Qualitative Belief Space Planning via Compositions

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***Abstract*— Planning under uncertainty is a fundamental problem in robotics. Classical approaches rely on a metrical representation of the world and robot's states to infer the next course of action. While these approaches are considered accurate, they are often susceptible to metric errors and tend to be costly regarding memory and time consumption. However, in some cases, relying on qualitative geometric information alone is sufficient. Hence, the issues described above become an unnecessary burden. This work presents a novel qualitative Belief Space Planning (BSP) approach, highly suitable for platforms with low-cost sensors and particularly appealing in sparse environment scenarios. Our algorithm generalizes its predecessors by avoiding any deterministic assumptions. Moreover, it smoothly incorporates spatial information propagation techniques, known as compositions. We demonstrate our algorithm in simulations and the advantage of using compositions in particular.**

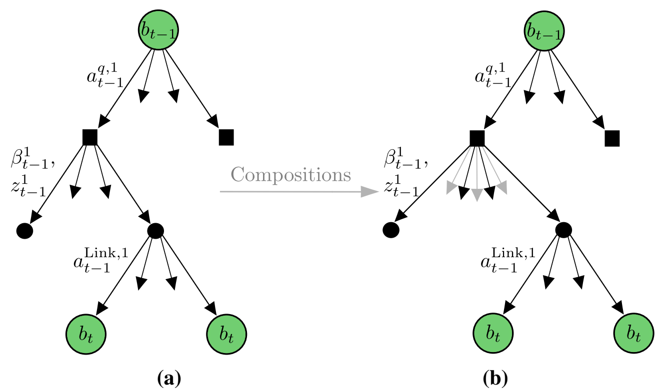

Fig. 1: (a) A belief tree, suitable for the qualitative framework, is illustrated. Each construction step between two consecutive times $t-1$ and t required updating the belief according to Eq. (19). (b) The same tree is illustrated, this time with compositions incorporated within the planning process. As shown, we can consider a wider data association space via compositions, thereby improving planning performance.

I. Introduction

Planning under uncertainty is a vital capability in various robotics applications, including autonomous car navigation, indoor navigation, surveillance, and medical devices. The planning problem is concerned with finding an optimal course of action to be carried out by some agent to achieve its goals. In the past three decades, extensive research efforts have been investigated in finding diverse solutions to this problem, among which the Belief Space Planning (BSP), and active Simultaneous Localization And Mapping (active SLAM) are particularly notable (see, e.g. [1], [2]).

Yet, some challenges remain. In the absence of high-quality sensors, accurate robot methods based on the above-mentioned solutions encounter significant difficulties. These methods (e.g., [3], [4]) are noise-sensitive and tend to accumulate errors as they rely on metrical estimates of map and robot's trajectory. Thus, noisy measurements can significantly impair their accuracy, cause undesirable drifts, and eventually lead to divergence if the loop-closer fails. Another concern that arises from the metric approaches is the need to maintain a dense, potentially large map representation, which often comes at the cost of substantial computational and memory resources. While the aforementioned is occasionally essential, it might be unnecessary in some cases, and therefore a burden. For instance, in long-term autonomous navigation missions, the robot is often required to travel long distances, so relying on a small number of critical landmarks along the way might be a good enough strategy. In other scenarios, the nature of the surrounding landscape is relatively monotonic and poor (for example, a desert or snowy terrain), and methods that depend on finding dense features are prone to fail.

Qualitative approaches have the potential to facilitate the issues mentioned above. In these methods, in contrast to the metrical, the environment and robot's poses are tracked using coarse, relative geometrical relations, known as qualitative spatial relationships (QSR). Each relationship fixes a coordinates system based on a small set of landmarks and discretizes space into disjoint regions, called qualitative states. Then, the location of a target landmark or robot pose is described in terms of these states. This coarse manner of reasoning about spatial information is potentially more noise-robust and suitable for low-cost platforms. Also, Qualitative Relational Mapping (QRM) algorithms produce QSR-based maps that sparsely represent the environment, in line with the motivation given above. Lastly, the robot can reason about the environment and even plan while accounting for partial information involved in a single or few QSR only, thus saving computational energy. While most of the research done in the qualitative field mainly addressed passive aspects such as localization and mapping, only few works (e.g., [5]–[7]) considered active planning.

In this paper, we introduce a novel BSP approach adapted to the qualitative framework (see Fig. 1). Before we discuss this paper's contributions, we briefly review the most relevant work done in the field to trace existing gaps.

