# Incorporating Compositions in Qualitative Approaches

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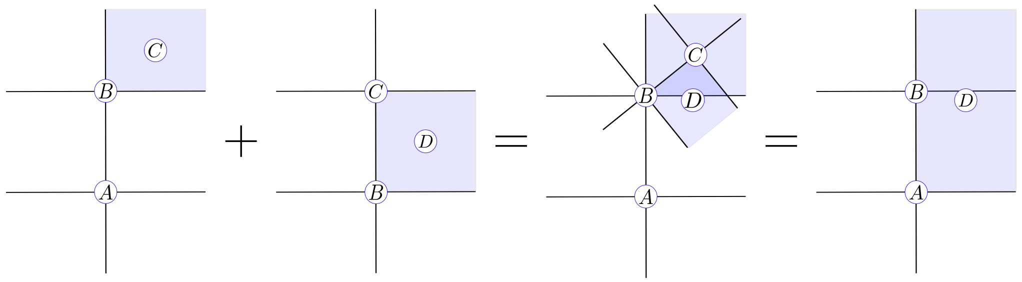
## Motivation

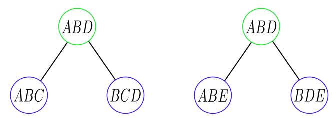
Qualitative approaches represent the environment through corase relationships between triplets of landmarks in independent local coordinate systems. An essential component in these approaches is the composition operator, enabling spatial information propagation between different triplets to infer new ones.



Compose(AB:C, BC:D) = AB:D

## Issues with compositions' calculi

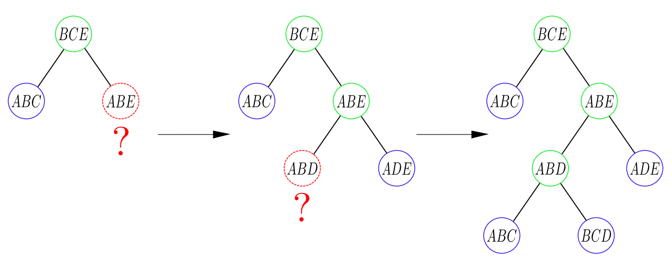
**Ambiguity**:
There are multiple ways to compose a target triplet.



(Illustration using Composition-Trees representation)

**Recursion**:
What if a source triplet required to compose a target one is not available?

Sourse set: {$ABC$, $BCD$, $ADE$}



## Our contributions

We address two questions arising from the above.

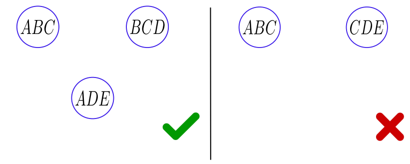
Given an initial set of source triplets:

1. What new triplets can be composed?
   We derive a sufficient topological condition attributed to the source set, whose existence ensures the ability to compose an entire space of triplets.
2. What is the optimal way to compose a target triplet?
   We develop a novel algorithm that finds the optimal way to compose a target triplet under an optimality criterion defined by the user.

## Composability as a sufficient condition

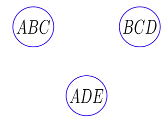
**Definition**: Let $\mathcal{T}$ be a set of triplets and let $\mathcal{L}$ be a *Landmark Space*. We say that $\mathcal{T}$ is *Composable* under $\mathcal{L}$, if $\mathcal{L} \subseteq \mathcal{L}(\mathcal{T})$, and one of the following holds:

1. $|\mathcal{T}| = 1$
2. $|\mathcal{T}| > 1$, and there is a cut $C = (\mathcal{T}_L, \mathcal{T}_R)$ of $\mathcal{T}$, s.t. $\mathcal{T}_L, \mathcal{T}_R$ are *Composable* under $\mathcal{L}(\mathcal{T}_L)$ and $\mathcal{L}(\mathcal{T}_R)$, respectively, and $|\mathcal{L}(\mathcal{T}_L) \cap \mathcal{L}(\mathcal{T}_R)| \geq 2$.



