Towards Planning in the Generalized Belief Space

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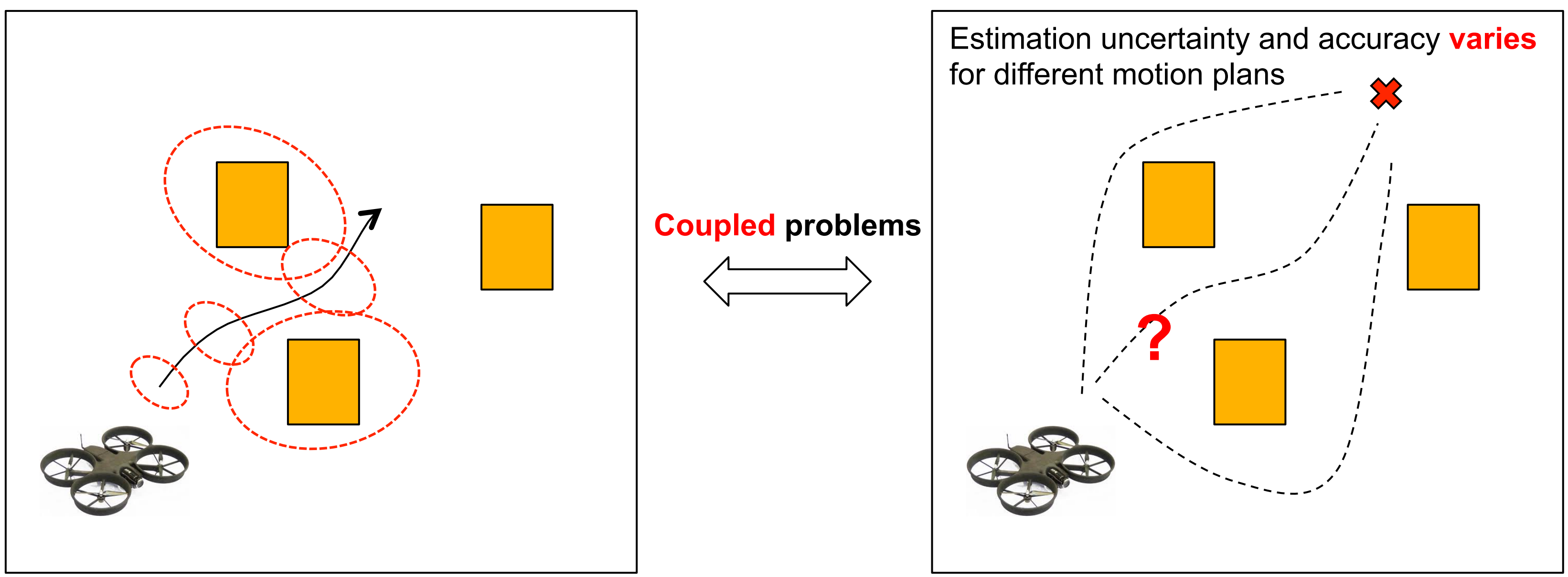
Introduction

- Key components for autonomous operation include
  - **Perception**: Where am I? What is the surrounding environment?
  - **Planning**: What to do next?

Localization and mapping given robot motion **Planning (e.g. reach a goal)**



Introduction

- Key components for autonomous operation include
  - **Perception**: Where am I? What is the surrounding environment?
  - **Planning**: What to do next?
- Reliable operation in complex scenarios
  - Planning should account for different sources of uncertainty
  - What if environment is partially unknown or uncertain?