A. *Related Work*

QSR applications for various robotics tasks began to emerge about three decades ago. Naturally, passive aspects

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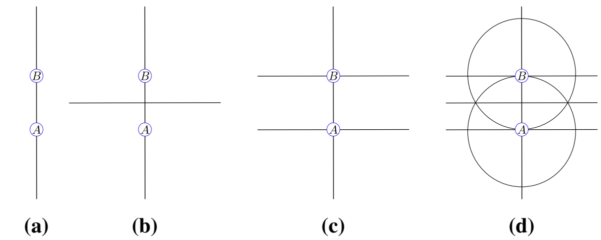


Fig. 2: Four different QSR partitions. (a) binary left-right [11]; (b) Freksa Single Cross [12]; (c) Freksa Double Cross [12]; (d) Extended Double Cross [6].

were the first to be addressed. A pioneering work by [8] presented a novel approach for egocentric robot localization based on the relative ordering of observed landmarks. [9] and [10] further improved this idea by encoding ordering views in a more complex hence distinguishable fashion, enabling localization ambiguity reduction.

Freksa suggested in [11] to represent the qualitative location of a landmark relative to a boundary line settled by a pair of other landmarks used as a reference. The location is described as "to the left" or "to the right" of the boundary. Freksa further refined this binary partitioning of space in [12], into quadratic and hexagonal ones, by adding extra boundary lines perpendicularly crossing the original one. The latter mentioned partitioning forms known as the "Freksa's Single Cross" (FSC) and "Freksa's Double Cross" (FDC). Moreover, in [11], Freksa introduced the binary composition operator that allows inference about the qualitative relationships between landmarks not directly observed together.

McClelland et al. took a step forward and introduced a more comprehensive QSR-based method for autonomous localization and mapping in [13]. The proposed algorithm constructs a graph-based map that encodes the environment using the relative geometrical layout of landmark triplets. For each triplet, one landmark is estimated in a local frame defined by the other two. The landmark is associated with one of several possible qualitative states, considering the FDC partitioning. McClelland extended his work in [6] by incorporating a method of determining the qualitative state for landmarks based on a novel set of geometric constraints. The above yielded a new qualitative spatial partitioning called the "Extended Double Cross" (EDC). This partition and those mentioned above are illustrated in Fig. 2. In addition, this work contributed a new composition operator, suitable for triplets, formulated as a look-up table. Another followed-up paper generalized the latter by developing a probabilistic QRM method (PQRM, [14]). Mor and Indelman were the first to incorporate stochastic motion model constraints in his formulations, reducing uncertainty levels of both landmarks and robot trajectory estimations [15]. In addition, the authors contributed a novel derivation of a probabilistic composition.

While many qualitative methods for localization and mapping have been developed, active planning approaches are much rarer. Previously mentioned [5] and [6] suggested applying the Dijkstra algorithm on graph-based maps they developed to find the shortest path to the desired destination. However, they both proposed only a general strategy rather than a detailed algorithm. In contrast, Padgett and Campbell developed a complete qualitative planning paradigm recently in [7]. The proposed *Q-Link* is a three-level planning architecture that generates high-level plans over QRM "links" (edges). It then uses local planners to execute trajectories to enable a robot to navigate from a start to a goal. However, even though the *Q-Link* considered some stochastic aspects, its high-level mechanism is essentially deterministic and does not exploit the potential of using compositions.

B. Contributions

We shall now briefly review the main contributions of this paper. (a) We develop a first-of-a-kind Qualitative Belief Space planning approach. Our planning mechanism accounts for possible future developments of the robot's and world's state by propagating a corresponding probabilistic distribution. It is not restricted by deterministic assumptions, in contrast to the methods discussed in Sec. I-A, and thus is more general; (b) We innovatively incorporate compositions in our algorithm, allowing our planning process to consider qualitative relationships between landmarks that have never been observed together. Building upon [16], we benefit from using these new relationships in two ways. Firstly, we can plan under challenging scenarios where other algorithms have failed. Secondly, we are able to find better plans in terms of expected cumulative cost, i.e., to find shortcuts; (c) We introduce the concept of qualitatively estimating global scales of frames. Namely, we reason about the distances between pairs of landmarks, which form different reference frames. The distances are evaluated qualitatively in global terms. This capability is crucial when estimating the likelihood to observe a specific triplet of landmarks, which may not have been viewed together so far, given a candidate action to execute; (d) We further take advantage of the latter capability and derive a new cost function, which globally measures metric path length.

