# Incremental Sparse GP Regression for Continuous-time Trajectory Estimation & Mapping

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### Introduction

- SLAM fundamental problem in robotics
- Challenges include long term autonomy how to operate online as more data is accumulated?
- Progress in recent years:
  - Sparsity-aware smoothing approaches (e.g. g2o, iSAM)
  - Incremental smoothing (iSAM2.0):
    - Identify and update only relevant part of the state
    - Fast, incremental
    - But discrete time formulation





# **Continuous-Time SLAM via GP Regression**

- Gaussian Processes have been recently incorporated within SLAM [Tong et al., IJRR '13]
  - Continuous time representation
  - Provide the ability to interpolate states while still using all measurements
  - Can be realized efficiently by exploiting sparsity of the inverse kernels
  - Naturally handles asynchronous measurements
- Key drawback: batch optimization





- We combine the benefits of Incremental smoothing (iSAM2.0) with the benefits of Continuous-time GP-SLAM
- This leads to:
  - State interpolation yields a major reduction in running time
  - Minor impact on accuracy



$$f(oldsymbol{ heta}) = \prod_i f_i(oldsymbol{ heta}_i)$$
  
 $f_j(oldsymbol{ heta}_j) \propto \exp\{-rac{1}{2} \|\mathbf{h}_k(oldsymbol{ heta}_k + \deltaoldsymbol{ heta}_k) - \mathbf{y}_k\|_{\mathbf{R}_k}^2\}$   
 $f_j(oldsymbol{ heta}_j) \propto \exp\{-rac{1}{2} \|\mathbf{h}_k(ar{\mathbf{x}}( au)) + \mathbf{H}_k \mathcal{K}( au) \mathcal{K}^{-1} \delta \mathbf{x} - \mathbf{y}_k\|_{\mathbf{R}_k}^2\}$ 





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Algorithm 2 Incremental Sparse GP Regression via the Bayes tree with Gaussian Process Priors (BTGP)

Assign the sets of *affected* variables, variables involved in *new factors*, and *relinearized* variables to empty sets,  $\theta_{aff} := \theta_{nf} := \theta_{rl} := \emptyset$ . while collecting data **do** 

1. Collect measurements, store as new factors. Set  $\boldsymbol{\theta}_{nf}$  to the set of variables involved in the *new factors*. If  $\mathbf{x}(\tau) \in \boldsymbol{\theta}_{nf}$  is a missing state, replace it by newby states (Eq. 19); If  $\mathbf{x}(\tau) \in \boldsymbol{\theta}_{nf}$  is a new state to estimate, a GP prior (Eq. 23) is stored, and  $\boldsymbol{\theta}_{nf} := \boldsymbol{\theta}_{nf} \cup \mathbf{x}_{i-1}$ .

2. For all  $\theta_i \in \theta_{aff} = \theta_{rl} \cup \theta_{nf}$ , remove the corresponding cliques and ascendants up to the root of the Bayes tree.

3. Relinearize the factors, using interpolation when missing states are involved (Eq. 30).

- 4. Add the cached marginal factors from the orphaned sub-trees of the removed cliques.
- 5. Eliminate the graph by a new ordering into a Bayes tree, attach back orphaned sub-trees.
- 6. Partially update estimate from the root, stop when updates are below a threshold.

7. Collect variables, for which the difference between the current estimate and the previous linearization point is above a threshold, into  $\theta_{rl}$ .

#### end while

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