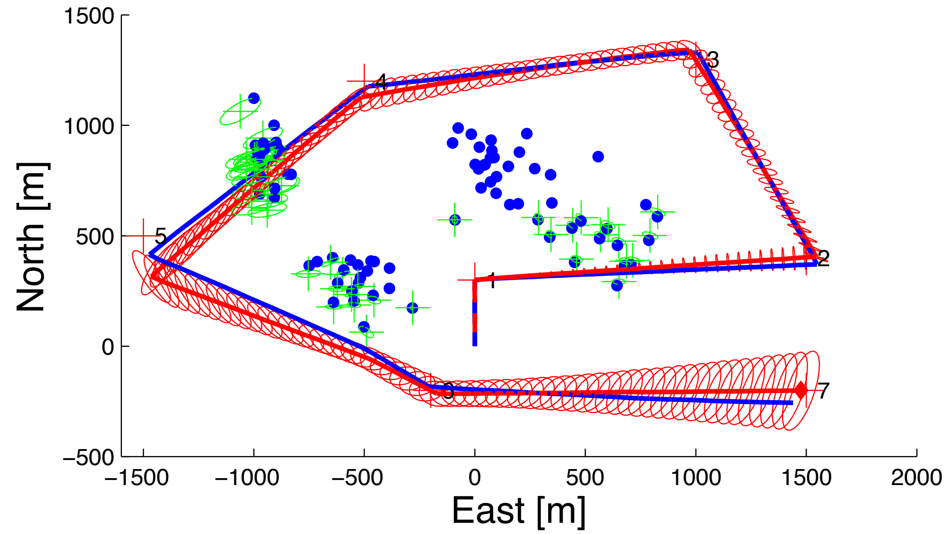
Integrated planning and perception

Perception ⇄ **Planning**

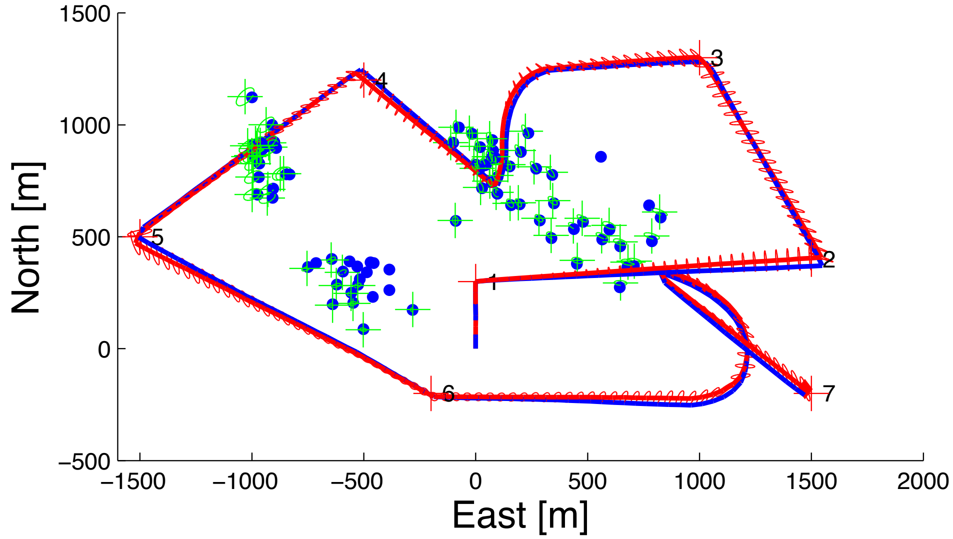
Introduction – Motivating Example

- **Autonomous navigation** to different goals in an unknown environment

Not accounting for uncertainty in planning



Accounting for uncertainty in planning



Related Work

- Existing approaches often
  - Assume environment (e.g. map) is a priori known
    [Prentice and Roy 2009], [Van den Berg et al. 2012], [Hollinger et al. 2013]
  - Discretize state and control space - performance depends on grid resolution
    [Stachniss et al. 2004], [Bryson and Sukkarieh 2008], [Valencia et al. 2012], [Kim and Eustice 2013]
  - Assume maximum likelihood observations
    [Miller et al. 2009], [Platt et al. 2010]
- This work - Planning in the **Generalized Belief Space (GBS)**
  - Probabilistic description of the **robot** <u>and</u> the **environment** states
  - General framework
  - Closely related to [Van den Berg et al. 2012]

- Environment is a priori unknown
- Planning is done in the continuous space
- Maximum likelihood observations assumption is avoided

Notations and Probabilistic Formulation

- **Joint** state vector $$X_k \doteq \{\underbrace{x_0, \ldots, x_k}_{\text{Past \& current robot states}}, \underbrace{L_k}_{\text{Mapped environment}}\}$$

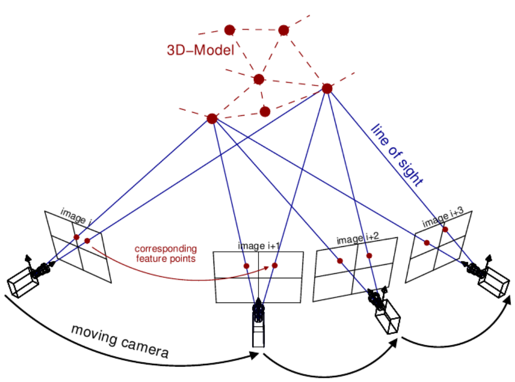
- **Joint** probability distribution function $p\left(X_k | \mathcal{Z}_k, \mathcal{U}_{k-1}\right)$