The rest of this paper is arranged as follows. First, in Sec. II, we define basic notations and state the problem we aim to solve. Then, in Sec. III, we describe our qualitative BSP approach, wherein in Sec. III-F, we explain how compositions can be integrated within it. Finally, we evaluate our approach in Sec. IV. This paper is accompanied by supplementary material [17] that provides further details and mathematical derivations.

II. Notations And Problem Formulation

A. Qualitative Belief Definition

We consider a robot operating in a partially known environment, consisting of a known set of key landmarks, denoted by $\mathbb{L}$. As it travels in space, the robot moves between different landmark-centric frames (see e.g. [6], [14]). Each frame sets a local coordinate system determined by two reference landmarks. The first fixes the origin, i.e., $(0, 0)$, whereas the second fixes an additional coordinate, in most cases, $(0, 1)$. Then, a predefined partition divides space into a finite set of non-overlapping and complementary regions, known as qualitative states (see Fig. 2). We denote by $\mathbb{F}$ the set of all available frames, based on landmarks from $\mathbb{L}$.

The robot maintains its self-poses and map through Qualitative Spatial Relationships (QSRs). Each QSR localizes a target point, either a landmark or a robot pose, relative to a chosen frame $F{\in}\mathbb{F}$ by associating it with one of the qualitative states discussed above. This ternary type of QSRs is often referred to as triplets. In this work, we consider only triplets, as this is the most basic and standard case, and richer QSRs can always be split into triplets. We exclude binary QSRs, as these are relevant when dealing with complex volumed landmarks (extended landmarks, see [5] and [18]), while we assume point landmarks. In the following, we denote an ordered triplet by τ, or explicitly by $F{:}L$, where F and L are the triplet's reference frame and target landmark, respectively (for example, $\tau{=}AB{:}C$). We further denote by $\mathcal{S}^{\tau}$ and $\mathcal{X}^{\tau}$, or explicitly by $\mathcal{S}_F^L$ and $\mathcal{X}_F^L$ the qualitative and metric location of L relative to F (i.e., of $\tau{=}F{:}L$). Similarly, $\mathcal{S}_F^{X_t}$ and $\mathcal{X}_F^{X_t}$ denote the robot's qualitative and metric location at time step t, both relative to F as well. Note that $\mathcal{S}_F^L$ and $\mathcal{S}_F^{X_t}$ are discrete variables, while $\mathcal{X}_F^L$ and $\mathcal{X}_F^{X_t}$ are continuous.

Apart from self-poses and triplets, the robot accounts also for *Global Frame Scale*s'. A *Global Frame Scale* is a new concept suggested in this work, which refers to the global metric distance between the two landmarks creating the frame. For example, the global frame scale of AB is the global metric distance between the landmarks A and B. We further clarify the usage of this term in Sections III-C and III-G. We denote the metric and qualitative scale of a frame F by $\mathcal{X}^F$ and $\mathcal{S}^F$, respectively. Furthermore, we denote by $n_F \in \mathbb{N}$ the resolution according to which we evaluate $\mathcal{S}^F$. Given a selected value of n_F (typical values are $4-6$), S^F is a discrete random variable (RV) equals to $s \in [1, 2, \ldots, n_F]$ if $\mathcal{X}^F{\in}\big[\frac{(s-1)\cdot 2R}{n_F}, \frac{s\cdot 2R}{n_F}\big]$ and equal to n_F+1 if $\mathcal{X}^F{\in}\big[\frac{s\cdot 2R}{n_F}, \infty)$. Here, R is the robot's sensing range given in some global frame units (for example, meters). In the case where $s^F{=}n_F+1$, the frame is considered unobservable since its global scale is bigger than $2R$.

