

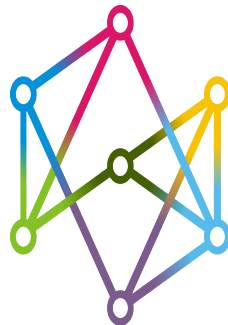
# Probabilistic Qualitative Geometry SLAM

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**TECHNION**  
Israel Institute  
of Technology



**ANPL**  
Autonomous Navigation and  
Perception Lab

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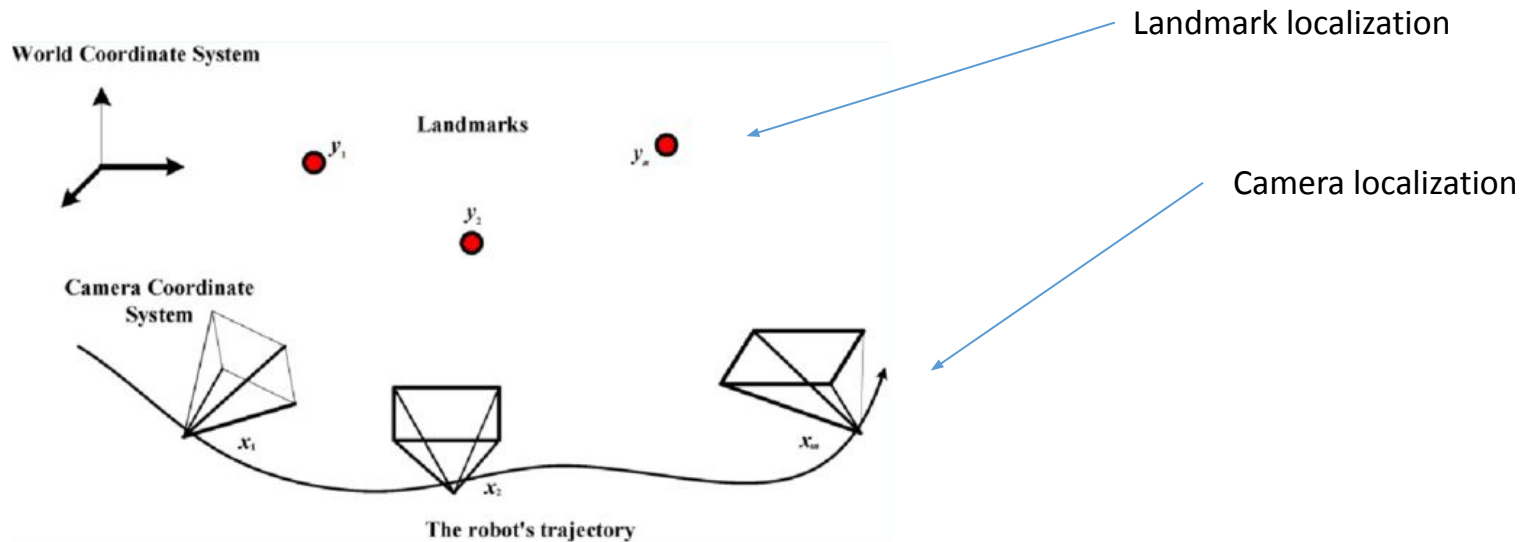
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# Introduction



# Introduction

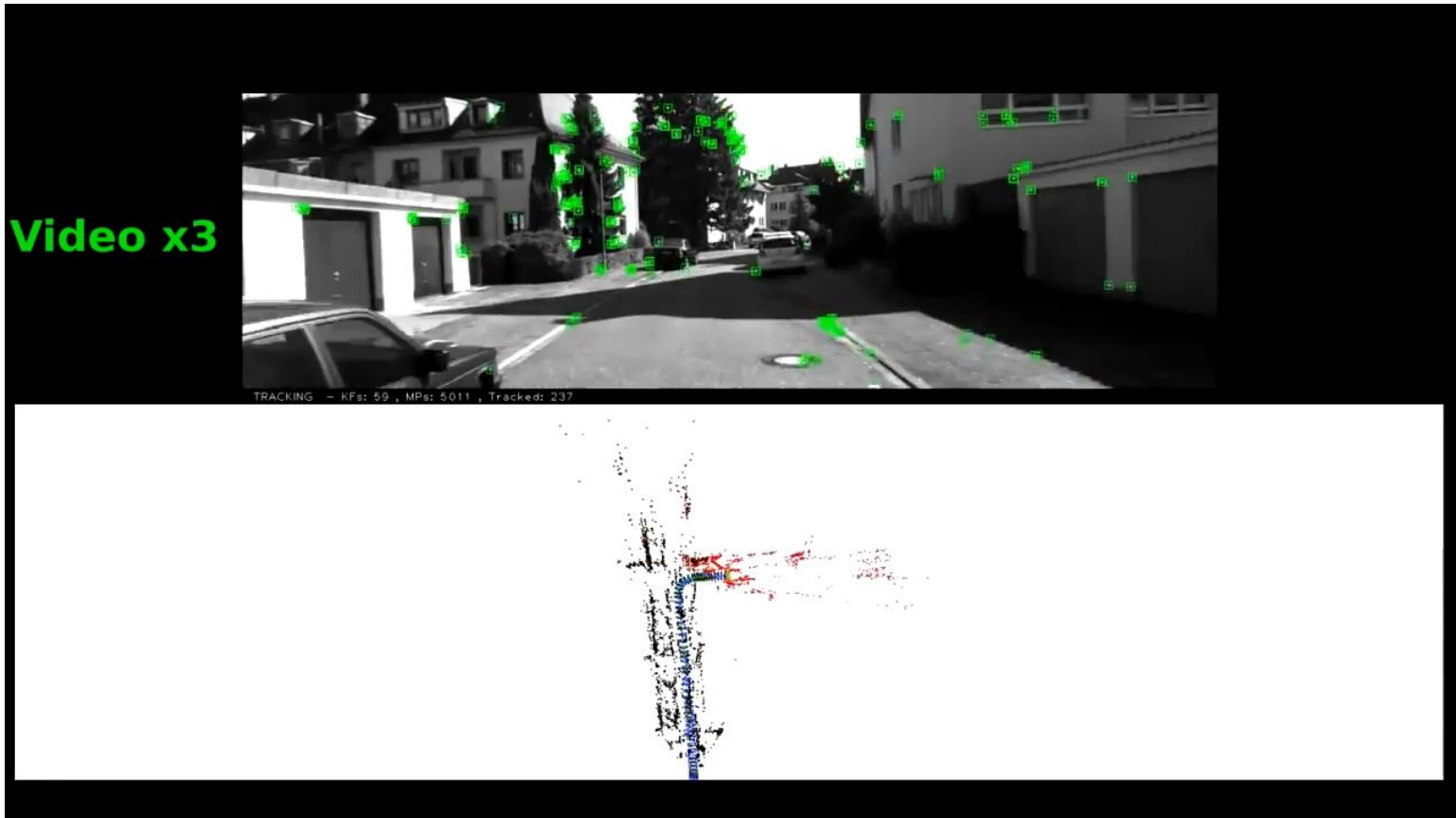
- SLAM – simultaneous localization and mapping:



# Introduction

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- SLAM – simultaneous localization and mapping:



# Introduction

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- SLAM – simultaneous localization and mapping:

## Status:

- Well researched (also today), many open-source libraries
- Partial success in real world autonomous systems
- Online performance

## Challenges:

- Accumulated error (Linearization, Measurement noise, miss identification)
- High complexity – not real-time. Uses much power.