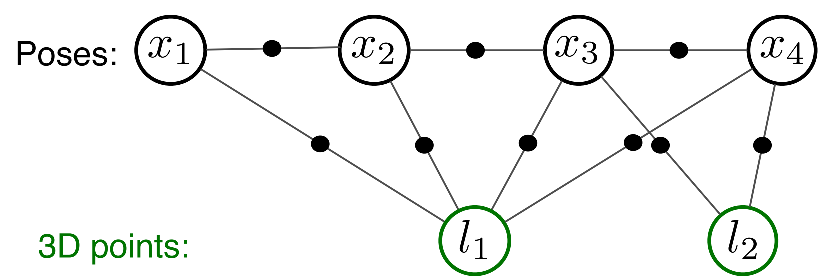
$$p\left(X_k | \mathcal{Z}_k, \mathcal{U}_{k-1}\right) = priors \cdot \prod_{i=1}^{k} p\left(x_i | x_{i-1}, u_{i-1}\right) \underline{p\left(z_i | X_i^o\right)}$$

General observation model $X_i^o \subseteq X_i$

- Computationally-efficient maximum a posteriori inference e.g. [Kaess et al. 2012]

$$p\left(X_k | \mathcal{Z}_k, \mathcal{U}_{k-1}\right) \sim N\left(X_k^*, \Sigma_k\right)$$





Planning in the Generalized Belief Space

- Plan (locally) optimal control sequence over L look-ahead steps: $u^*_{k:k+L-1}$
  - By minimizing an objective function (can now include uncertainty)
  - Operating over the **generalized belief**
  - Model predictive control framework

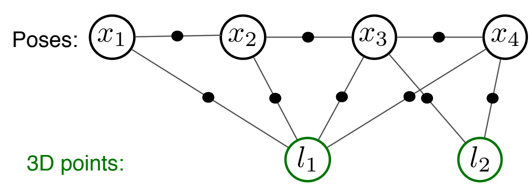
X_k : Joint state at time t_k

$\mathcal{U}_{k-1}$: All past controls

- **What is the generalized belief?**
  - Probabilistic description of the **robot** and the **environment** states
  - Generalized belief at planning time t_k: $gb(X_k) \doteq p\left(X_k | \mathcal{Z}_k, \mathcal{U}_{k-1}\right) \sim N\left(X_k^*, \Sigma_k\right)$

Known (from perception)

Generalized belief at planning time = joint pdf



Planning in the Generalized Belief Space

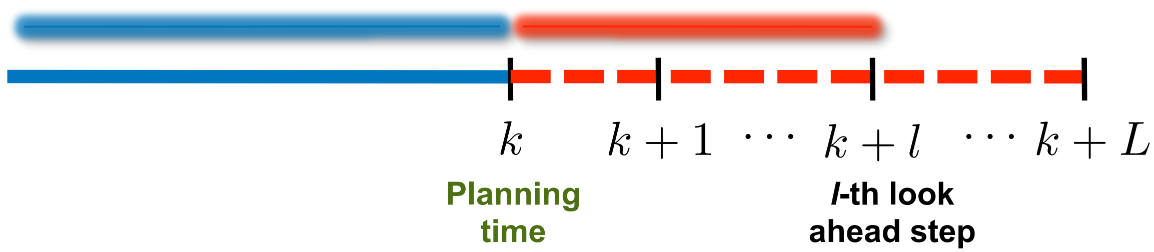
- Generalized belief at the l-th look-ahead step
  - Describes the joint pdf (robot and environment states) at that time

$$gb\left(X_{k+l}\right) \doteq p\left(X_{k+l} | \mathcal{Z}_k, \mathcal{U}_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1}\right)$$

Joint state at the *l*-th look ahead step

Past controls & measurements

Controls & measurements at the first *l* look-ahead steps



Planning in the Generalized Belief Space

- Generalized belief at the l-th look-ahead step
  - Describes the joint pdf (robot and environment states) at that time

$$gb\left(X_{k+l}\right) \doteq p\left(\underbrace{X_{k+l}}_{\text{Joint state at the } l\text{-th look ahead step}} \middle| \underbrace{\mathcal{Z}_k, \mathcal{U}_{k-1}}_{\text{Past controls \& measurements}}, \underbrace{Z_{k+1:k+l}, u_{k:k+l-1}}_{\text{Controls \& measurements at the first } l \text{ look-ahead steps}}\right)$$