Due to the nature of the robot's mobility, we consider two types of actions. The first, referred to as *qualitative action*, allows the robot to travel between different qualitative states w.r.t. a given frame. We shall denote by a_t^{q} a *qualitative action* taken at time step t. In contrast, the second type, referred to as *link action* [7], enables the robot to switch from one local frame to another. We shall denote by a_t^{Link} a *link action* taken at time step t. The robot executes the abovementioned actions alternately. Namely, consider the robot's frame at time step t, F_t, and the corresponding robot's state $\mathcal{S}_{F_t}^{X_t}$. After executing a_t^{q}, the robot moves to a new state, $\mathcal{S}_{F_t}^{X_{t+1}}$. Then, it links to a new frame by executing the action a_t^{Link}, which results in the state $\mathcal{S}_{F_{t+1}}^{X_{t+1}}$. Note that a_t^{Link} is not a natural action in the sense that the robot makes no actual movement. In fact, the latter can equivalently be written as a tuple of source and destination frames, i.e., $a_t^{\mathit{Link}}{\triangleq}(F_t, F_{t+1})$. Note that a_t^{Link} may be degenerated, in case $F_t{=}F_{t+1}$. Each time the robot completes a qualitative action, it acquires a new measurement. Let z_t denote a measurement acquired by the robot at time step t. We further denote by β_t the data association from time step t, that is, the identity of the landmarks captured in z_t. Of course, determining the data association is a very challenging problem in itself. In this work, we assume it to be solved.

Consider k as the current time step. We denote by $\mathcal{H}_k$ the history of applied actions, measurements and data associations up to that time step. That is, $\mathcal{H}_k{\triangleq}\{a_{1:k-1}, z_{1:k}, \beta_{1:k}\}$, where a_i represents a consecutive pair of qualitative and link actions $\{a_i^{\text{q}}, a_i^{\mathit{Link}}\}, \forall i{\in}\{1, \ldots, k\}$. The index $t{:}t'$ compactly refers to a series of elements between time steps t and t'. Due to the stochastic nature of the problem, the robot maintains a qualitative belief, i.e., a posterior probability over the states of the robot, landmark triplets, and frames' scales, given by:

$$b_k \triangleq \mathbb{P}(\mathcal{S}^{X_{1:k}}, \mathcal{S}^{\mathcal{M}_k}, \mathcal{S}^{\mathbb{F}_k} | \mathcal{H}_k), \tag{1}$$

where $\mathcal{S}^{X_{1:k}}$ represents the set of robot states $\mathcal{S}^{X_{1:k}}{\triangleq}\{\mathcal{S}_{F_i}^{X_i}\}_{i=1}^k$, $\mathcal{S}^{\mathcal{M}_k}$ represents all available landmark triplets states at time step k, $\mathcal{S}^{\mathcal{M}_k}{\triangleq}\{\mathcal{S}^{\tau_j}\}_{j=1}^{m_k}$, with m_k being the set size, and finally, $\mathcal{S}^{\mathbb{F}_k}$ represents all available frames' scales at time step k, i.e., $\mathcal{S}^{\mathbb{F}_k}{\triangleq}\{\mathcal{S}^{F_q}\}_{q=1}^{p_k}$, with p_k being the set size.

Inspired by [7], [14], we maintain marginals over qualitative states conditioned on only local information, prioritizing computational speedup over an accurate model. Accordingly, we maintain the belief as a product of individual posteriors:

$$b_k{\approx}\prod_{i=1}^{k}\mathbb{P}(\mathcal{S}_{F_i}^{X_i}|\mathcal{H}_k^{X_i})\prod_{j=1}^{m_k}\mathbb{P}(\mathcal{S}^{\tau_j}|\mathcal{H}_k^{j})\prod_{q=1}^{p_k}\mathbb{P}(\mathcal{S}^{F_q}|\mathcal{H}_k^{q}). \tag{2}$$

In the above, $\mathcal{H}_k^{X_i}, \mathcal{H}_k^{j}, \mathcal{H}_k^{q}$ denote relevant part of the history used to evaluate $\mathcal{S}_{F_i}^{X_i}, \mathcal{S}^{\tau_j}$, and $\mathcal{S}^{F_q}$, respectively. While Eq. (2) is beneficial for implementation needs, the theoretical formulations in this paper are derived using Eq. (1) to stay as general as possible.

B. Qualitative Belief Space Planning

We now introduce a belief space planning (BSP) formulation considering the qualitative framework discussed above.

BSP, in essence, is the problem of finding an optimal sequence of actions, or policy, that minimizes a meaningful objective function. In this work, we consider actions sequences rather than policies. Assuming a future horizon of L look-ahead steps, we compactly represent by a_{k+} a candidate sequence of actions from time step k to the predefined horizon, that is, $a_{k+}{\triangleq}a_{k:k+L-1}$. The objective function maps the current belief, b_k, and a candidate actions sequence, a_{k+}, to an expected cumulative cost:

$$J(b_k, a_{k+}){\triangleq} \underset{z_{k+1:k+L}}{\mathbb{E}}\left[\sum_{l=1}^{L} c_l(b_{k+l}, a_{k+l-1})\right], \tag{3}$$

where c_l is the l-th cost function with the appropriate arguments, with $l{\in}\{1, 2, \ldots, L\}$.