# Motivation

## Qualitative spatial reasoning – easier, and good enough

Human navigation:

- Landmark Relative path
- Qualitative geometry
- Local accurate navigation for minimal effort



● landmarks

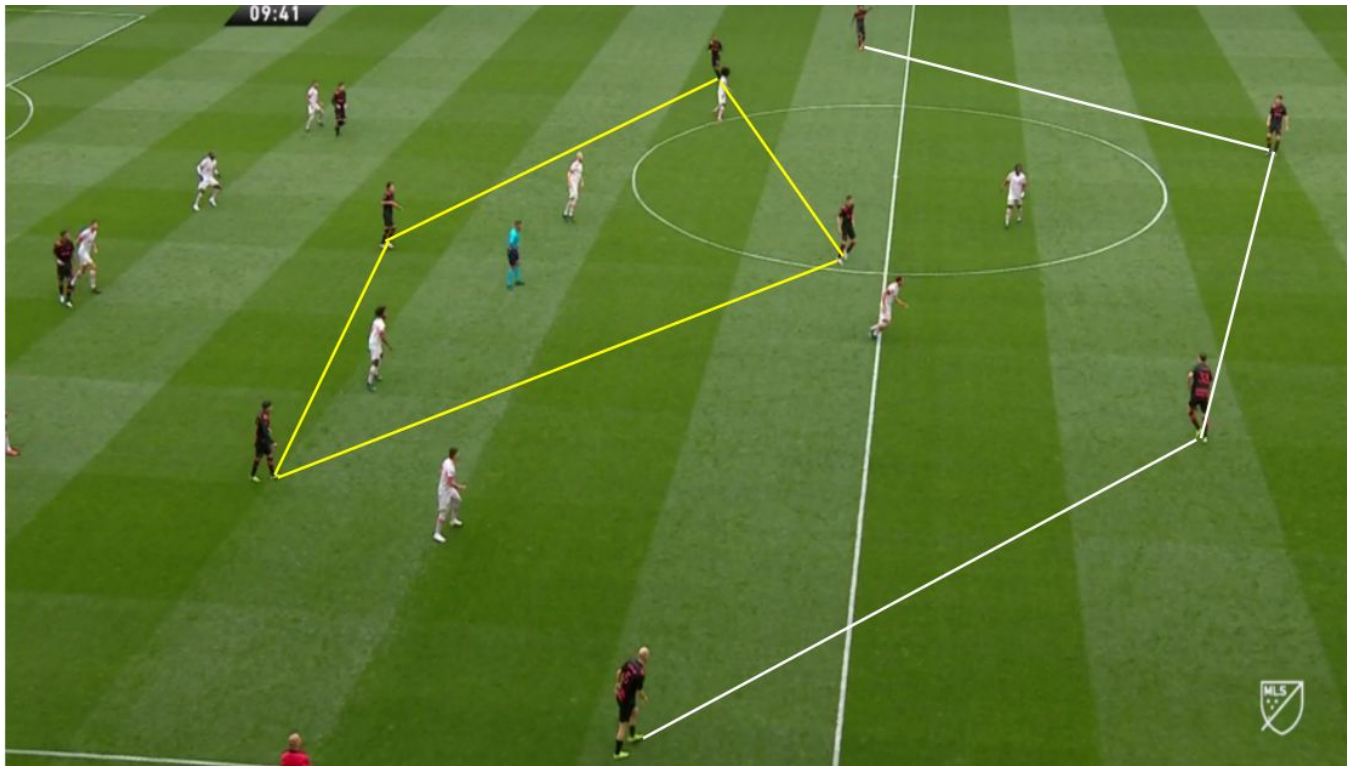
Relative path

Qualitative path Vs metric path

# Motivation

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**Qualitative spatial reasoning** – easier, and good enough



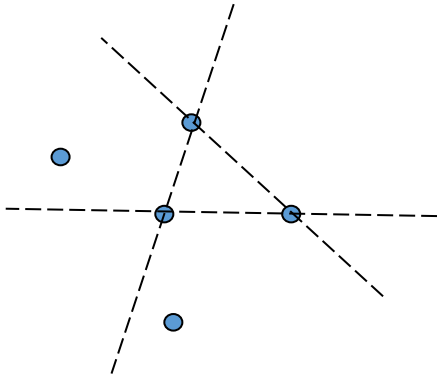


# Motivation

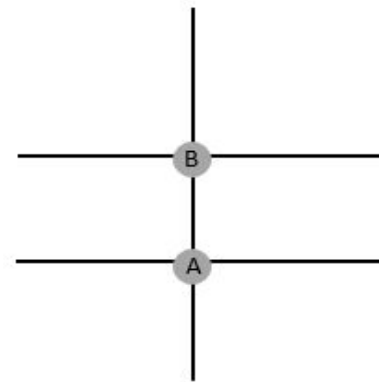
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**Qualitative spatial reasoning** – easier, and good enough

relative location  
(no global frame)



qualitative localization  
(qualitative geometric relations)



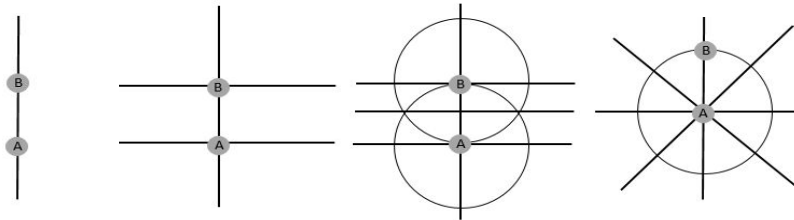
- ✓ less sensitive to noise
- ✓ No Long term error accumulation
- ✓ Low complexity

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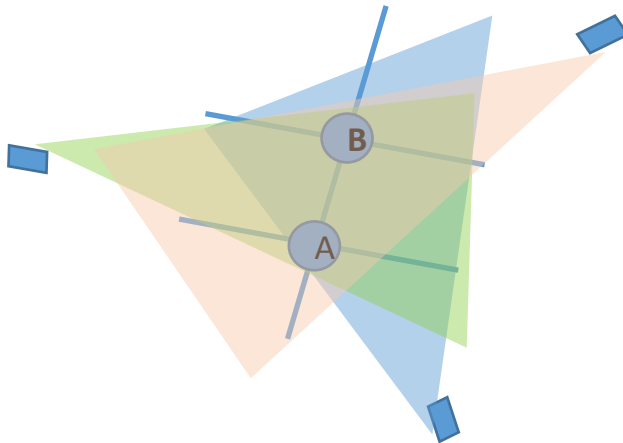
# Concept Overview

# Concept - Intuition

- Many small two-landmark relative frames of reference – no global frame
- Qualitative spatial partition instead of metric location



- Estimate state from landmark relative measurements



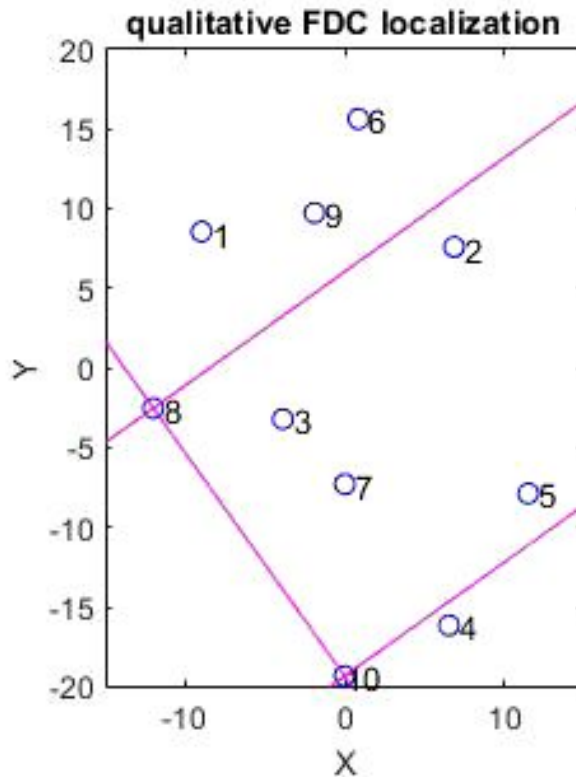
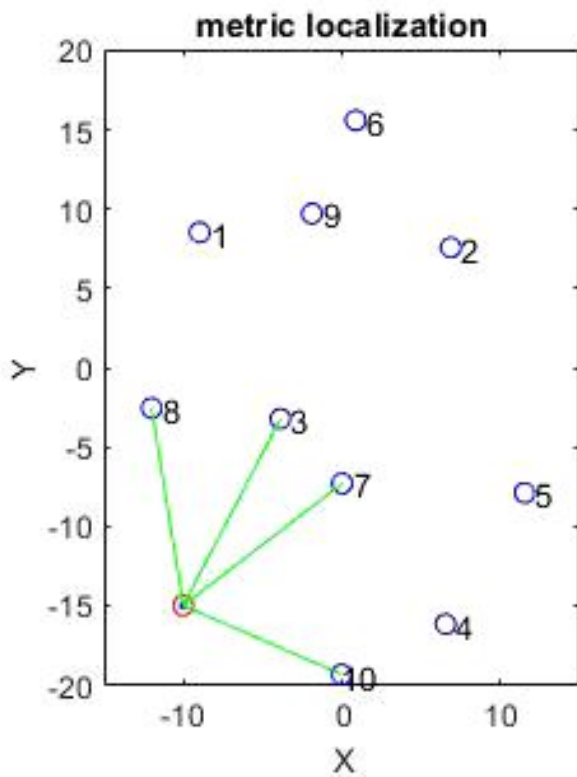
[1] Freksa 1992 . On the utilization of spatial structures for cognitively plausible and efficient reasoning.