- Objective function can now involve **uncertainty** (e.g. covariance) in robot and environment states

$$J_k\left(u_{k:k+L-1}\right) \doteq \underset{Z_{k+1:k+L}}{\mathbb{E}} \left\{ \sum_{l=0}^{L-1} c_l\left(gb\left(X_{k+l}\right), u_{k+l}\right) + c_L\left(gb\left(X_{k+L}\right)\right) \right\}$$

- For example, plan motion to minimize uncertainty in robot state

Planning in the Generalized Belief Space

$$gb\left(X_{k+l}\right) \doteq p\left(X_{k+l}|\mathcal{Z}_k, \mathcal{U}_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1}\right) \sim N\left(X^*_{k+l}, \Sigma_{k+l}\right)$$

$$J_k\left(u_{k:k+L-1}\right) \doteq \underset{Z_{k+1:k+L}}{\mathbb{E}} \left\{\sum_{l=0}^{L-1} c_l\left(gb\left(X_{k+l}\right), u_{k+l}\right) + c_L\left(gb\left(X_{k+L}\right)\right)\right\}$$

- How to calculate (locally) optimal control policy?

$$u^*_{k:k+L-1} = \pi\left(gb\left(X_k\right)\right) = \underset{u_{k:k+L-1}}{\arg\min}\, J_k\left(u_{k:k+L-1}\right)$$

Dual-layer iterative optimization

Outer layer

Inference over the control

$u_{k:k+L-1}$

– Starting from initial guess

current $u_{k:k+L-1}$

Inner layer:

Inference over the **belief**

$gb(X_{k+l}) \sim N\left(X^*_{k+l}, \Sigma_{k+l}\right)$

– For **each** look-ahead step, for a **given** control

– As a function of random variables $Z_{k+1:k+l}$

$\forall l$ **calculate:** $\underset{Z_{k+1:k+l}}{\mathbb{E}}\left[c_l\left(gb\left(X_{k+l}\right), u_{k+l}\right)\right]$

Outer Layer: Inference over the Control

Iterative optimization over the nonlinear objective function $J_k\left(u_{k:k+L-1}\right)$

$$J_k\left(u_{k:k+L-1}\right) \doteq \underset{Z_{k+1:k+L}}{\mathbb{E}}\left\{\sum_{l=0}^{L-1} c_l\left(gb\left(X_{k+l}\right), u_{k+l}\right) + c_L\left(gb\left(X_{k+L}\right)\right)\right\}$$

- In each iteration:
  - Look for $\Delta u_{k:k+L-1}$
  - Update $u_{k:k+L-1}^{(i+1)} \leftarrow u_{k:k+L-1}^{(i)} + \Delta u_{k:k+L-1}$

Outer layer

Inference over the control

$u_{k:k+L-1}$

- Starting from initial guess

current $u_{k:k+L-1}$

Inner layer:

Inference over the **belief**

$$gb(X_{k+l}) \sim N\left(X_{k+l}^{*}, \Sigma_{k+l}\right)$$

- For **each** look-ahead step, for a **given** control
- As a function of random variables $Z_{k+1:k+l}$

$\forall l$ **calculate:** $\underset{Z_{k+1:k+l}}{\mathbb{E}}\left[c_l\left(gb\left(X_{k+l}\right), u_{k+l}\right)\right]$

Inner Layer: Inference Over the Belief

Given **current** controls $u_{k:k+L-1}$, for each look ahead step l :

- Compute the Gaussian approximation X^*_{k+l}, Σ_{k+l} such that

$$gb\left(X_{k+l}\right) \doteq p\left(X_{k+l} | \mathcal{Z}_k, \mathcal{U}_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1}\right) \sim N\left(X^*_{k+l}, \Sigma_{k+l}\right)$$

  – Maximum a posteriori (MAP) estimate:

$$X^*_{k+l} = \underset{X_{k+l}}{\arg\max}\, gb\left(X_{k+l}\right)$$

  – Typically solved by iterative optimization methods (e.g. Gauss Newton)

