

# Multi-Robot Pose Graph Localization and Data Association from Unknown Initial Relative Poses

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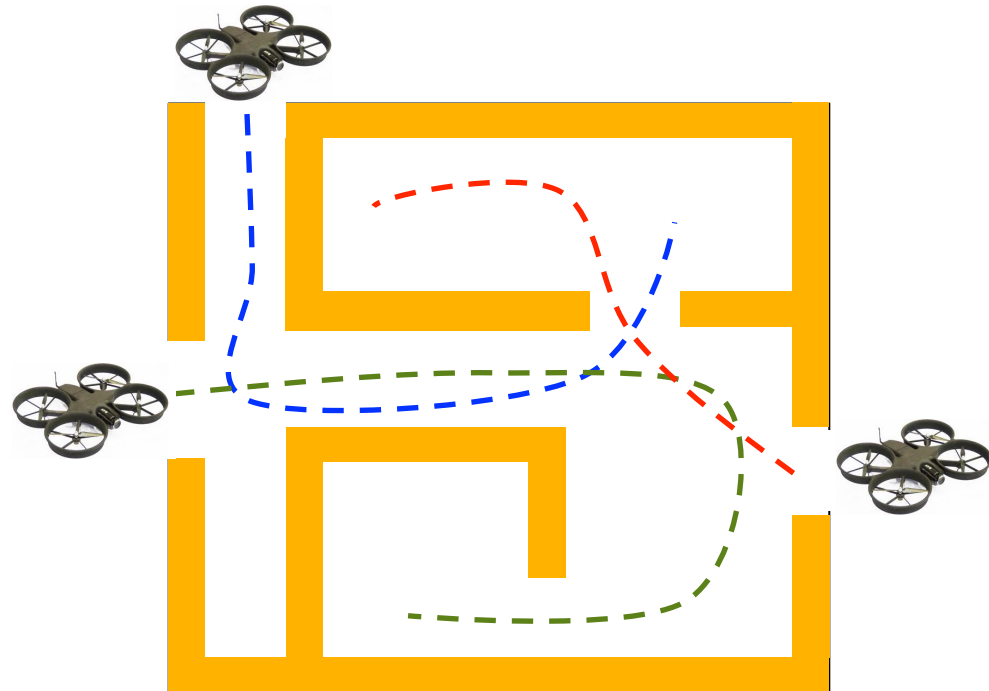
**Carnegie Mellon**  
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# Collaborative Localization and Mapping

- Important in a variety of scenarios
  - Exploration in unknown/uncertain, dangerous environments
  - Search and rescue
  - Surveillance, tracking ...
- Cooperative inference requires
  - Sharing relevant information (observations, marginals over variables of interest)
  - Correct interpretation (data association)
  - Robustness to outliers

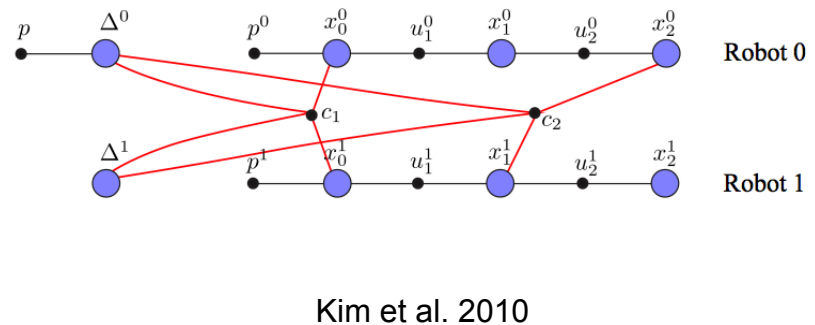
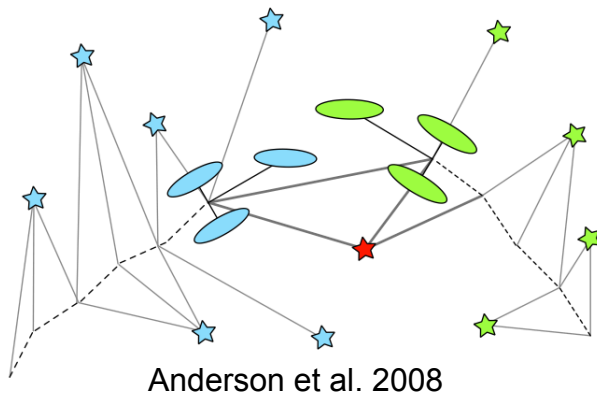
# Motivating Scenario

