Topological Aspects in Information-Theoretic Belief Space Planning

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May 24, 2019

Classical Belief Space Planning

- determine optimal non-myopic control action $\mathcal{U}^{\star}_{k:k+L-1} = \arg\min_{\mathcal{U}} J_k(\mathcal{U})$ given objective

$$J_k(\mathcal{U}) = \mathop{\mathbb{E}}_{\mathcal{Z}} \left\{ \sum_{l=0}^{L-1} c_l \left[b(X_{k+l}), \mathcal{U}_{k+l} \right] + c_L \left[b(X_{k+L}) \right] \right\}$$

X_{k+l} - state of the system at time t_{k+l}

$b(X_{k+l})$ - future posterior belief at time t_{k+l} based on observations $\mathcal{Z}_{k+l} \subseteq \mathcal{Z}$

$\mathcal{U}_{k+l}$ - control applied at time t_{k+l}

- instantiation of a Partially-Observable Markov Decision Process (POMDP)
- finding optimal solution to POMDP in the most general form is computationally intractable

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  Shannon joint entropy of the Gaussian belief

$$J(\mathcal{U}) = \frac{N}{2}\ln(2\pi e) + \frac{1}{2}\ln|\Sigma(X_{k+L})|$$

$\Sigma(X_{k+L})$ - marginal posterior covariance

N - dimension of the state X_{k+L}

- instantiation of a Partially-Observable Markov Decision Process (POMDP)
- finding optimal solution to POMDP in the most general form is computationally intractable
- in information-theoretic BSP, J_k is a function of state uncertainty

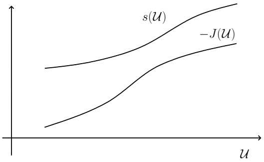
Topological Belief Space Planning

Decision making via t-bsp

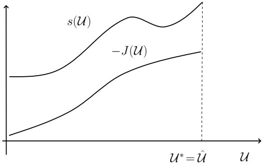
1. topological representation of a belief $\mathcal{T}$, e.g. a graph associated to its factor graph
2. topological metric $s : \mathcal{T} \rightarrow \mathbb{R}$ highly correlated with J_k, e.g. graph entropy, a number of spanning trees of a graph

$$\hat{\mathcal{U}} = \arg\max_{\mathcal{U}} s(\mathcal{U})$$

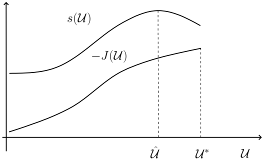
3. providing bounds on the error $|J_k(\hat{\mathcal{U}}) - J_k(\mathcal{U}^{\star})|$



(a) action consistent on the whole domain, $\gamma_f = 0$



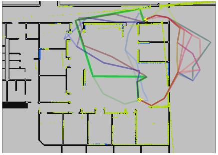
(b) action consistent, $\gamma_f \neq 0$



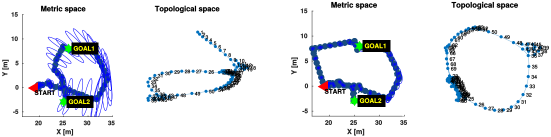
(c) non action consistent

Figure: Different topological metrics s and their influence on decision making

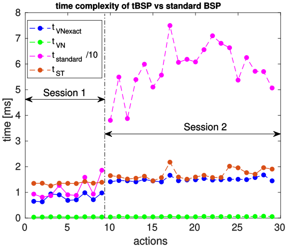
Results



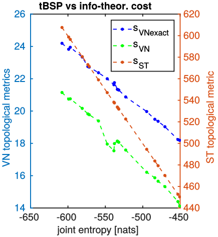
(a) candidate actions



(b) trajectory uncertainty and topology of the worst (left) and best (right) action



(c) time complexity



(d) actions consistency