Outer layer

Inference over the control

$u_{k:k+L-1}$

– Starting from initial guess

current

$u_{k:k+L-1}$

Inner layer:

Inference over the **belief**

$$gb(X_{k+l}) \sim N\left(X^*_{k+l}, \Sigma_{k+l}\right)$$

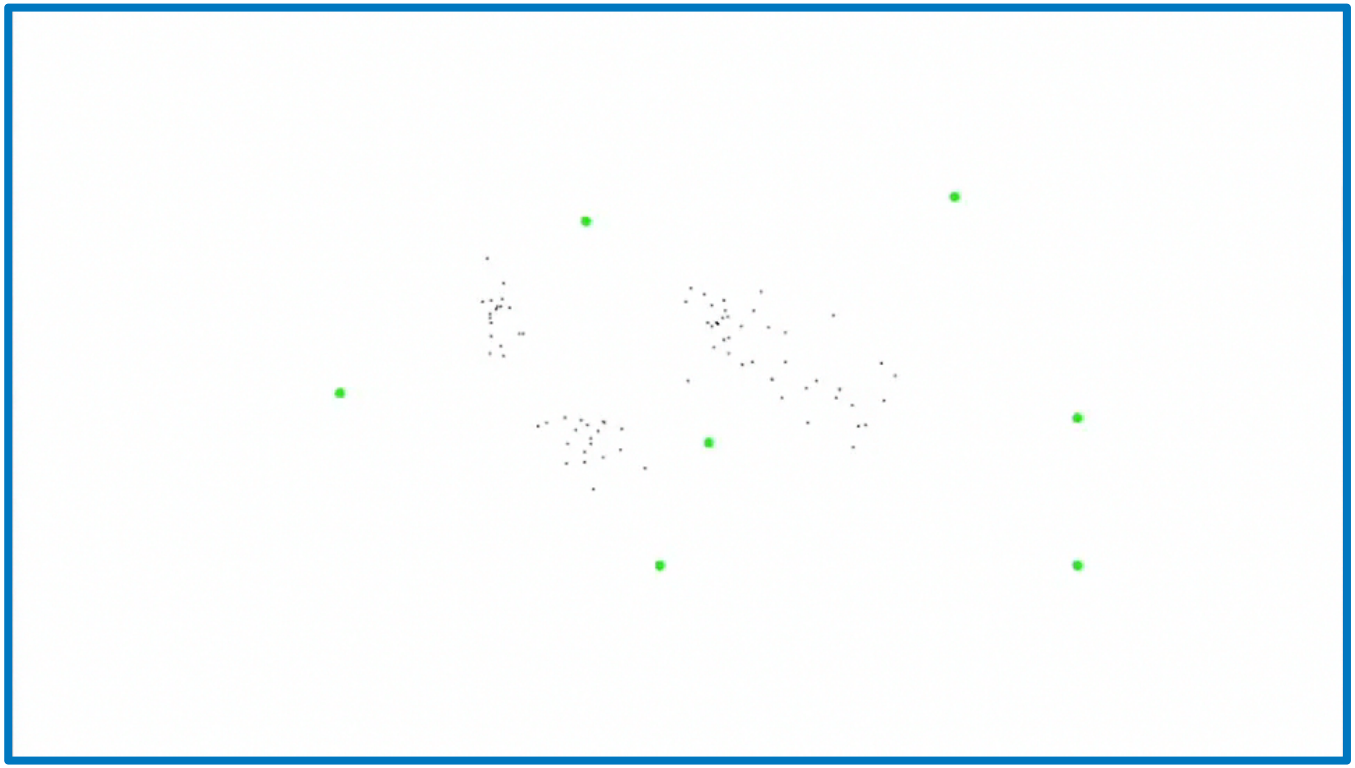
– For **each** look-ahead step, for a **given** control

– As a function of random variables $Z_{k+1:k+l}$

$\forall l$ **calculate:** $\underset{Z_{k+1:k+l}}{\mathbb{E}}\left[c_l\left(gb\left(X_{k+l}\right), u_{k+l}\right)\right]$