- Robots/sensors are deployed in an environment (e.g. building)
- Initially **unaware** of each others' location
- How to establish collaboration and perform multi-robot localization?
  - **Unknown** multi-robot data association
  - **Unknown** initial relative poses between robots



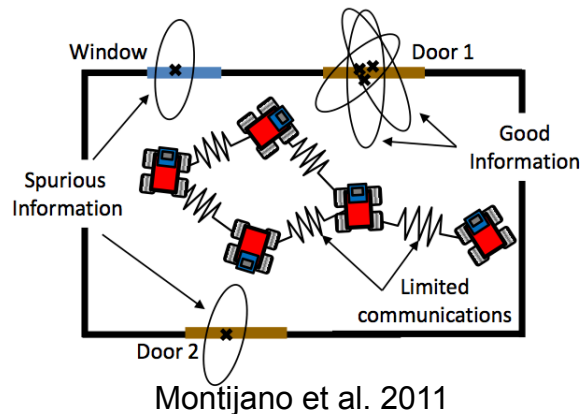
# Related Work

- **Known** data association and common reference frames
  - Full SLAM [Howard et al. 2006], [Andersson et al. 2008]
  - Pose SLAM (direct, indirect) [Roumeliotis et al. 2002], [Kim et al. 2010], [Indelman et al. 2012]



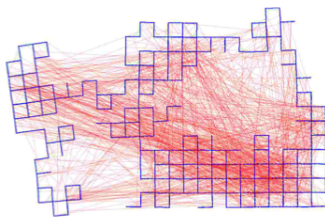
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- **Unknown** multi-robot data association **and** common reference frame
  - Full SLAM [Montijano et al. 2011], [Cunningham et al. 2012]

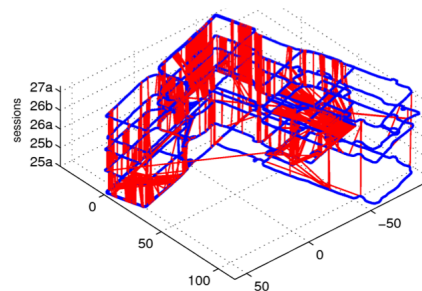


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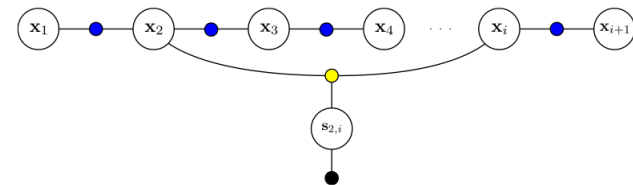
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- **Robust** graph optimization (**single** robot case – loop closures)
  - [Sunderhauf and Protzel 2012, 2013], [Latif et al. 2012], [Lee et al. 2013]