[2] Schlieder 1993 Representing visible locations for qualitative navigation.

[3] Scivos 2004 The finest of its class: The natural pointbased ternary calculus Ir for qualitative spatial reasoning.

# Concept - Intuition

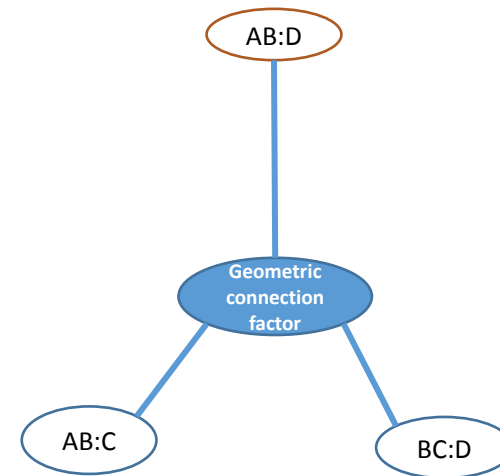
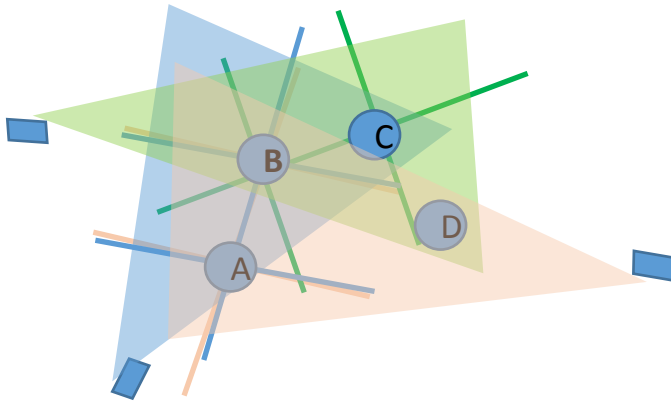
- Qualitative relational localization



# Concept - Intuition

- Qualitative relational mapping

Map -> connected graph of landmark triplets

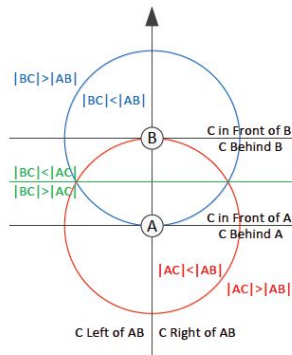


# QSR related work

- Spatial Qualitative Reasoning (QSR) approaches:

## McClelland,2013

- Typically assume data association is given
- Address mainly mapping, less localization
- Not probabilistic
- Extended double cross



(a) Region Boundaries

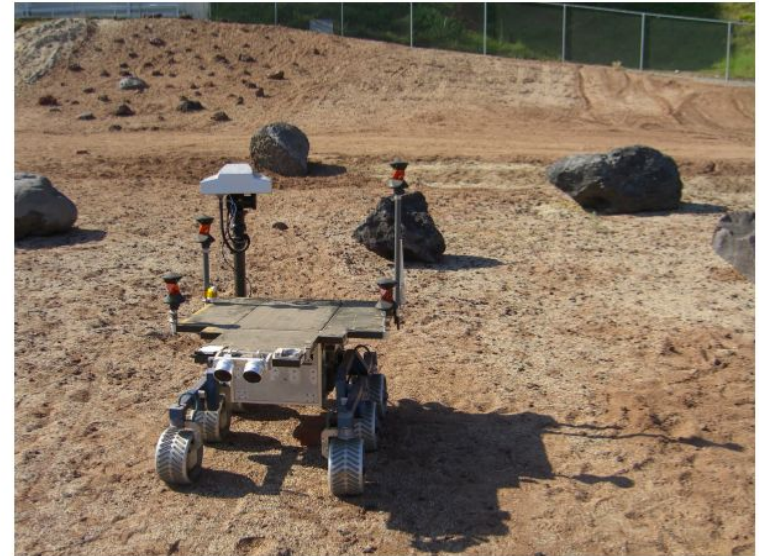


Image taken from McClelland,2013 [5]

McClelland,2013, Qualitative relational mapping for planetary rovers

# QSR related work

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- Spatial Qualitative Reasoning (QSR) approaches:

## Padgett 2016+2017

- Probabilistic
- Passive + Active planning
- Not a full SLAM framework

## Zilberman & Indelman 2022

- Composition in qualitative approaches (RA-L + ICRA 2022)
- Active planning (ongoing)

Padgett, 2016, Probabilistic qualitative mapping for robots  
Zilberman, 2022, Incorporating Compositions in Qualitative

# Contributions

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**Our approach:** – probabilistic time and spatial dependent QSR:

- Full probabilistic SLAM framework:
  - Localization
  - mapping
- Incorporating Motion model
- Factor graph propagation

**publications:**

- IROS 2020
- Journal paper(in progress)
- Open-source repo (in progress)



# Contributions

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## Benefits Vs previous QSR work:

- improve accuracy
- improve performance complexity
- estimate sets of landmarks that weren't seen together

## Benefits Vs metric SLAM:

- Low computation
- Robustness to noise / sensor quality
- Simpler computational process

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# Single Triplet Qualitative Estimation

# Our Approach – single triplet

Estimate each triplet separately:

- Landmark relative coordinate frames
- Small 3 landmark – multiple view SLAM problems

Fusing data:

- Build qualitative map and propagate data

# Formulation

2D navigation:

- Metric state

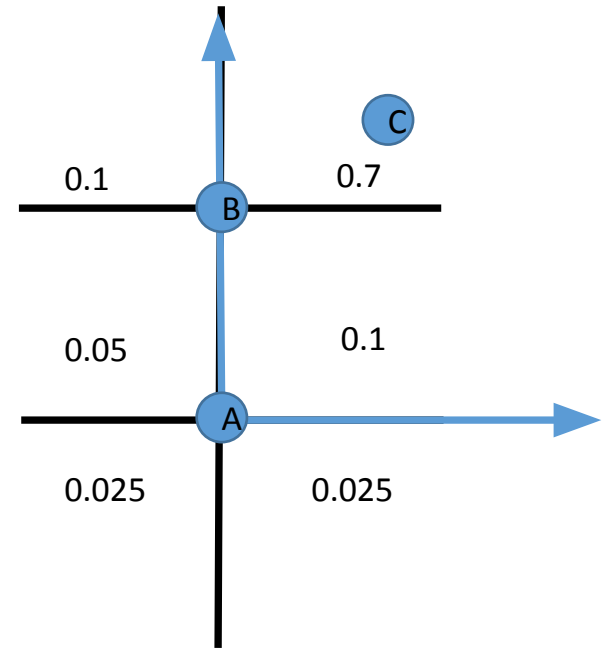
$X_{1:n}$  - Camera pose at times 1:n

$L^{AB:C}$  - Metric location of landmark C in AB frame

$H_n^{AB:C} = \{Z_1, \dots, Z_n\}$  - All A,B,C measurements up to time n

- qualitative state probability:  $\mathbb{P}(S^{AB:C} | H_n)$

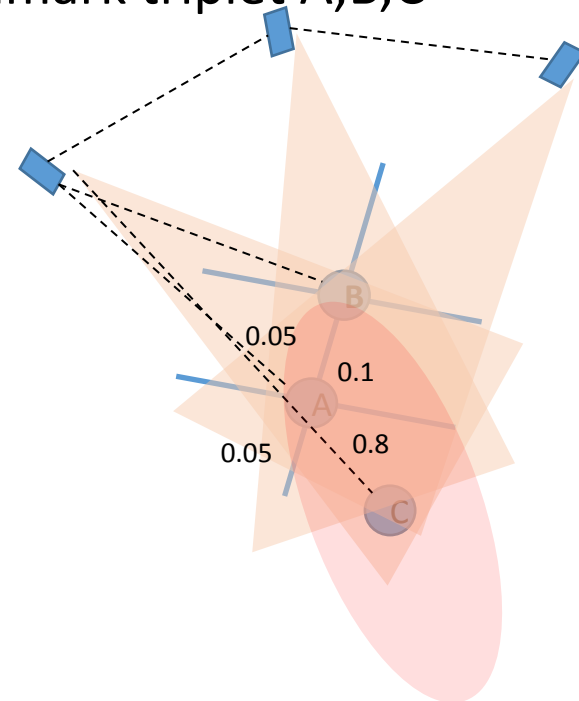
$S^{AB:C}$  - Qualitative state of landmark C in AB frame