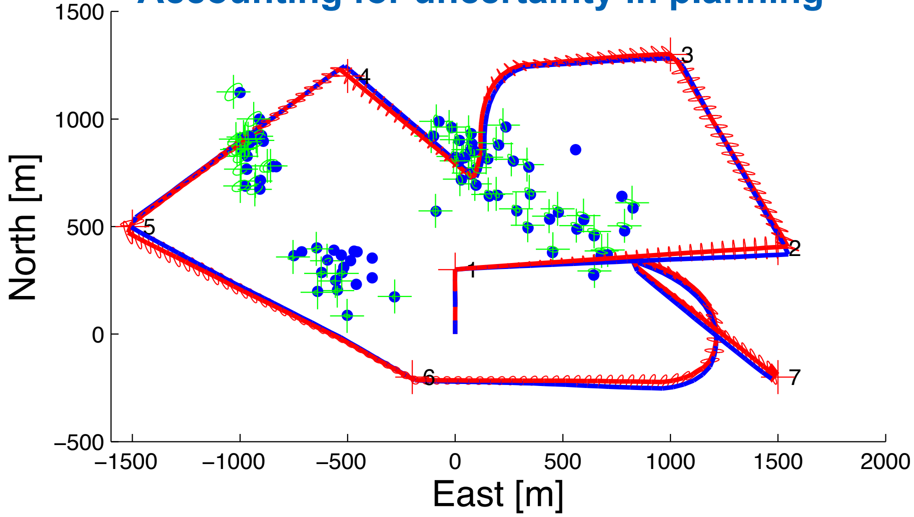
Planning in the Generalized Belief Space

- **Autonomous navigation** to different goals in an unknown environment
  - **Objective function**: penalize **control usage, uncertainty and distance to goal**
  - No absolute information
  - Onboard sensors: camera and range sensor
  - Control: heading angle

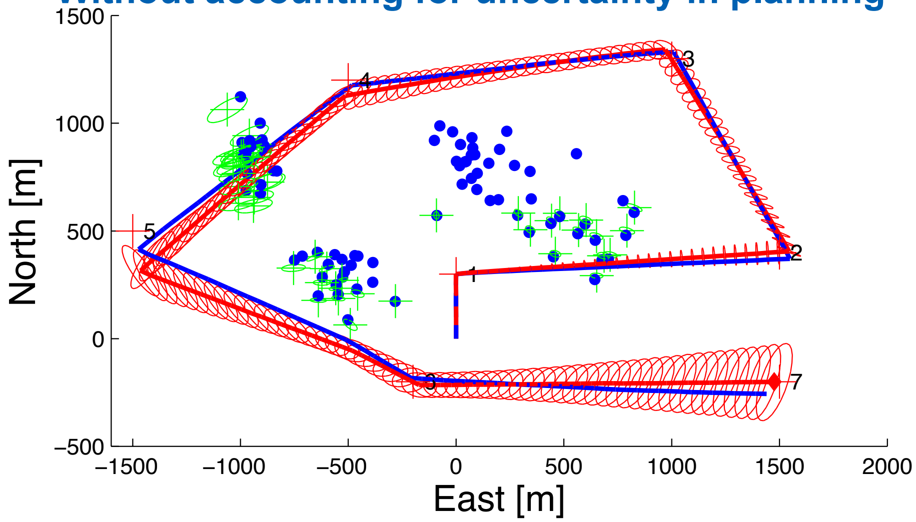


Results

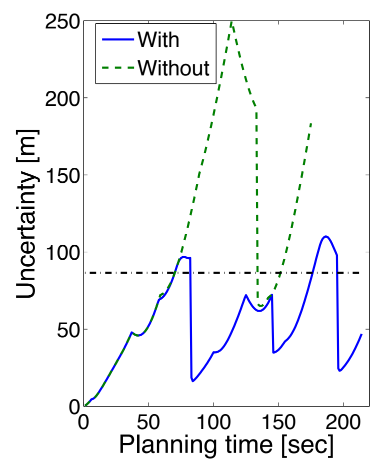
Accounting for uncertainty in planning



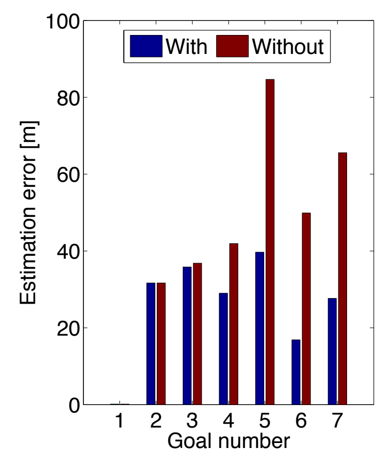
Without accounting for uncertainty in planning