Lee et al. 2013



Latif et al. 2012



Sunderhauf and Protzel 2012, 2013

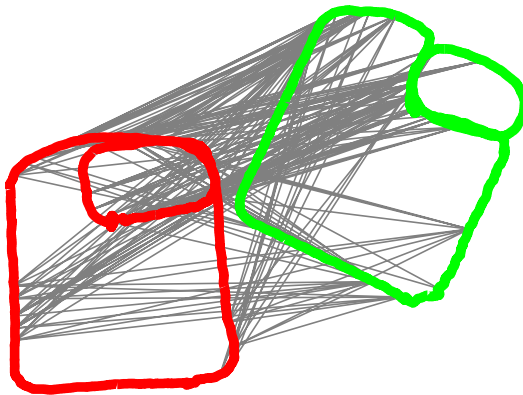
# This Work

- Multi-robot framework with
  - **Unknown** multi-robot data association
  - **Unknown** initial relative poses between robots
  - **Pose SLAM** approach
- How to establish multi-robot data association when robots start operating from unknown locations?

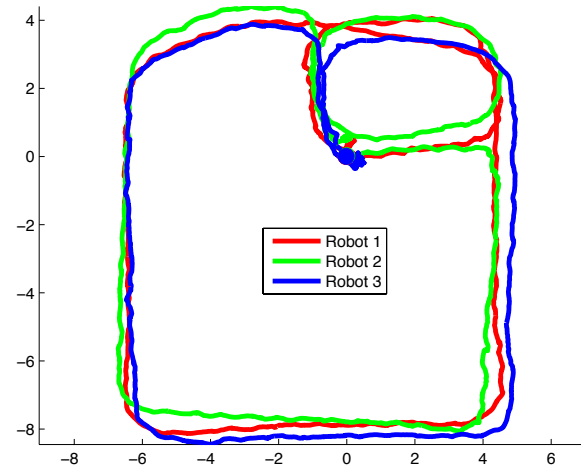
# Multi-Robot Correspondences

- If **no** common reference frame is available, what information to share?
  - Robots share **informative** observations (e.g. laser scans)
  - Calculate **candidate** multi-robot relative pose constraints
    - Collect into set  $\mathcal{F}$
    - Includes (many) outliers

Arbitrary common reference frame



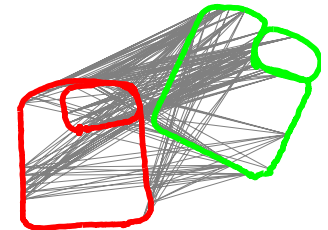
Ground truth





# Probabilistic Formulation

- Notations:
  - $\mathcal{F}$ : Multi-robot correspondences set
  - $\mathcal{J}$ : Latent variables to indicate inliers/outliers



- Joint pdf over robot trajectories and multi-robot data association:

$$p(X, \mathcal{J} | Z) \propto \prod_r p(X^r | Z^r) \prod_{(r_1, r_2, k, l) \in \mathcal{F}} p(j_{k,l}^{r_1, r_2}) p(u_{k,l}^{r_1, r_2} | x_k^{r_1}, x_l^{r_2}, j_{k,l}^{r_1, r_2})$$

Only local measurements
Data association
Multi-robot measurement likelihood, given data association

Each multi-robot correspondence

# Measurement likelihood

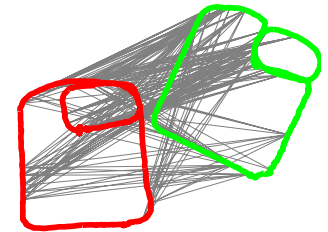
$$p\left(u_{k,l}^{r_1,r_2} | x_k^{r_1}, x_l^{r_2}\right) \propto \exp\left(-\frac{1}{2} \left\| \text{err}\left(u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2}\right) \right\|_{\Sigma}^2\right)$$

with

$$\text{err}\left(u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2}\right) \doteq \underbrace{u_{k,l}^{r_1,r_2}}_{\text{measured}} \ominus \underbrace{h\left(x_k^{r_1}, x_l^{r_2}\right)}_{\text{predicted}}$$

$$\doteq x_k^{r_1} \ominus \left(\underline{T_{r_2}^{r_1}} \oplus x_l^{r_2}\right)$$

