Probabilistic Qualitative Localization and Mapping
Supplementary Material

Roee Mor and Vadim Indelman

This document provides supplementary material to the paper [2]. Therefore, it should not be considered a self-contained document, but instead regarded as an appendix of [2]. Throughout this report, all notations and definitions are with compliance to the ones presented in [2].

1 Extended Geometrical Analysis

In this section we analyze the geometric properties of our problem, and the effect of incorporating a motion model on the solution quality.

As specified in [2, Section 3b], our approach considers multiple landmark-centric triplet frames. For each local frame, the underlying basic metric problem determines camera poses $X_{1:n}^{AB}$ and landmark C location $L^{AB:C}$ in the local AB frame, given a set of measurements $Z_{1:n}^{ABC}$.

When considering measurement noise, the problem $\mathbb{P}(X_{1:n}^{AB}, L^{AB:C}|Z_{1:n}^{ABC})$ is actually a SLAM problem. The SLAM problem is observable only with enough landmark measurements and camera poses. It can also be solved with less measurements, if priors are available. Remembering we solve many small local 3 landmark and few cameras problems, achieving enough measurements or priors for each local frame, can be a difficulty.

Incorporating a motion model, $\mathbb{P}(X_{1:n}^{AB}, L^{AB:C}|Z_{1:n}^{ABC}, a_{1:n-1})$, makes the problem easier to solve, and requieres less measurements. This can be demonstrated through a simple degrees of freedom analysis for the 2D case. The unknowns are landmark C location (x_C, y_C), and camera poses $(x_{1:n}, y_{1:n}, \alpha_{1:n})$. For n time steps, we have $3n + 2$ unknowns. When using only azimuth measurements $(\phi_{1:n}^A, \phi_{1:n}^B, \phi_{1:n}^C)$ to landmarks A, B and C, the number of equations is $3n$. This problem is therefore under-determined. When considering actions $a_{1:n-1}$ (i.e. the azimuth angles from one camera pose to the next, $\psi_{1:n-1}$) and

R. Mor is with the Department of Computer Science, Technion - Israel Institute of Technology, Haifa 32000, Israel. V. Indelman is with the Department of Aerospace Engineering, Technion - Israel Institute of Technology, Haifa 32000, Israel. roeeki.mor@gmail.com vadim.indelman@technion.ac.il. This work was partially supported by the Israel Ministry of Science & Technology (MOST).

the corresponding motion model (specified in [2, Section 2], the number of equations is $4n-1$. For $n \geq 3$, the number of equations is equal or greater than the number of unknowns, and the problem becomes fully observable. The intuition is that, incorporating a motion model enables to solve landmark C location by triangulating line of sight vectors from multiple views.

This well-known insight related to the metric problem is also valid for the qualitative problem. Therefore we expect incorporating a motion model will enable solving the qualitative problem with less measurements and prior knowledge. This is our motivation for incooperating a motion model into the qualitative inference. Here we will look at this effect in more detail.

In this section we consider the basic geometry of our problem, and do a metric deterministic analysis. Thus, for the sake of explanation, we consider the measurement and motion models ideal, noise-free observations that we shall denote by $\bar{Z}$.

First, we shall look at the single view case. Landmarks A,B are known (we work in the AB frame), while the camera pose and landmark C are unknown. Determining camera pose by 2D azimuth measurements to two known landmarks is a well known 2D 2-point camera resection problem. A full and efficient analytic solution to this problem can be found in [3]. Given these azimuth measurements, the camera's location must be on a *circle* that goes through the two landmarks A,B. The "locus circle" center and radius calculation is given in [3]. Azimuth ordering of ϕ_n^A, ϕ_n^B further confines the camera location to left or right to the AB vector, see Figure 1b. It is not difficult to show that camera orientation α_n is directly determined by its location:

$$\alpha_n = \text{atan}_2(y_n^X, x_n^X) + \phi_A = \text{atan}_2(y_n^X - 1, x_n^X) + \phi_B. \tag{1}$$

Per every possible camera pose, landmark C can be located somewhere on the line of sight corresponding to the azimuth measurement ϕ_n^C. A diagram illustrating these aspects is given in Figure 1b, while a simulative example can be seen in Figures 2a and 2b. A crucial insight is that even in this deterministic setting the problem is not fully observable, i.e. solutions for metric camera poses and landmark locations are continuously distributed. We denote this joint distribution by $\mathbb{P}(X_n^{AB}, L^{AB:C} | \bar{Z}_n^{ABC})$.

If measurements from several time instances are available, the joint pdf is:

$$\mathbb{P}(X^{AB}, L^{AB:C} | \bar{Z}_{1:n}^{ABC}) = \prod_{i=1}^{n} \mathbb{P}(X^{AB}, L^{AB:C} | \bar{Z}_i^{ABC}) \tag{2}$$

As described in [2], the posterior distribution over qualitative states can be extracted by integration.