# Our Approach – single triplet

## Estimation of a single landmark triplet:

- Measurements:
  - Azimuth to landmark triplet A,B:C
  - Heading between camera poses
- Metric SLAM For camera poses and landmark triplet A,B,C
  - Uses several separate camera poses
  - Incremental
- Integrate qualitative state probability



# Our Approach – single triplet

- Probabilistic formulation

$$\mathbb{P}(S^{AB:C} = i | H_n^{AB:C}) = \int \dots \int_{X_{1:n}, L^{AB:C}} \mathbb{P}(L^{AB:C} | S^{AB:C} = i) \mathbb{P}(X_{1:n}, L_C | H_n^{AB:C}) dL^{AB:C} dX_{1:n}$$

Integrate over  
metric states

Metric SLAM

For landmark triplet

A,B,C

$$\mathbb{P}(X_{1:n} L^{AB:C} | H_n) = \frac{\mathbb{P}(Z_1 | X_1, L^{AB:C}) \mathbb{P}(X_1 | L^{AB:C})}{\mathbb{P}(Z_1)} \prod_{i=2}^n \frac{1}{\zeta_i} \mathbb{P}(Z_i | X_i, L^{AB:C}) \mathbb{P}(X_i | X_{i-1}, a_{i-1})$$

Measurement model

Motion model

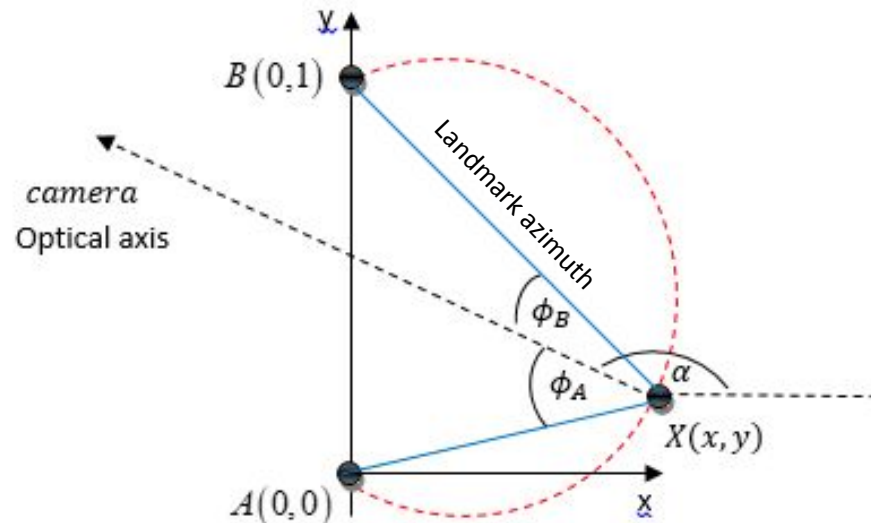
# Our Approach – single triplet

## Solving the 3 landmark SLAM problem:

- Non linear sample based SLAM approach
- Measurements
  - Measurements – azimuth to landmarks ( $\varphi$ )
  - Motion model – heading to next pose ( $\Psi$ )

## Single view:

Camera on locus circle

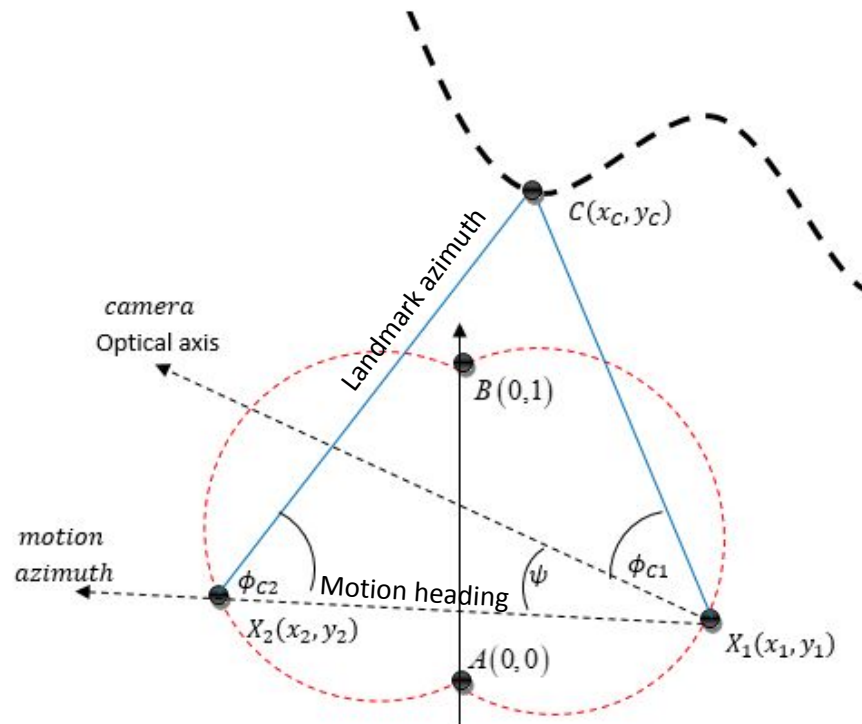


# Our Approach – single triplet

## Solving the 3 landmark SLAM problem:

### Two views:

- Cameras on locus circles
- Landmark C can be triangulated to a curve



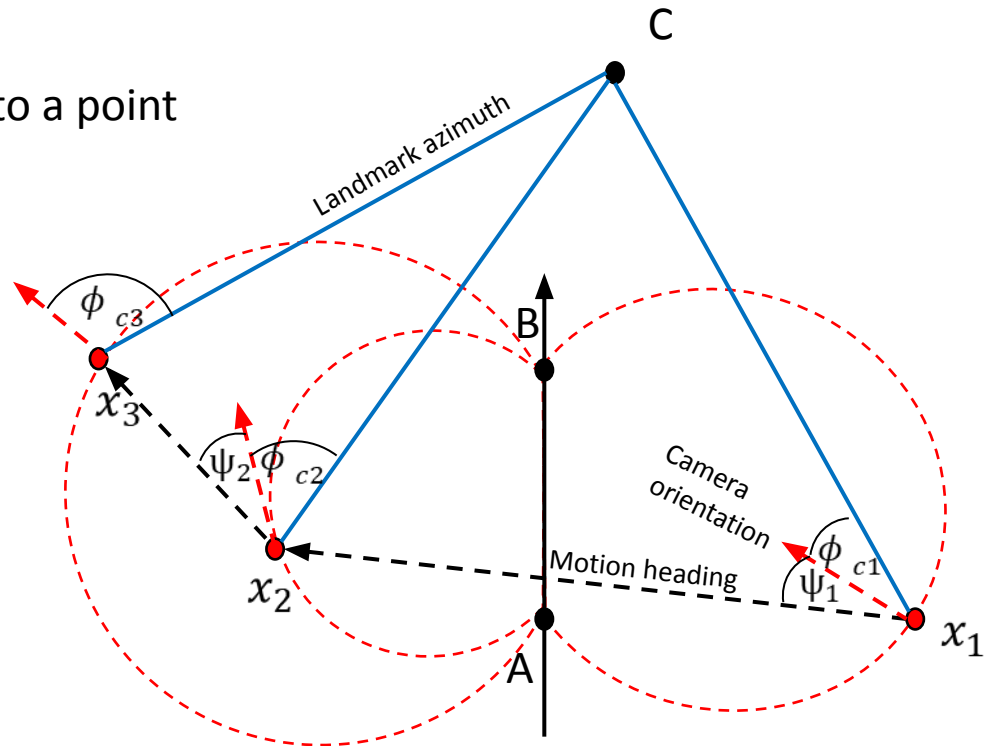


# Our Approach – single triplet

**Solving the 3 landmark SLAM problem:**

Three views or more:

- Cameras on locus circles
- Landmark C can be triangulated to a point



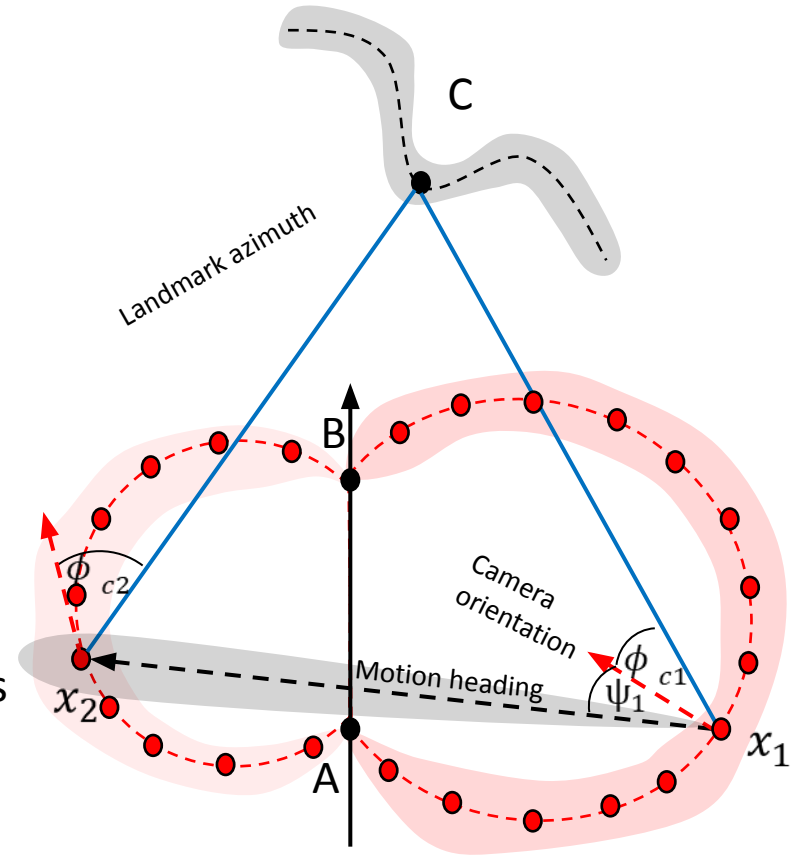
# Our Approach – single triplet

## Solving the 3 landmark SLAM problem:

- Non linear sample based SLAM approach
- A,B locus circle
- A,B azimuth measurements noise
- Motion heading noise

## Number of samples:

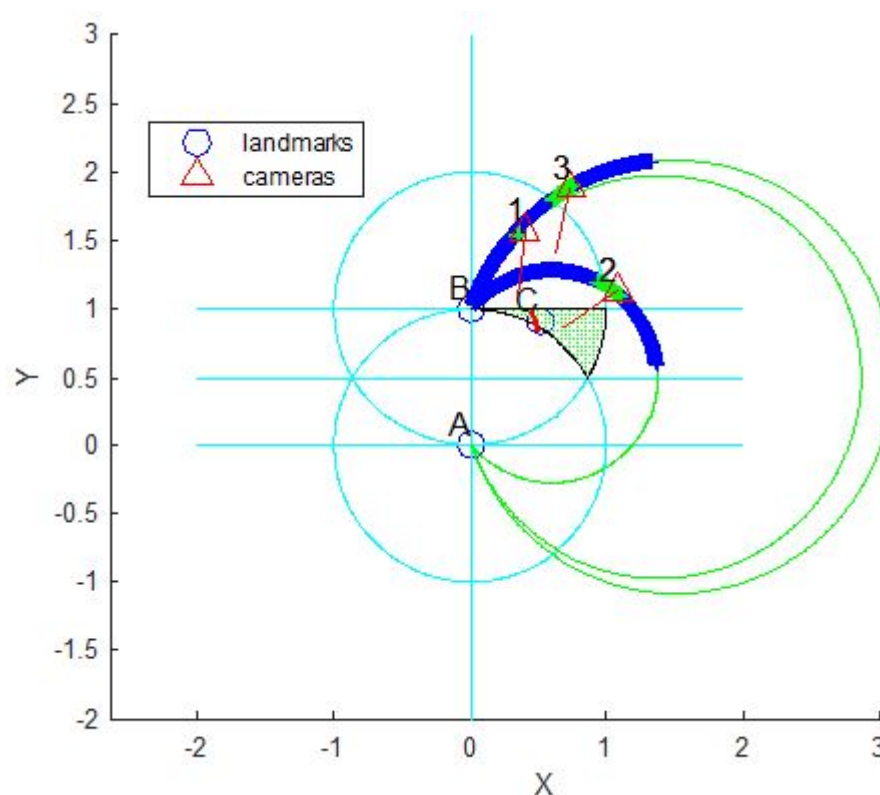
- Exponential in camera poses
- Practically reduces fast by consistency tests
- Very small for 3 camera poses or more
- Good for incremental algorithm



# Our Approach – single triplet

## Solving the 3 landmark SLAM problem:

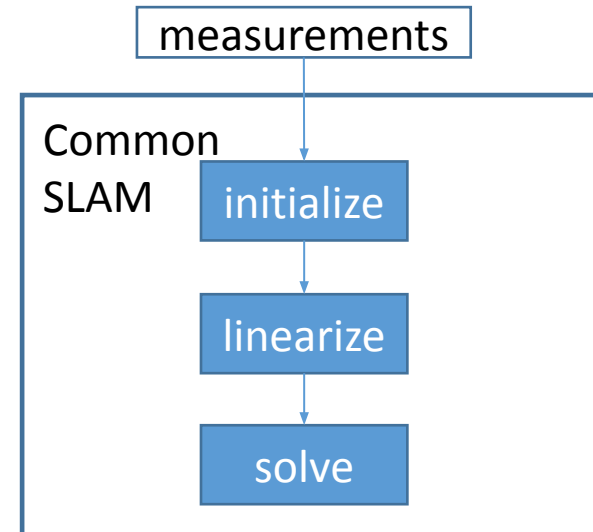
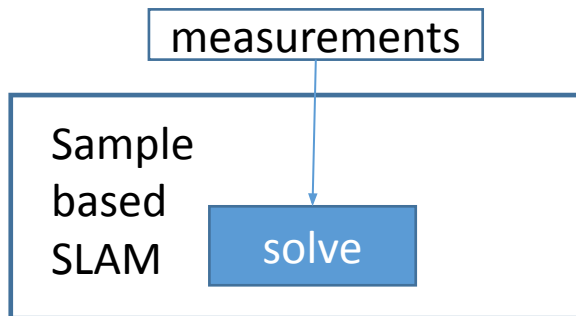
- 3 view Simulation example



# Our Approach – single triplet

## Our approach Vs regular SLAM

- Non linear
  - no linearization errors
  - No need for linearization
  - No initialization process
- General - variables are not assumed to be Gaussian



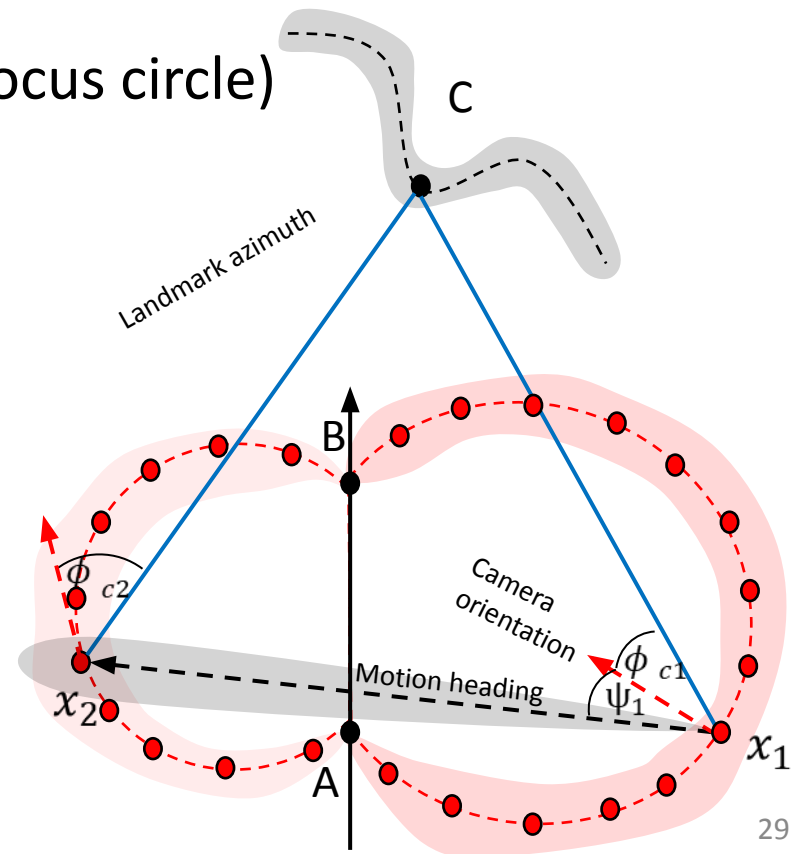
# Our Approach – single triplet

## Solving the 3 landmark SLAM problem – Fast approximation

Trying to capitalize on QSR coarse spatial partition

Fast solver variant:

