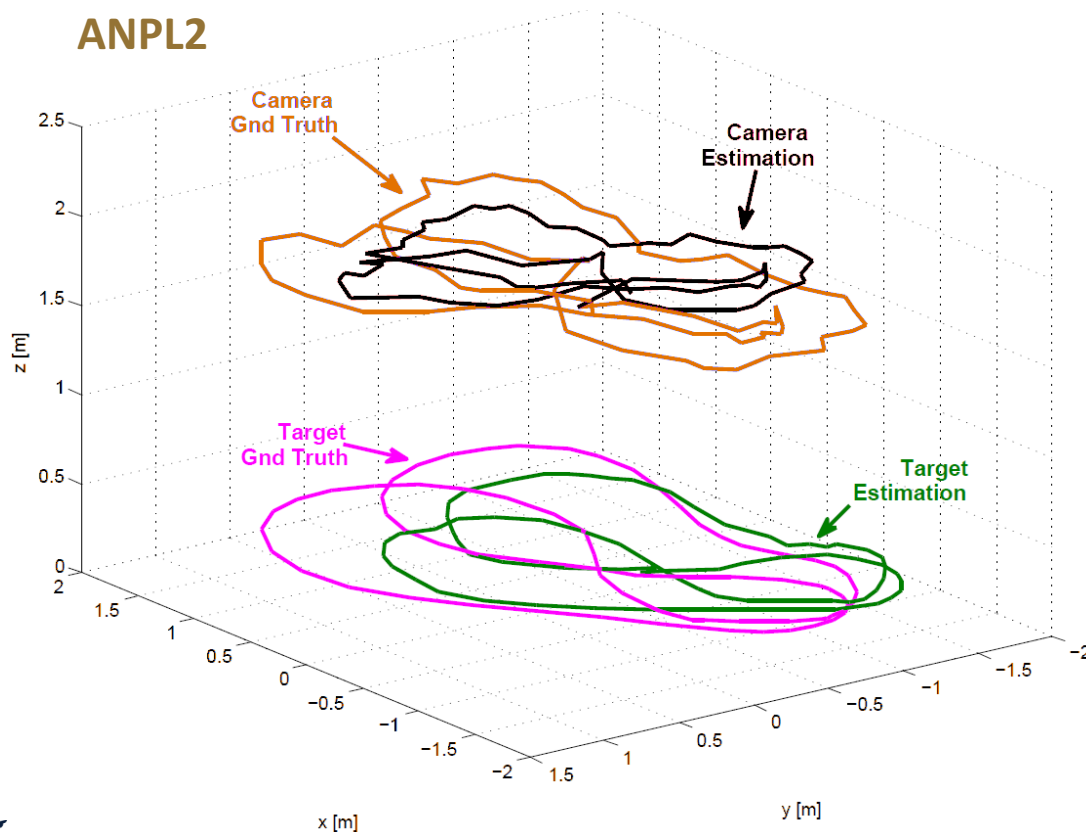
- Circular recurrent path
- Synchronized movements
- Frequent loop closer
- Target always in sight

### 3. Real-Imagery Experiments

- 2 different datasets

	Camera Resolution [pix]	Frames	Duration [sec]	Landmarks	Observations
<i>ANPL1</i>	1280 × 960	80	40	2439	31333
<i>ANPL2</i>	1920 × 1080	73	117	3366	25631