**Unknown!!**



# Measurement likelihood

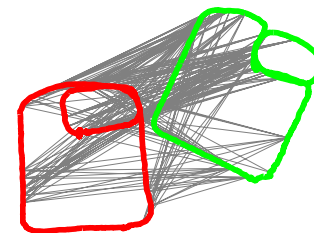
$$p\left(u_{k,l}^{r_1,r_2} | x_k^{r_1}, x_l^{r_2}\right) \propto \exp\left(-\frac{1}{2} \left\| \text{err}\left(u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2}\right) \right\|_{\Sigma}^2\right)$$

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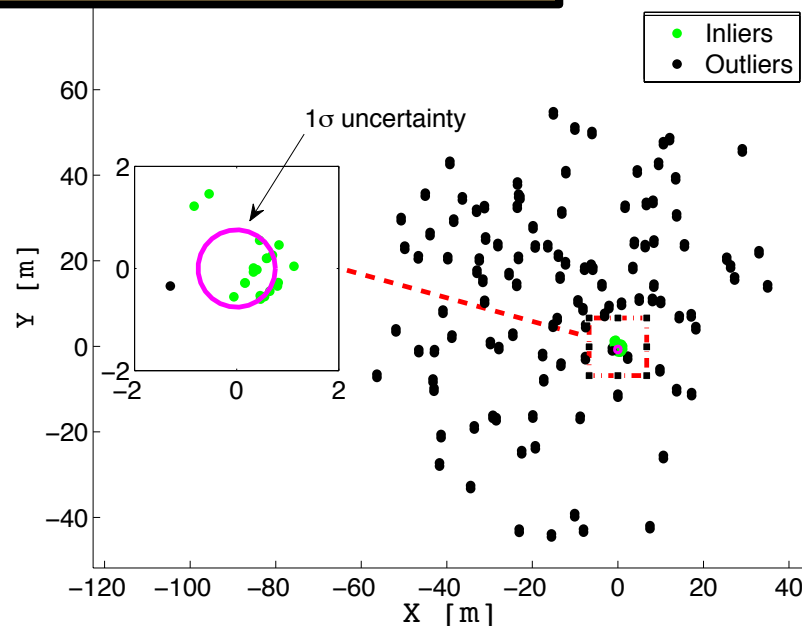
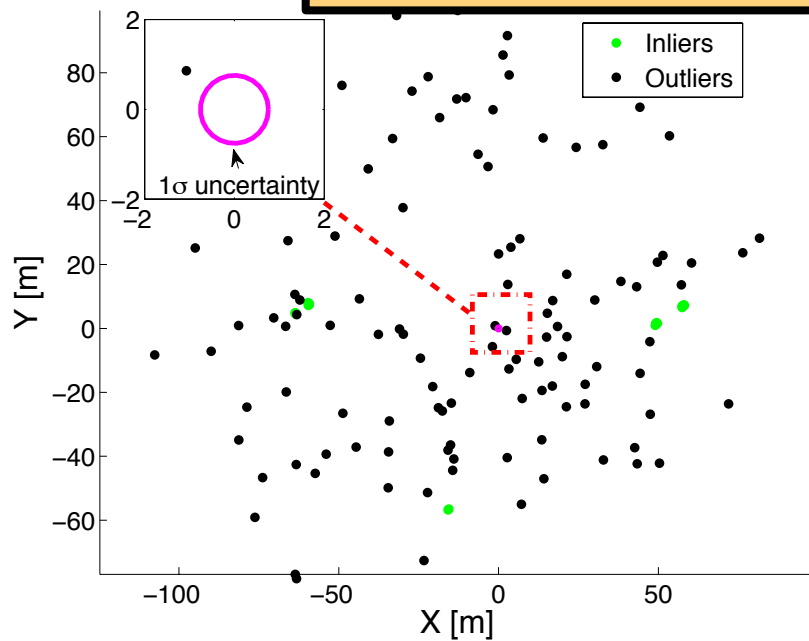
$$\doteq x_k^{r_1} \ominus \left(\underline{T_{r_2}^{r_1}} \oplus x_l^{r_2}\right)$$

Unknown!!