- Sample only geometry (camera locus circle)
- No noise samples



# Our Approach – single triplet

## Single triplet results

single triplet EDC estimation results			
	baseline	ours	ours-fast
DMSE	0.39, 0.63, 0.71	<b>0, 0.16, 0.63</b>	0, 0.21, 0.62
geometric distance	0.28, 1.10, 2.30	<b>0, 0.25, 1.15</b>	0, 0.27, 1.16
Entropy	0.28, 0.66, 0.87	<b>0, 5e-3, 0.58</b>	0, 0.07, 0.64
time[sec]	26	18	<b>0.05</b>

**Motion Model makes a difference!**

### Metrics:

- DMSE – probabilistic correctness
- Geometric distance – geometric correctness
- Entropy – distribution steepness
- GT rating – the position of the GT qualitative state when states are ordered by probability (1 – most probable)

$$DMSE = \sqrt{\sum_{i=1}^m (\mathbb{P}(s_i) - \mathbb{P}(s_{GT}))^2}$$

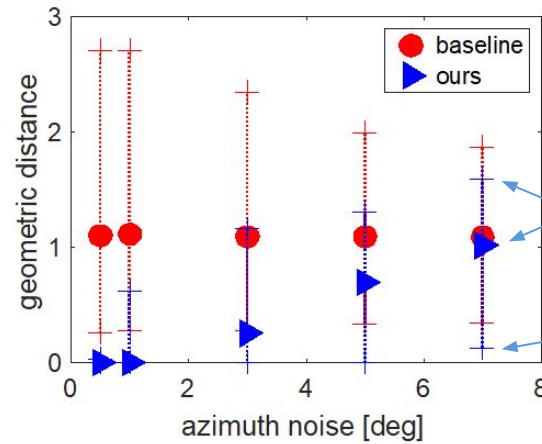
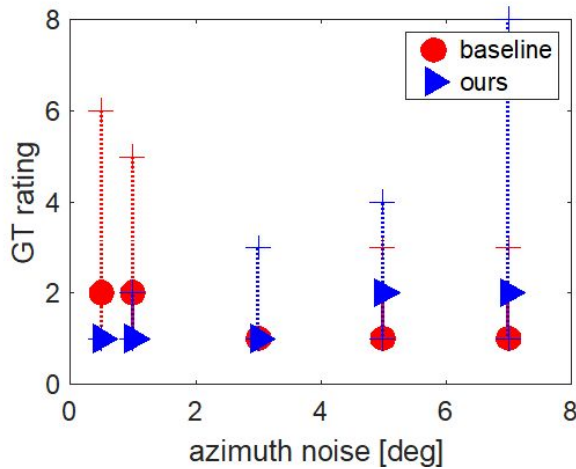
$$gmd = \sum_{i=1}^m \mathbb{P}(s_i) \|c_i - c_{GT}\|_2$$

$$e = - \sum_{i=1}^m \mathbb{P}(s_i) \log(\mathbb{P}(s_i))$$

Baseline = padget

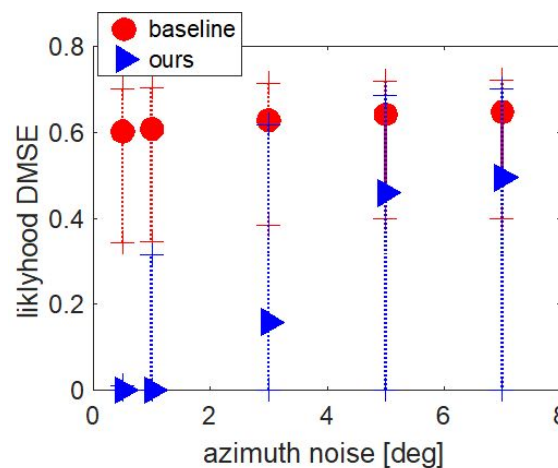
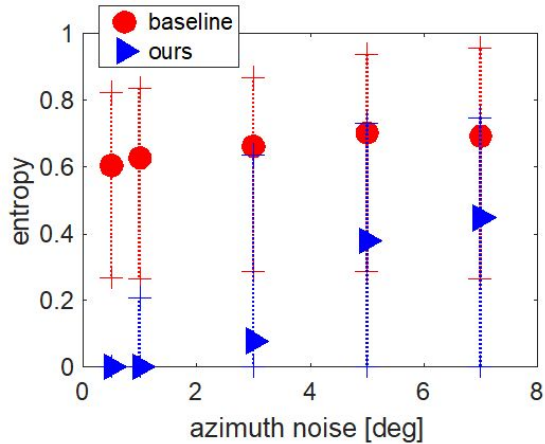
# Results

## Single triplet results



median

25/75 percentiles



**Motion Model makes a difference!**

# Results

## MRCLAM dataset

- Autonomous Space Robotics Lab (ASRL) at the University of Toronto
- Cylindrical landmarks
- Occlusions
- Sensors
  - Camera azimuth measurements
  - Odometry



MRCLAM dataset EDC estimation		
	ours-fast	uniform
DMSE	0.03, 0.45, 0.69	0.97
gmd	5e-3, 0.27, 0.71	2.2
Entropy	4e-3, 0.38, 0.69	3
GT rating	1, 1, 2	-

[Autonomous Space Robotics Lab: MR.CLAM Dataset \(utoronto.ca\)](http://utoronto.ca)

median      25/75 percentiles



# Our Approach – single triplet

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## Conclusions

- Adding motion model:
  - Better performance
  - Better complexity (feasibility tests reduce samples faster)
- fast approximation
  - Much faster
  - Performance very close to full algorithm
  - uses qualitative inherent course spatial partition
- General performance
  - Up-to azimuth measurement noise of 3deg – very close to GT
- (Published in IROS 2020)

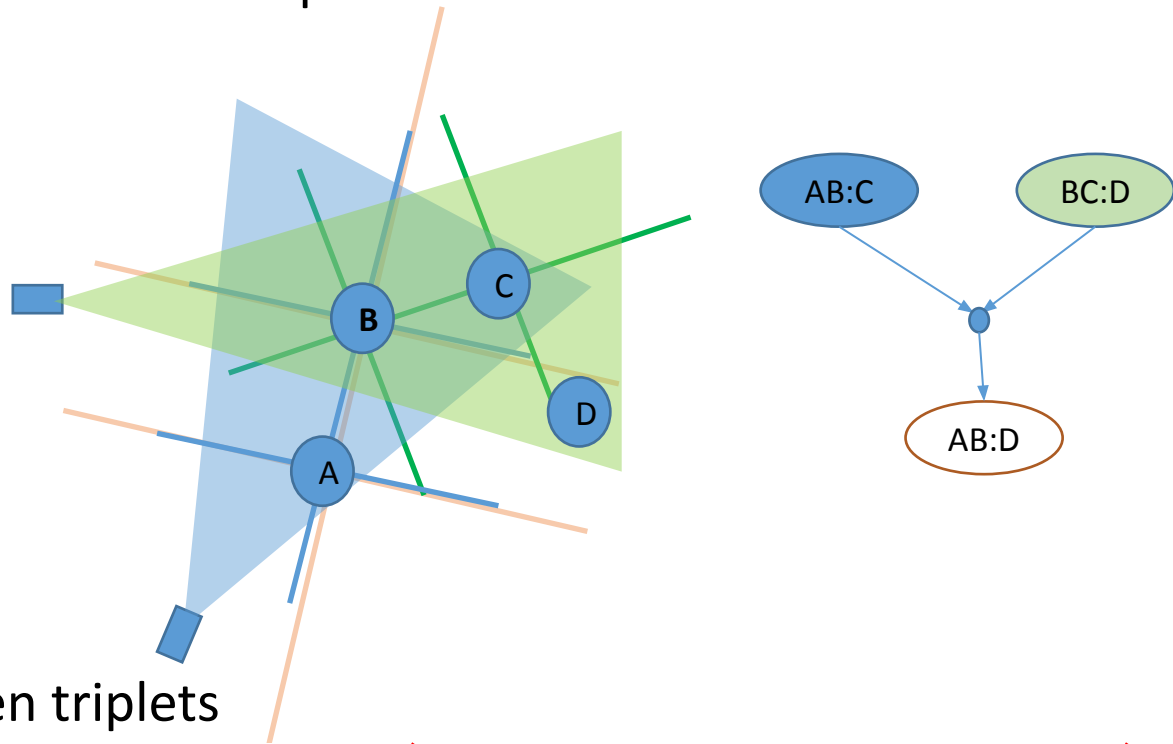
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# Qualitative Composition