Uncertainty development



Estimation errors

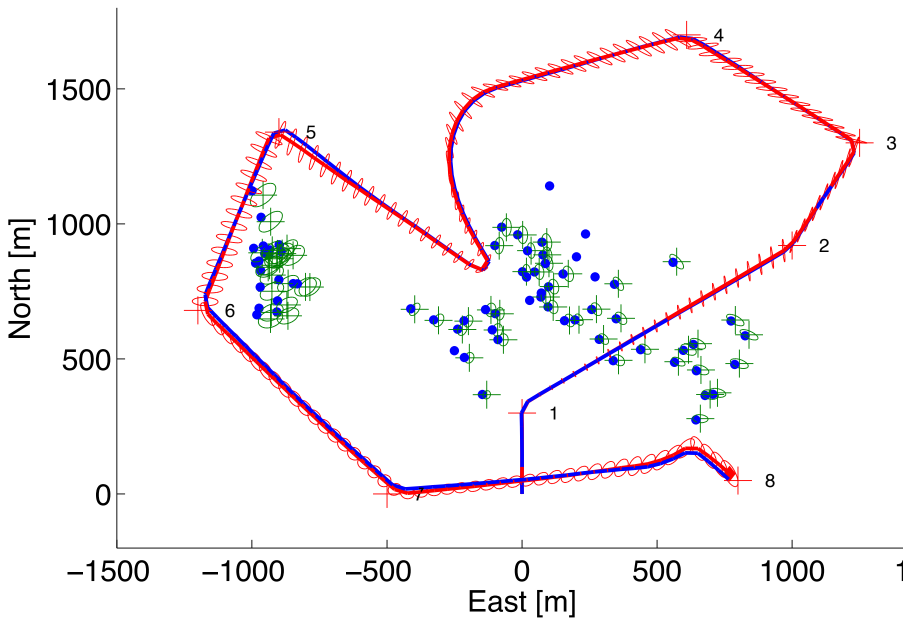


Indelman et al., Planning in Generalized Belief Space

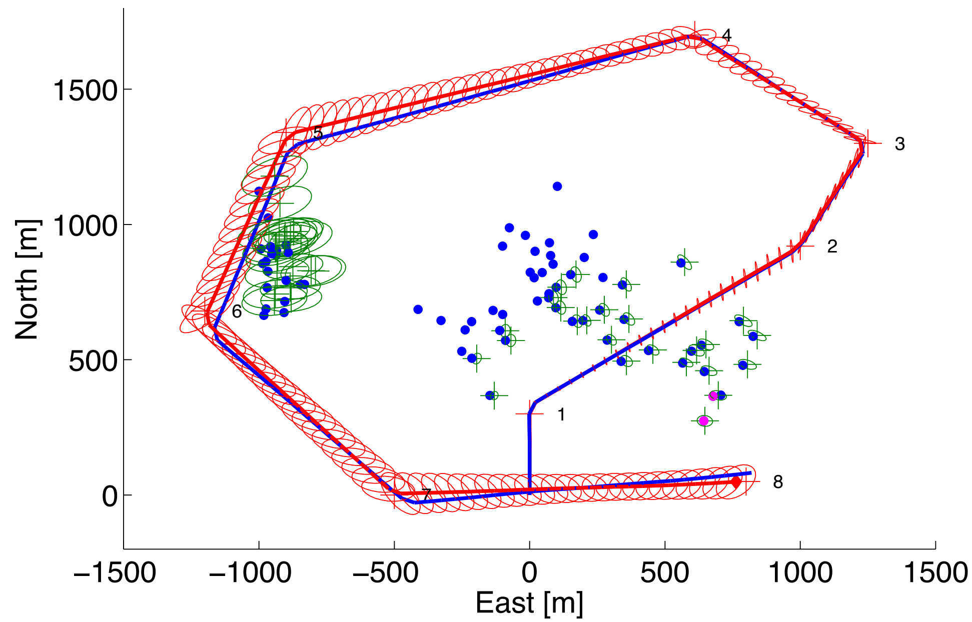
Results

- Another example

Accounting for uncertainty in planning



Without accounting for uncertainty in planning



Conclusions

- **Planning in the Generalized Belief Space**
  - General framework for planning under uncertainty
    - Including uncertainty in perception and state estimation
    - Does not assume known environment
    - Planning over continuous control space
  - Computational efficiency
    - Perception – computationally efficient (exploit sparsity, re-use information)
    - Planning – Future work