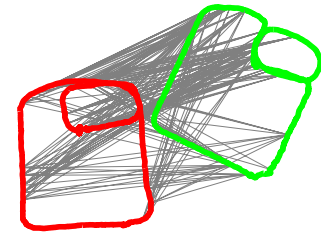
Error distribution for all correspondences:

Must **first** infer a common reference frame  $T_{r_2}^{r_1}$  !

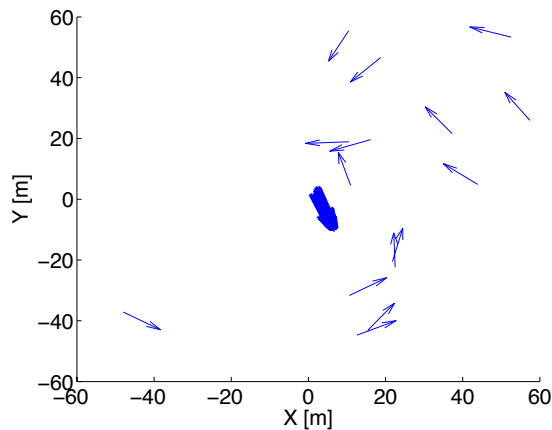


# Key Observation

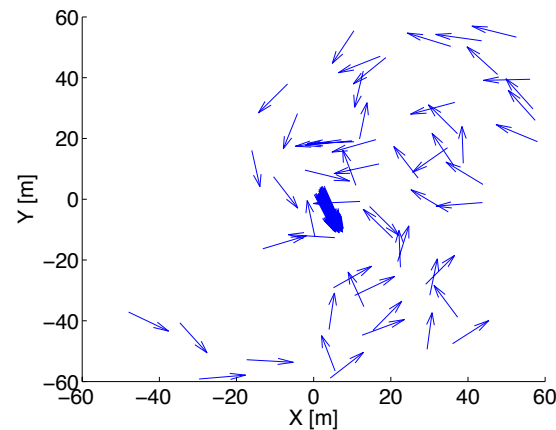
- Given robot local trajectories, relative initial pose can be calculated from **each** candidate multi-robot correspondence
  - **Only** inliers produce similar transformations
  - Objective: identify cluster



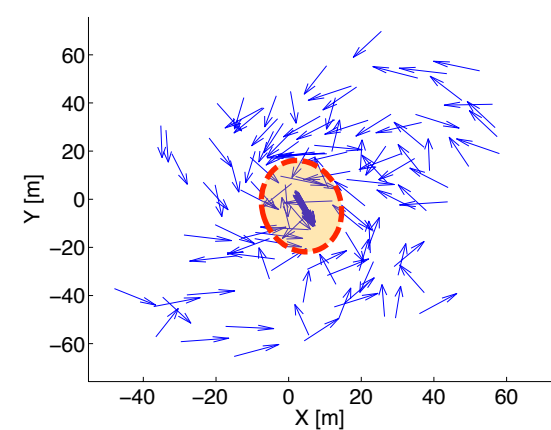
Initial relative pose between two robots (planar case:  $x, y, \theta$ )  
[synthetic data]



10% outliers



40% outliers



85% outliers

# Inference Over Common Reference Frame via EM

- MAP estimate of  $T_{r_2}^{r_1}$  given robot **local** trajectories (using only local data):

$$\hat{T}_{r_2}^{r_1} = \arg \max_{T_{r_2}^{r_1}} p \left( T_{r_2}^{r_1} | \hat{X}^{SR}, Z \right) = \arg \max_{T_{r_2}^{r_1}} \sum_{\mathcal{J}} p \left( T_{r_2}^{r_1}, \mathcal{J} | \hat{X}^{SR}, Z \right)$$