# Our Approach - composition

## Novel probabilistic Composition:

- Propagate data between triplets



- Estimate unseen triplets

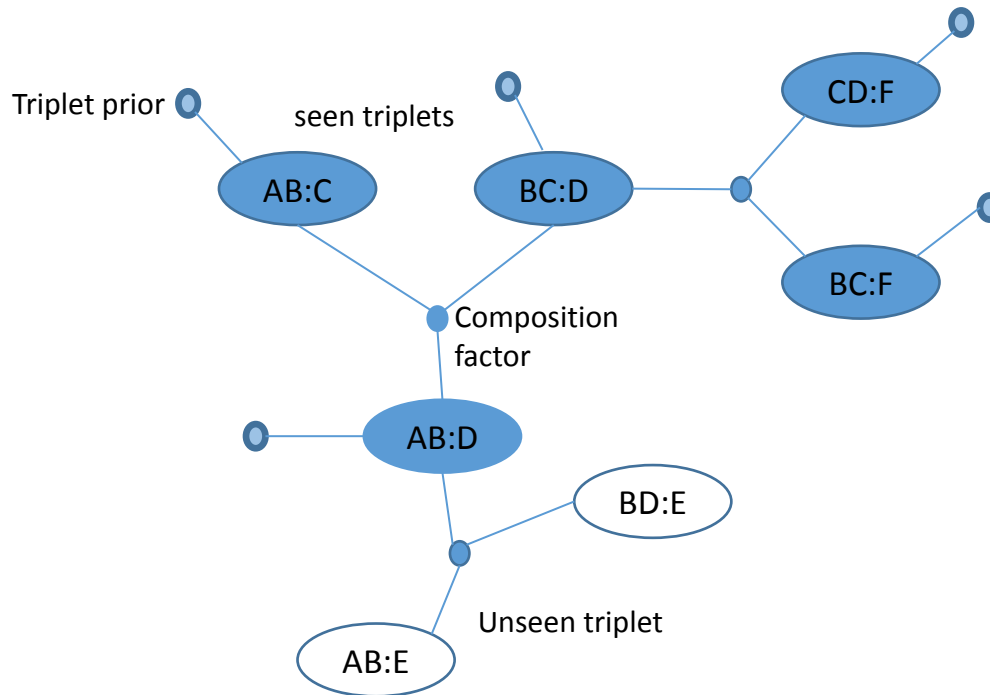
- Improve estimation

$$\mathbb{P}(S^t | \cancel{H^t}, H^{p1}, H^{p2}) = \sum_{s_i^{p1}} \sum_{s_j^t} p(S^t, s_i^{p1}, s_j^{p2} | \cancel{H^t}, H^{p1}, H^{p2})$$

# Concept - Intuition

## Composition:

Qualitative map propagation by composition factor graph

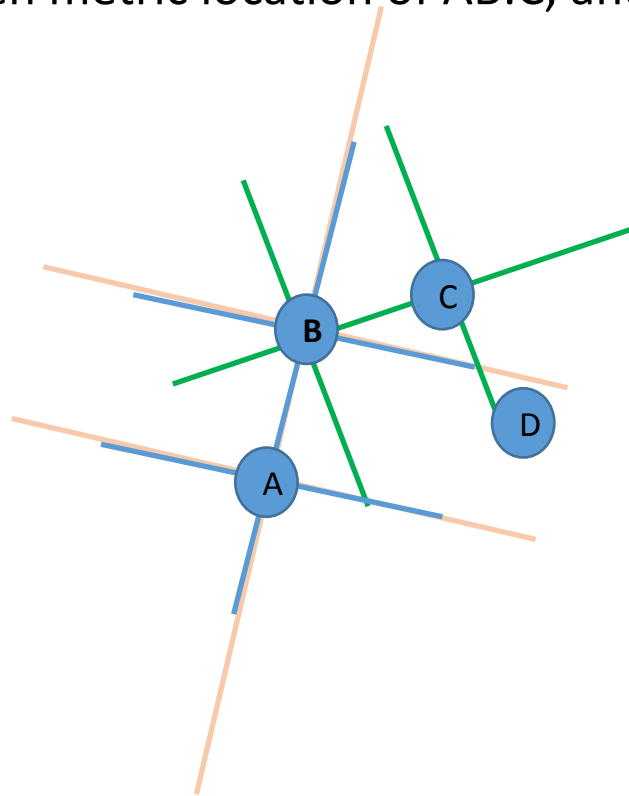


# Our Approach - composition

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## Composition:

- Calculate  $AB:D$  given metric location of  $AB:C$ , and  $BC:D$



# Our Approach - composition

## Composition:

- Composition factor – pure qualitative approximation

- Remember only qualitative state
- Forget metric data

$$\mathbb{P}(L|s_i, H) \approx \mathbb{P}(L|s_i)$$

## Formulation:

AB:C = p1

BC:D = p2

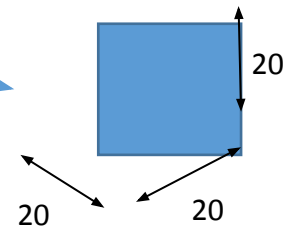
AN:D = t

$$\mathbb{P}(S^t | H^{p1}, H^{p2}) \approx \sum_{s_i^{p1}} \sum_{s_j^t} \mathbb{P}(s_i^{p1} | H^{p1}) \mathbb{P}(s_j^t | H^{p2}).$$

$$\iint_{L^{p1} \in s_i^{p1}, L^{p2} \in s_j^t} \mathbb{P}(S^t | L^{p1}, L^{p2}) dL^{p1} dL^{p2}.$$

Calculate offline  
Same for all factors

Single triplet estimation



# Our Approach - composition

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## Composition:

- Composition factor pure qualitative approximation:
  - Fast graph propagation
  - Very efficient in HW accelerators
  - Low memory consumption

- (Published in IROS 2020)

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# Factor Graph Propagation

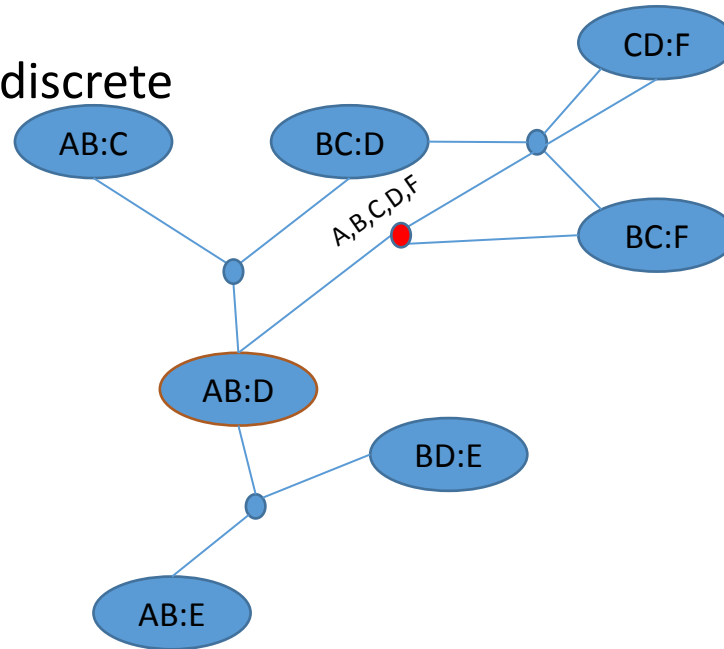


# Our Approach - composition

## Factor graph propagation algorithm:

Accurate method :

- Elimination - trinary factors  $\rightarrow$  multiple node factor
- Calculation is exponential in number of nodes
- Runtime Not feasible
- Implemented in GTSAM-discrete