While the above formulation is expressed in terms of a general cost function, we choose a specific one that best serves our purposes. We elaborate on the different types of costs in the Approach section. The optimal action sequence is defined by:

$$a_{k+}^{*}=\underset{a_{k+}}{\arg\min}\, J(b_k,a_{k+}). \tag{4}$$

III. APPROACH

A. Approach Overview

We present an end-to-end algorithm to address the qualitative BSP problem defined in Sec. II-B. Our algorithm operates in two steps.

First, we construct a belief tree, reflecting the future posterior beliefs considering various possible future developments. We describe the construction process and the different models assumed in this work in Secs. III-B-III-D. We explain how to update the belief between two adjacent tree nodes in Sec. III-E. In Sec. III-F, we further explain how compositions can be incorporated within the updating step. We construct the tree considering a predefined depth $L\in\mathbb{N}$.

In the second step, we utilize the constructed belief tree to evaluate the objective (7) for each candidate action sequence. Then, we search for the action sequence that minimizes the objective through a proper optimization process. In Sec. III-G, we suggest two types of costs to consider while evaluating the objective.

B. Qualitative Action and Transition Model

The qualitative action enables the robot to move from one qualitative state to another, considering a specific reference frame. Mathematically, we assume a probabilistic transition model,

$$\mathbb{P}(\mathcal{S}_{F_{t-1}}^{X_t}|\mathcal{S}_{F_{t-1}}^{X_{t-1}},a_t^{\text{q}}), \tag{5}$$

which maps any realization of the pair $\mathcal{S}_{F_{t-1}}^{X_{t-1}},a_t^{\text{q}}$ to a Q dimensional vector which describes the outcome distribution of the new robot's state, $\mathcal{S}_{F_{t-1}}^{X_t}$. The values of each vector are chosen or learned offline according to the level of noise that characterizes the robot's control system. We consider this transition model to be available and assume that a proper low-level controller exists. Furthermore, we assume a finite set of $(Q-1)^2$ qualitative actions, where for any mutual realization $\mathcal{S}_{F_{t-1}}^{X_t}{=}i,\mathcal{S}_{F_{t-1}}^{X_{t-1}}{=}j$, where $i{\neq}j$, there is a corresponding qualitative action that transforms the robot to state j with high probability, given that its current state is i.

C. Data Association and Measurement Likelihood

The objective requires evaluating the likelihood of capturing a measurement z_t and a corresponding data association β_t for any time step $t{\in}\{k{+}1,\ldots,k{+}L\}$. In this section, we rigorously derive the abovementioned likelihood term.

Formally, we can rewrite Eq. (3) recursively, as follows:

$$J(b_k,a_{k+}){=}\underset{z_{k+1}}{\mathbb{E}}\Big[c_1(b_{k+1},a_k){+}J(b_{k+1},a_{(k+1)+})\Big], \tag{6}$$

where the expectation is with respect to $\mathbb{P}(z_{k+1}|\mathcal{H}_{k+1}^{-})$. In the above, and throughout the rest of this paper, $\mathcal{H}_t^{-}{\triangleq}\mathcal{H}_{t-1}{\cup}\{a_{t-1}\},\forall t{\in}\{k{+}1,\ldots,k{+}L\}$. Uniquely in this work, data associations play a major role as they dictate link actions considered by the robot, as we shall see in Sec. III-D. Therefore, explicitly incorporating them within the objective is useful. We can rewrite (6) as:

$$J(b_k,a_{k+}){=}\underset{\beta_{k+1}}{\mathbb{E}}\Bigg[\underset{z_{k+1}|\beta_{k+1}}{\mathbb{E}}\Big[c_1{+}J(b_{k+1},a_{(k+1)+})\Big]\Bigg], \tag{7}$$

where $c_1{\triangleq}c_1(b_{k+1},a_k)$. As the above implies, the law of total probability relates between z_t and β_t, for any $t{\in}\{k{+}1,\ldots,k{+}L\}$. That is:

$$\mathbb{P}(z_t|\mathcal{H}_t^{-}){=}\sum_{\beta_t}\mathbb{P}(z_t,\beta_t|\mathcal{H}_t^{-}). \tag{8}$$

We aim to derive the joint likelihood term $\mathbb{P}(z_t,\beta_t|\mathcal{H}_t^{-})$, given specific realizations of z_t and β_t. In contrast to [19], we do so considering a qualitative framework.