- $\mathcal{J}$ : Latent **binary** variables to indicate inliers/outliers

- EM formulation:

Local trajectories

$$\hat{X}^r = \arg \max_{X^r} p(X^r | Z^r)$$

$$\hat{X}^{SR} \doteq \left\{ \hat{X}^r \right\}_{r=1}^R$$

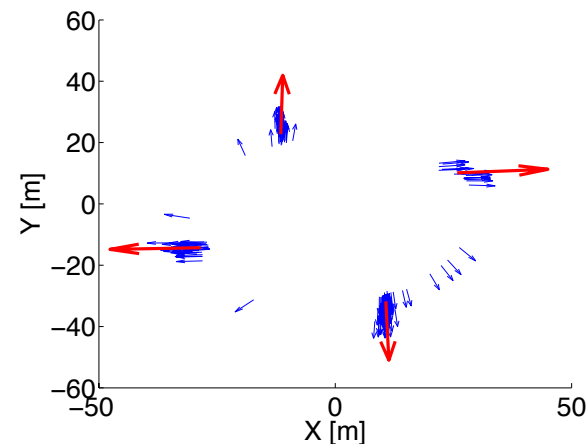
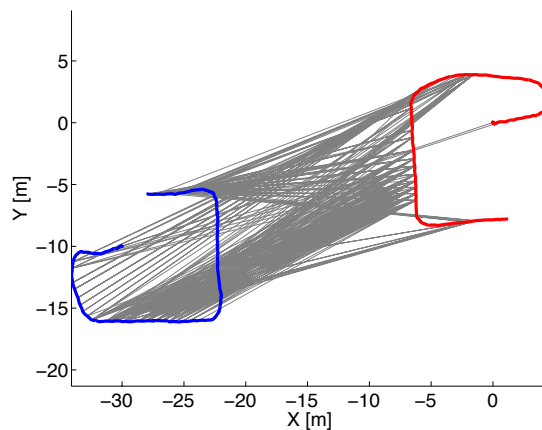
$$\hat{T}_{r_2}^{r_1} = \arg \max_{T_{r_2}^{r_1}} p \left( \mathcal{J} | \hat{T}_{r_2}^{r_1}, \hat{X}^{SR}, Z \right) \log p \left( T_{r_2}^{r_1}, \mathcal{J} | \hat{X}^{SR}, Z \right)$$

**E step**

**M step**

# Inference Over Common Reference Frame via EM (Cont.)

- Convergence only to **local** minima
- Therefore:
  - Start process from several initial guesses of  $T_{r_2}^{r_1}$
  - Results in several locally-optimal solutions (inliers/outliers, estimated  $T_{r_2}^{r_1}$ )
  - Choose most likely solution (best support)
    - Ongoing research: model selection, sensitivity to perceptual aliasing



# Inference Over Robot Trajectories

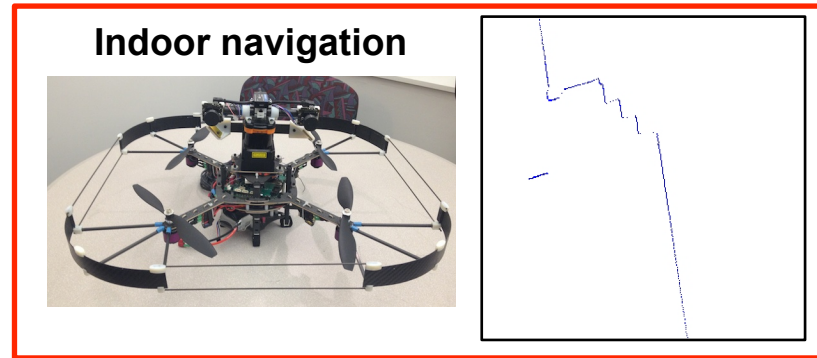
- Once a common reference frame is established:
  - Multi-robot localization becomes possible
  - Robot trajectories can be expressed in the same frame
- Infer robot trajectories via EM (see paper for full derivation):