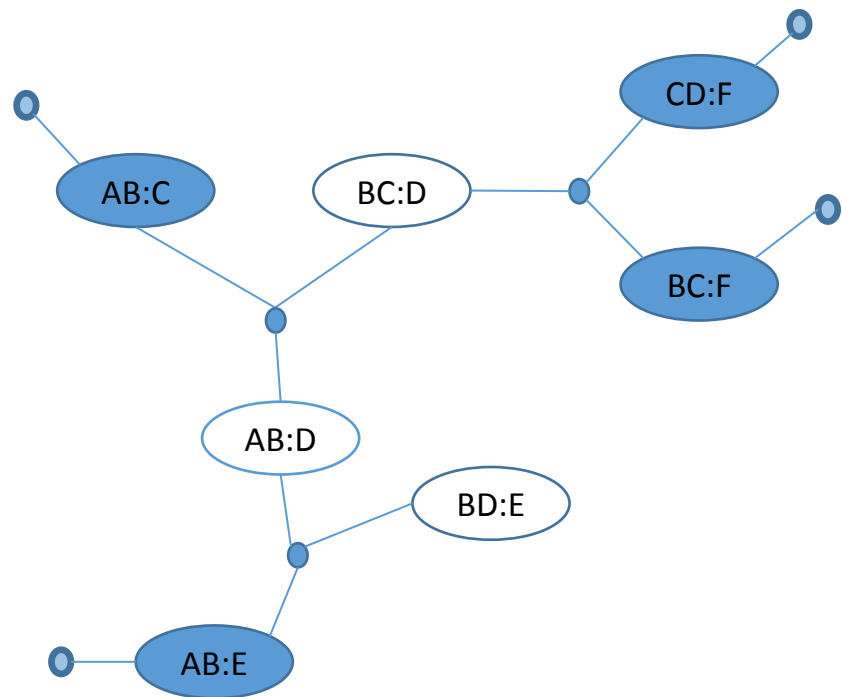


# Our Approach - composition

## Factor graph propagation:

Fast Approximated algorithm:

- Greedy – one most informative step
- Single best path
- One pass over each node



# Our Approach - composition

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## Factor graph propagation:

### Information score (ISC):

A metric to measure how informative is the probability distribution for a specific landmark triplet qualitative state:

- $0 < \text{ISC} < 1$
- Higher is better (more informative)

$$\text{ISC} = \frac{H_{\max} - H_n}{H_{\max} - H_{\min}}$$

$H_n$  = node entropy

$H_{\max}$  = uniform (max) entropy

$H_{\min}$  = perfect (min) entropy

# Our Approach - composition

## Factor graph propagation:

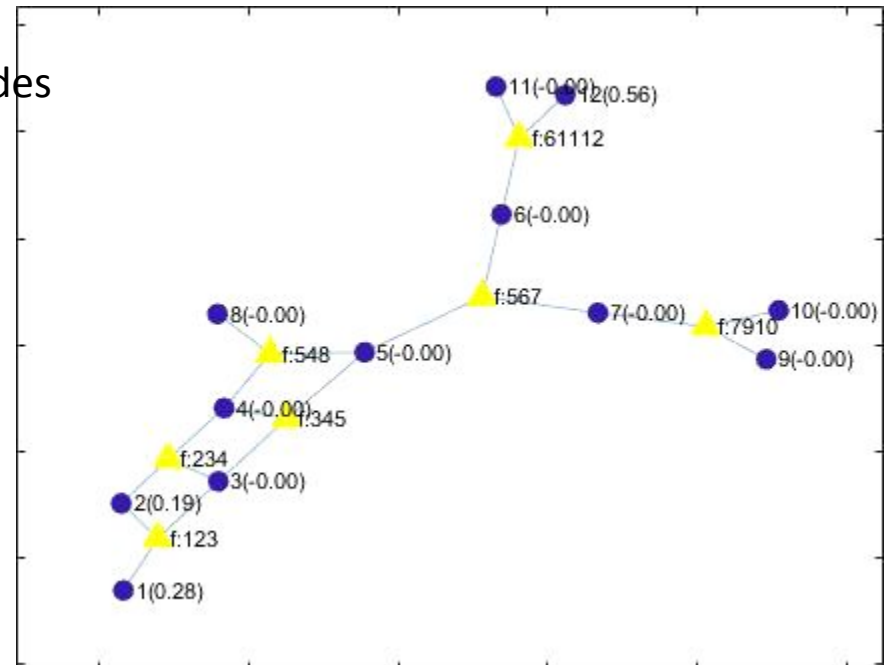
Fast Approximated algorithm:

- Observed nodes = Source nodes
- Loop:
  - Propagate any factor that has 1 or 2 'done' nodes
  - Calculate ISC for all newly calculated nodes
  - Keep best ISC node, and mark as 'done'
- Break when no factor has 1 or 2 'done' nodes

Example:

Fast Approximated algorithm:

- Node text: id (ISC)
- Priors on nodes 1,2,12
- Update order: 6,5,3,4,11,7,9,10,8



# Our Approach - composition

## Factor graph propagation:

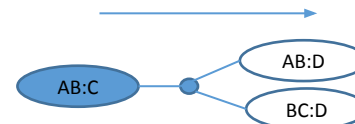
### Composition level (CL):

- A tool to study composition behavior in correlation to:
  - Graph topology
  - prior information
- Propagation in graph:
  - CL = ISC for observed nodes
  - Same graph propagation algorithm
  - ISC decay Factor:

$$ISC_{AB:D} = (1 - \alpha^2) \frac{ISC_{AB:C} + ISC_{BC:D}}{2}$$



$$ISC_{AB:D} = ISC_{BC:D} = (1 - \alpha) ISC_{AB:C}$$



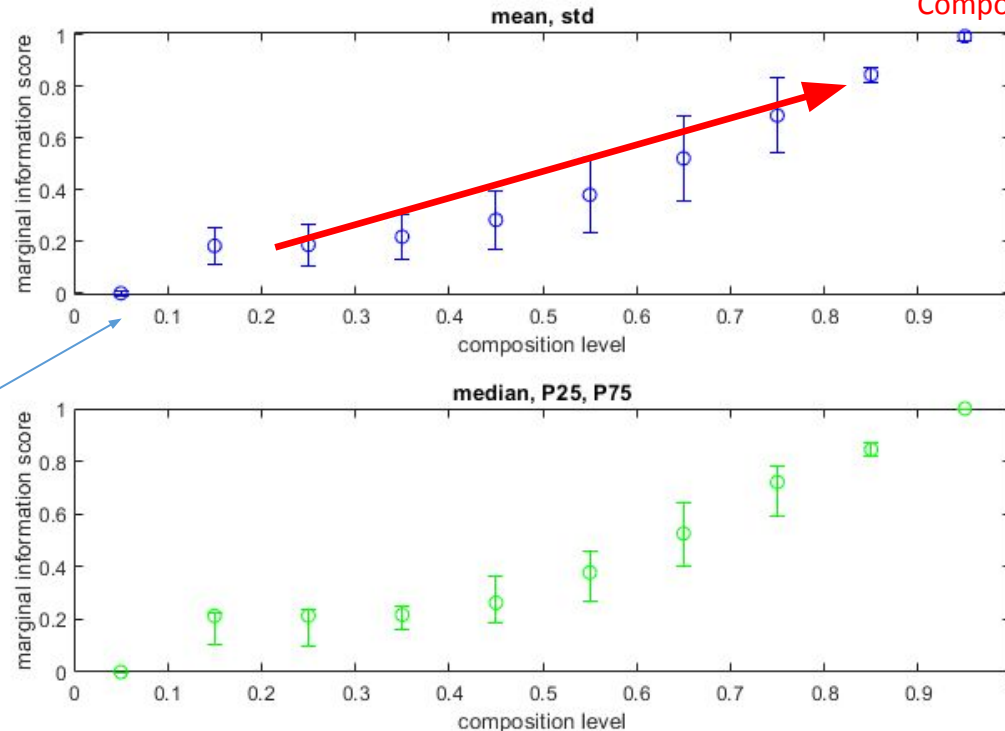
$\alpha=0.5$  - information decay factor

# Our Approach - composition

## Composition results:

composition level Vs marginal information score

actual composition informative  
↕  
Composition level informative



Marginal ICS  
Calculated by actual  
composition

ICS Calculated by  
"composition level"  
propagation

# Our Approach – composition

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## Conclusions

- Composition propagates significant information
- Information propagated is correlated to graph topography (composition level)
- Might be practical for:
  - Estimating unseen nodes (for planning / landmark recognition)
  - Improving existing estimation
  
- (will be published soon)

---

# Conclusions



# Conclusions

- Good Performance
  - Low measurement noise -> almost perfect results (up to 3°)
  - High measurement noise -> Better performance than state of the art (up to 7°)
- Low complexity (practical for low compute systems)
  - Good performance for fast approximation
- Good for fast active planning
  - Composition is fast and informative