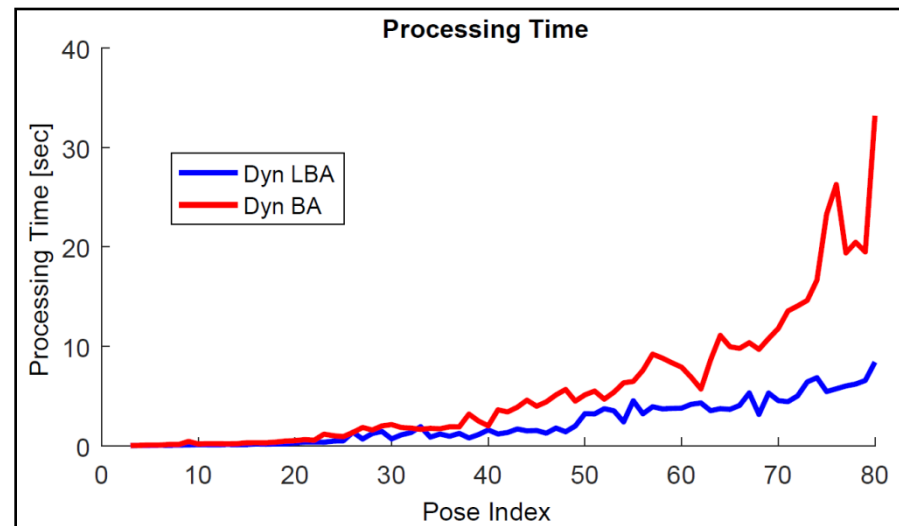
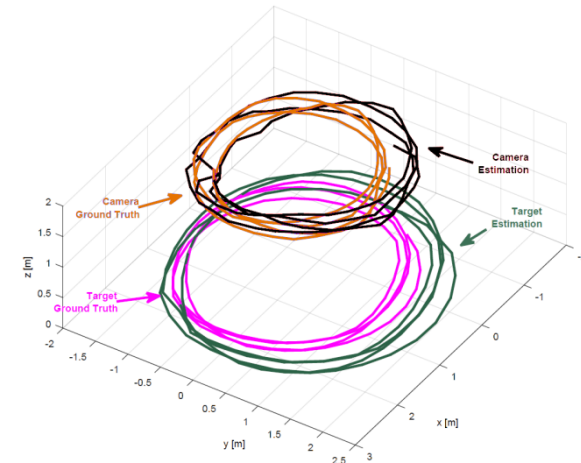
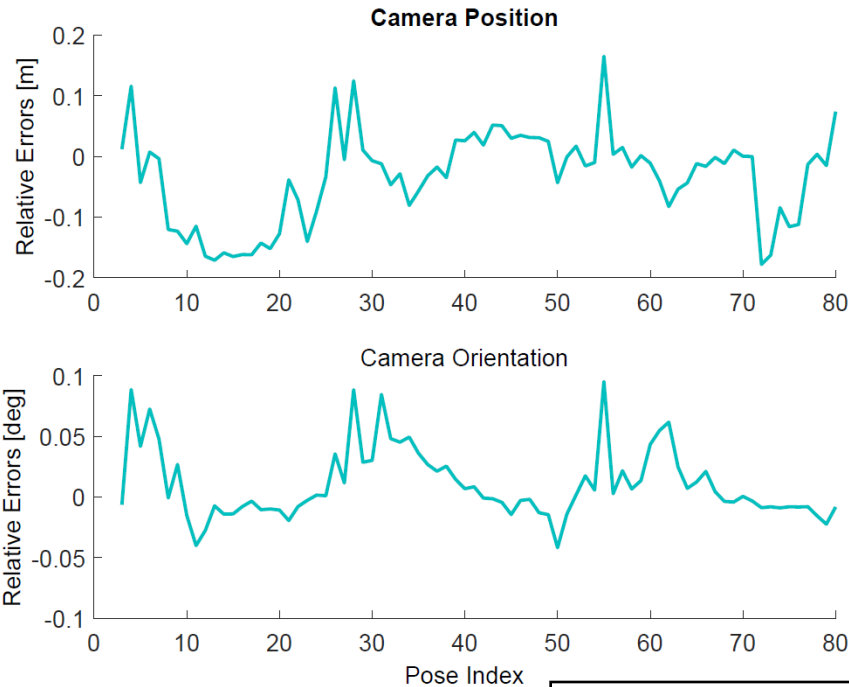
#### ANPL2



- More complex path
- Unsynchronized movements
- Target not always in sight

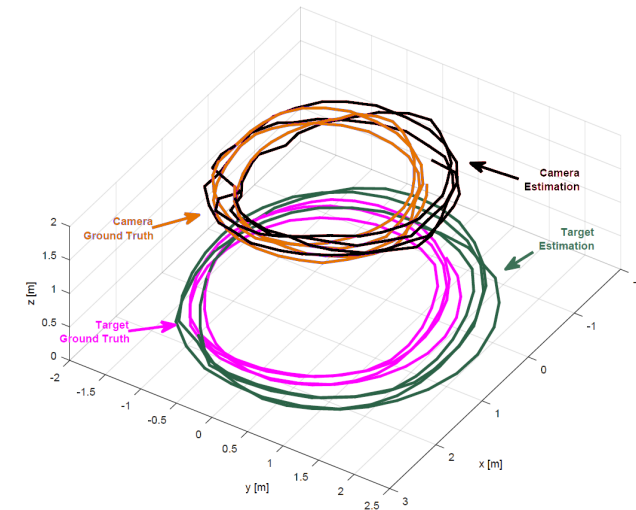


## ANPL 1

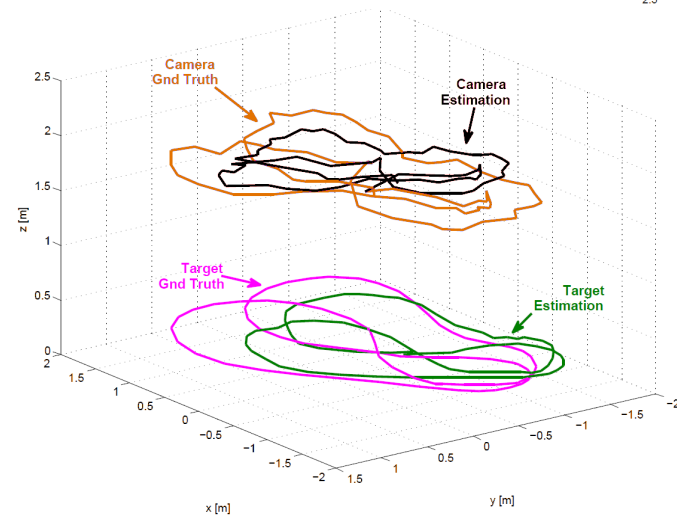


## Real-Imagery results summary

	Target Rel. Error [m]		Camera Rel. Error [m]	
	Mean	Max	Mean	Max
<b>ANPL1</b>	0.06	0.19	0.01	0.09
<b>ANPL2</b>	0.01	0.42	0.01	0.23



		Processing Time [sec]	
		Mean	Total
<b>ANPL1</b>	BA	5.6	447.8
	LBA	2.2	177.1
<b>ANPL2</b>	BA	3.1	222.9
	LBA	1.9	139.4



## Conclusions / Future Work

### Contributions

- An efficient method for vision-based ego-motion and target trajectory estimation  
    —→ Target tracking problem is integrated into the iLBA framework
- Simulations/Tests show :
  - Considerable gain in computational efforts compared to BA
  - Similar levels of accuracy for both methods
- Publicly available datasets online

### Future Work

- Problem extension to multi-robot / multi-target cases  
Challenge : data association





# THANK YOU