$$\hat{X} = \arg \max_X p(\mathcal{J} | \hat{X}, Z) \log p(X, \mathcal{J} | Z)$$

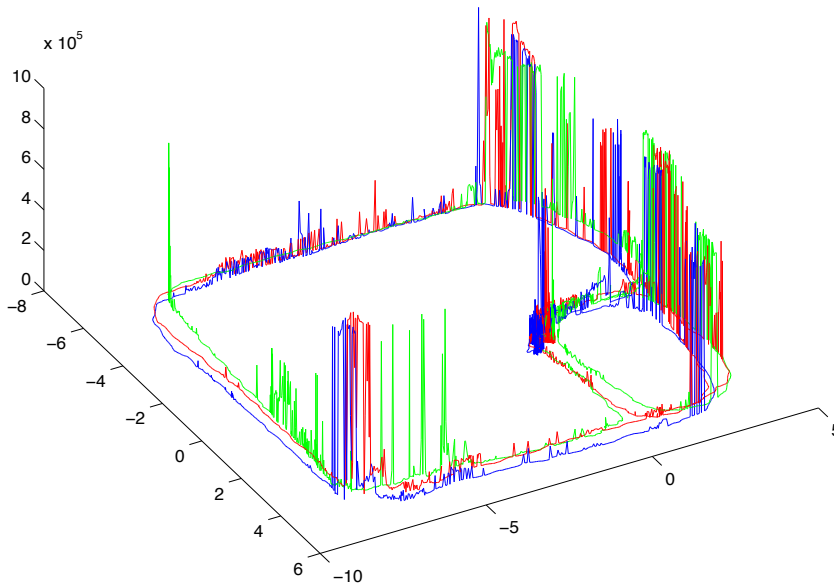
- Identified common reference frame is used as **initial guess** within measurement likelihood

$$p\left(u_{k,l}^{r_1,r_2} | x_k^{r_1}, x_l^{r_2}\right) \propto \exp\left(-\frac{1}{2} \left\| \text{err}\left(u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2}\right) \right\|_{\Sigma}^2\right)$$