Landmark Space: $\{A,B,C,D,E\}$

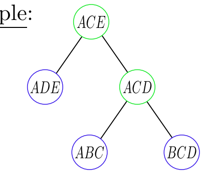
**Theorem**: Let $\mathcal{T}$ be a *Composable* set of triplets under the *Landmark Space* $\mathcal{L}$. Then any triplet $\tau \subseteq \mathcal{L}$ can be composed based on triplets from $\mathcal{T}$.



Composable under $\mathcal{L}(\mathcal{T})=\{A,B,C,D,E\}$

$\xrightarrow{\text{Thm.}}$ **Any** triplet $\tau \subseteq \mathcal{L}(\mathcal{T})$ can be composed!

For example: $ACE$



## Optimal Composition Sequence

We suggest a graph-based algorithm to address the following problem:

$$T^* = \underset{T \in \mathbb{T}_{\tau_0}}{\operatorname{argmin}} \sum_{\tau \in T} C(\tau)$$

The cost function takes the following form:

$$C(\tau) = \begin{cases} C^{\text{Source}}(\tau) & \text{, if } \tau \text{ is a source triplet} \\ C^{\text{Comp}}(\tau_L, \tau_R) & \text{, if } \tau \text{ is composed directly using } \tau_L, \tau_R \end{cases}$$

For example, the following cost accumulates the number of composition operations required to form a target triplet:

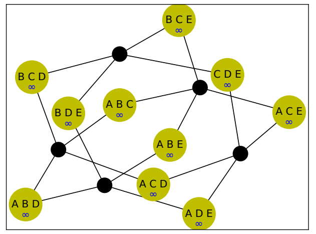
$$C(\tau) = \begin{cases} 0 & \text{, if } \tau \text{ is a source triplet} \\ 1 & \text{, if } \tau \text{ is composed directly using } \tau_L, \tau_R \end{cases}$$

## Running Example

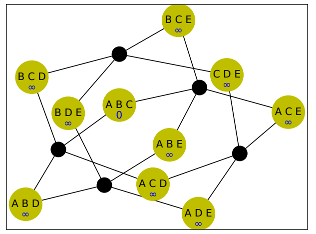
Landmark Space: $\{A,B,C,D,E\}$

Sourse set: {$ABC$, $BCD$, $ADE$}

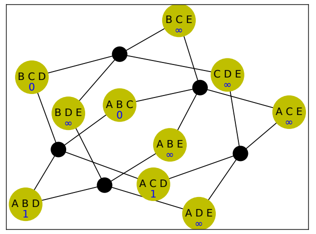
(1) Initialization



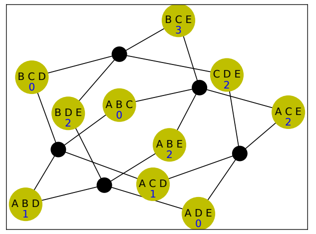
(2) Update $ABC$



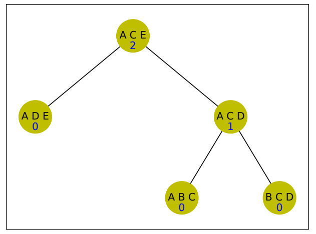
(3) Update $BCD$

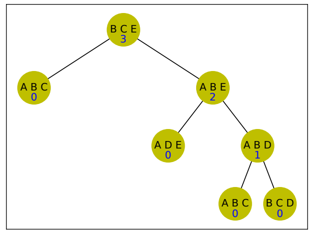


(4) Update $ADE$



Output Examples:





## Summary

We suggested two main contributions regarding compositions' calculi:

1. Composability - a sufficient condition to compose triplets.
2. A first-of-its-kind algorithm that finds the optimal composition sequence to create a target triplet

**We encourage incorporating our algorithm as a component in future qualitative approaches for a veriety of tasks, such as localization, mapping, and active planning.**