Now we examine the impact of incorporating a motion model. Given a specific pose for the first camera $(x_1^*, y_1^*, \alpha_1^*)$, the action ψ_1, and the second measurement $Z_2 = \{\phi_2^A, \phi_2^B, \phi_2^C\}$ obtained after executing the action, the second camera location (x_2^*, y_2^*) can be calculated as the intersection of the line of motion with the valid part of the second measurement locus circle (see Figure

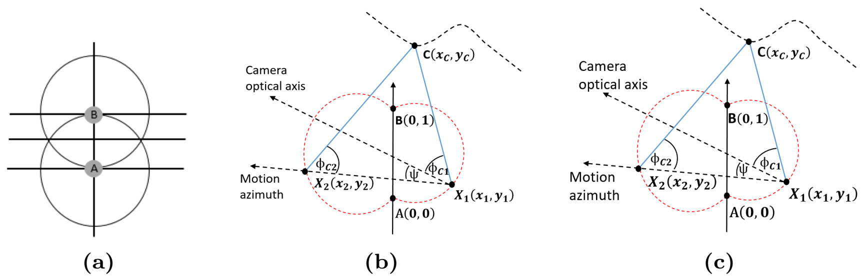


Figure 1: (a) Extended double cross spatial partition sugested in [1]. (b) 2D 2-point camera resection. (c) camera resection and landmark triangulation with two measurements

1c). As this intersection can occur once, twice or not at all. As a result, some camera poses are disqualified. Another consequence is geometric ambiguity: some measurements can support two solutions for landmark location and the corresponding second camera pose.

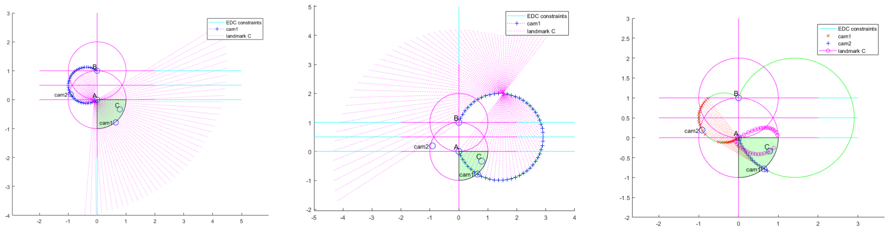
Given the two camera poses $(x_1^*, y_1^*, \alpha_1^*), (x_2^*, y_2^*, \alpha_2^*)$ and azimuth measurements ϕ_1^C, ϕ_2^C to landmark C, its location (x_c^*, y_c^*) can be triangulated. Considering all possible poses for the first camera, the mapping of landmark C is reduced to a curve (might be split into two curves) - see Figure 1c. A simulation example that displays how our motion model allows to disqualify certain camera poses, and triangulate the landmark C can be seen in Figure 2c.

When considering three or more measurements, triangulation consistency further disqualifies most camera poses and landmark locations, leaving only one or few discrete possible locations for landmark C, and the corresponding camera trajectories. The surviving landmark C locations are in fact an intersection of the curve estimates for pairs of consecutive views.

These geometric properties of the problem are the main motivation of our work. The deterministic analysis reveals that incorporating a motion model into the formulation improves results in the metric problem. Since probability for qualitative states is an integration on the metric pdf, it should improve as well. We also notice that the geometric ambiguity enables several distinct solutions to the problem. Therefore probabilistic solutions based on linearization might converge to local minima and achieve bad results. A global solution, or a hypothesis based approach is therefore needed.

2 Extended Result Analysis

In [2] we present part of the results to best explain the our main insights. Actually we used more metrics, and various noise level instances to get a good understanding of the meaning of the results. In this section we give some additional details to support out conclusions.



(a) **(b)** **(c)**

Figure 2: Simulation example for first camera resection and landmark estimation. True camera positions, and landmark positions marked in circles.(a) First view. Estimated camera position marked by blue +. Estimated landmark C location marked in magenta (.) - see legend. (b) Second view. Same markings. (c) Estimation by both views using motion model. Estimated camera position marked by +,x. Estimated landmark C location marked in magenta circles (see legend).

2.1 Result Metrics

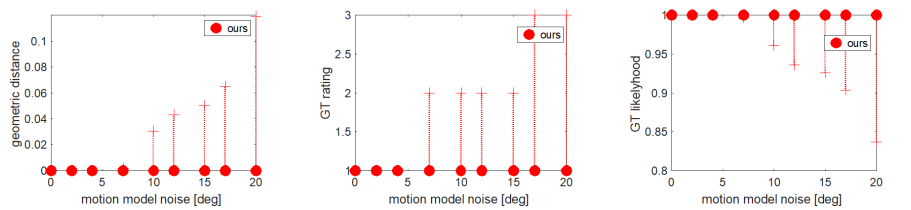
In our work [2] we use various metrics to analyses inference performance. This is actually an important addition for QSR SLAM, since it gives us deeper understanding than what was presented in previous works. We use two types of metrics:

ground truth (GT) related metrics:

- Probability DMSE: This is the mean square error for the difference between estimated qualitative state vector to GT state vector $DMSE = \sqrt{\sum_{i=1}^{m}(s_i^{AB:C} - s_{GT}^{AB:C})^2}$. It tests how accurate is the estimation.
- Ground truth rating: This is the position of the GT qualitative state when ordering qualitative states by their estimation likelihood (1 is most likely)
- Geometric distance: The mean distance from qualitative state centroids to GT state centroind weighted by the state estimated probability. $gmd = \sum_{i=1}^{m} ||c_i^{AB:C} - c_{GT}^{AB:C}||_2$ This metric tells us how close are the estimated states to GT (1 is the distance between A,B).
- Ground truth likelihood: estimated posterior probability of GT qualitative state $\mathbb{P}(s_{GT}^{AB:C})$. This measures the accuracy of estimation, but ignores false qualitative states.
- Ground truth likelihood ratio: the ration between estimated probability of GT qualitative state to estimated probability of the most likely state $\frac{\mathbb{P}(s_{GT})}{max(S^{AB:C})}$. This metric measures how close the GT state is to being most likely.

information (or entropy) related metrics:

- Entropy: estimation probability entropy $E = -\sum_{i=1}^{m}\mathbb{P}(s_i^{AB:C}\log s_i^{AB:C})$ measures how distributed is the qualitative state distribution (or how much information is in the distribution).



(a) **(b)** **(c)**

Figure 3: Sensitivity to motion model error. The plot shows median and percentiles 25 and 75. (a) mean geometric distance Vs motion model noise. (b) ground truth rating Vs motion model noise (c) ground truth likelihood Vs motion model noise.

- Likelihood ration: the ration between the second most likely qualitative state to the most likely qualitative state. This measure how "decisive" the estimation result is. very important to maximal likelihood approaches.

Using all these metrics in different stages of result analysis teaches us much about the quality of our algorithms.

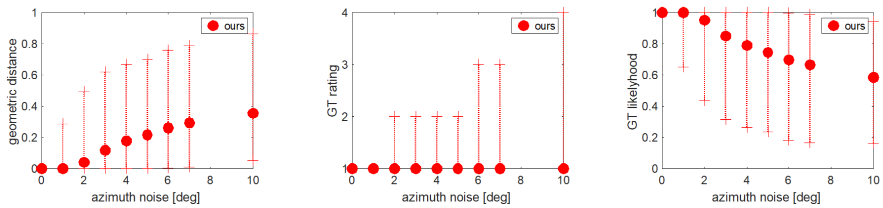
2.2 Motion Model Inference Extended Results

After noticing that the qualitative localization and mapping fast approximation algorithm presented in our main paper [2] gives results close to the full algorithm, we made a more extensive analysis of the approximated algorithm performance. In this section we'll test the different effects of motion model and azimuth noise separately. We use the same framework, and scenarios as specified in [2], but compare different sets of noise levels.

To test motion model noise effect on our estimation, we use scenarios with measurement noise 0, and various motion model noise levels. results are presented in figure 3. It can be seen that motion model noise affect weekly ground truth likelihood and geometric distance. It does however have more effect on the ground truth rating (but only on the 75 percentile).

This is an intuitive result. Scenarios with harder geometry (landmark close to qualitative state edges, or ill conditioned camera poses) are the first to suffer from the motion model errors. It gives more weight to false qualitative states, but does not significantly change probability distribution. Generally the estimation handles motion model noise well. Even with very high noise of 20°, the estimated probabilities are just minorly affected. GT rating is affected, but GT is still on the top 3 (and most likely in most cases - median).

To test measurement model noise effect on out estimation, we use scenarios with motion model noise 0, and various measurement model noise levels. results can be seen in figure 4. The effect of measurement noise is stronger. Both geometric distance, and ground truth likelihood respond harder as noise increases. GT rating responds a little harder on 75 percentile but stable on median.



(a) **(b)** **(c)**

Figure 4: Sensitivity to measurement model error. The plot shows median and percentiles 25 and 75. (a) mean geometric distance Vs measurement model noise. (b) ground truth rating Vs measurement model noise (c) ground truth likelihood Vs measurement model noise.

Generaly it seem that estimation handles the noise pretty well in most scenarios. We note for camera based platforms a measurement noise of up to $2°$ is very reasonable. under these conditions estimation handles noise well.

In summary, it seems that our approximated qualitative inference algorithm is more sensitive to measurement model noise than to motion model noise. This result is encouraging, since measurement noise is usually smaller in real world scenarios. In addition, we note response to high levels of noise is gradual and generally stable. In addition, we also see that our algorithm can handle reasonable amounts of noise well. Remembering that this is a low compute algorithm, this is another demonstration to support our conclusion that our method has potential to be practical for qualitative autonomous navigation and mapping, for online qualitative active planning. A more comprehensive analysis should be done to evaluate more realistic scenarios.

References

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[3] Vincent Pierlot and Marc Van Droogenbroeck. A new three object triangulation algorithm for mobile robot positioning. *IEEE Transactions on Robotics*, 30(3):566–577, 2014.