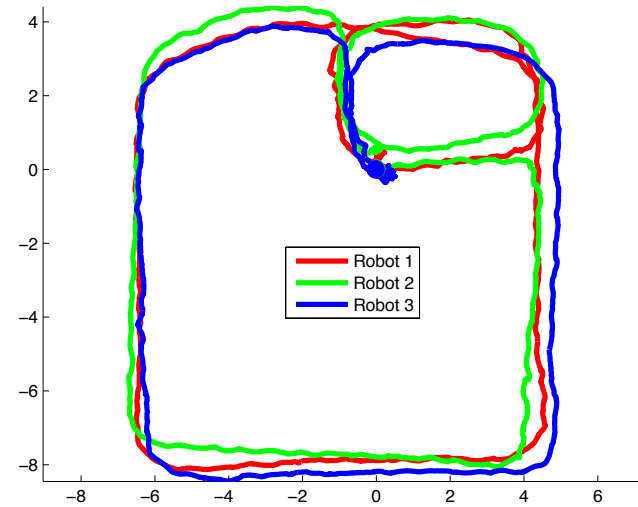
# Results



Shared **salient** laser scans



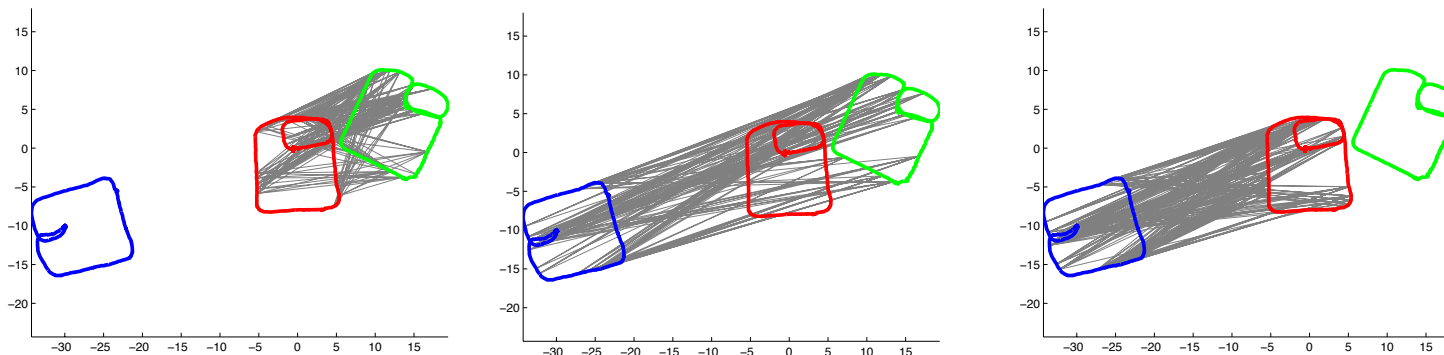
Ground truth





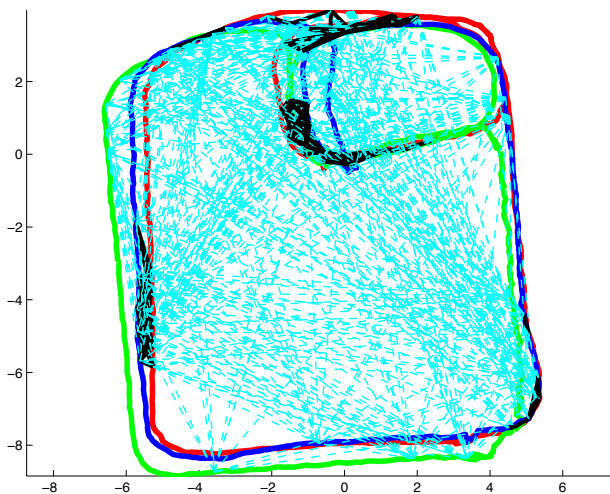
# Results (Cont.)

Local trajectories of 3 robots; Arbitrary common reference frame

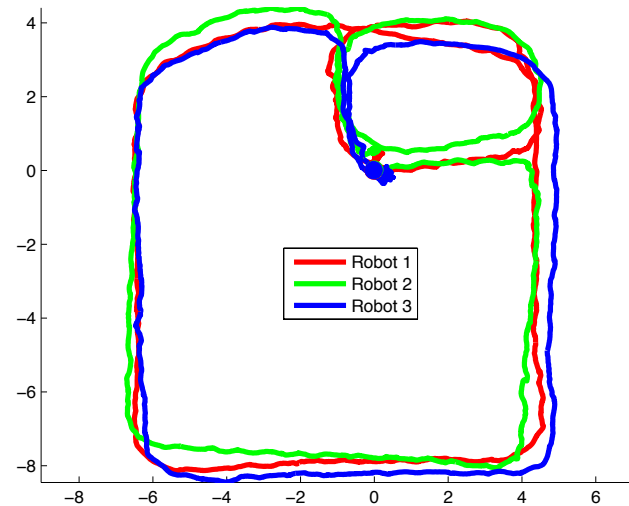


Estimated

Inliers  
Outliers



Ground truth



# Conclusions and Future Work

- Collaborative inference from unknown initial poses and data association
  - Key observation (clusters for inlier correspondences)
  - EM approach to infer common reference frames and data association
  - Once established, EM approach for inference over robot poses
  - Extensive experimental study to appear in ISER 2014

## Future Work

- Distributed and incremental framework
  - Perceptual aliasing
  - How to know when to make a decision?
- Vision sensors