First, we marginalize over the robot states $\mathcal{S}_{F_{t-1}}^{X_t}$ and $\mathcal{S}_{F_{t-1}}^{X_{t-1}}$, global frame scale $\mathcal{S}^{F_{t-1}}$ and, the qualitative state $\mathcal{S}^{\tau_{\beta_t}}$ that corresponds to the considered data association realization β_t. To simplify the exposition, we consider the latter to be a single landmark triplet, although this is not a limitation of our formulation:

$$\mathbb{P}(z_t,\beta_t|\mathcal{H}_t^{-}){=} \tag{9}$$
$$\sum_{\mathcal{S}_{F_{t-1}}^{X_t}}\sum_{\mathcal{S}_{F_{t-1}}^{X_{t-1}}}\sum_{\mathcal{S}^{F_{t-1}}}\sum_{\mathcal{S}^{\tau_{\beta_t}}}\mathbb{P}(z_t,\beta_t,\mathcal{S}_{F_{t-1}}^{X_t},\mathcal{S}_{F_{t-1}}^{X_{t-1}},\mathcal{S}^{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}}|\mathcal{H}_t^{-}).$$

Continuing with chain rule over the inner expression, we get:

$$\begin{aligned}&\mathbb{P}(z_t,\beta_t,\mathcal{S}_{F_{t-1}}^{X_t},\mathcal{S}_{F_{t-1}}^{X_{t-1}},\mathcal{S}^{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}}|\mathcal{H}_t^{-}){=}\\&\quad\mathbb{P}(z_t|\mathcal{S}_{F_{t-1}}^{X_t},\mathcal{S}^{\tau_{\beta_t}},\beta_t,\mathcal{H}_t^{-})\mathbb{P}(\beta_t|\mathcal{S}_{F_{t-1}}^{X_t},\mathcal{S}^{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\mathcal{H}_t^{-})\\&\quad\mathbb{P}(\mathcal{S}_{F_{t-1}}^{X_t}|\mathcal{S}_{F_{t-1}}^{X_{t-1}},a_{t-1}^{\text{q}})\mathbb{P}(\mathcal{S}_{F_{t-1}}^{X_{t-1}},\mathcal{S}^{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}}|\mathcal{H}_{t-1}).\end{aligned} \tag{10}$$

The term $\mathbb{P}(z_t|\mathcal{S}_{F_{t-1}}^{X_t},\mathcal{S}^{\tau_{\beta_t}},\beta_t,\mathcal{H}_t^{-})$, known as the measurement model, describes the probability of capturing the measurement z_t, given the robot's state $\mathcal{S}_{F_{t-1}}^{X_t}$, the data association β_t with its corresponding state $\mathcal{S}^{\tau_{\beta_t}}$, and history. The term $\mathbb{P}(\beta_t|\mathcal{S}_{F_{t-1}}^{X_t},\mathcal{S}^{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\mathcal{H}_t^{-})$, known as the association model, describes the probability to observe the triplet β_t, given the robot's and β_t's states and F_{t-1}'s global scale. Intuitively, to evaluate this probability, the robot must estimate its sensing range, R, in local terms of F_{t-1}. Using F_{t-1}'s scale, this can be done via a simple normalization process, described in [17, Sec. 1].

To calculate the abovementioned qualitative models in practice, one can further marginalize over the relevant metric state and use standard non qualitative models. See [17, Sec. 1-2] for more details. Moreover, in (10), the term $\mathbb{P}(\mathcal{S}_{F_{t-1}}^{X_t}|\mathcal{S}_{F_{t-1}}^{X_{t-1}},a_{t-1}^{\text{q}})$ is the qualitative motion model stated in (5), and finally, the term $\mathbb{P}(\mathcal{S}_{F_{t-1}}^{X_{t-1}},\mathcal{S}^{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}}|\mathcal{H}_{t-1})$ can be calculated via marginalization from the belief from time instant $t{-}1$, b_{t-1}.

Generally speaking, we need to consider all possible realizations for both z_t and β_t, for any $t{\in}\{k{+}1,\ldots,k{+}L\}$, to calculate the objective precisely. However, in practice, this is intractable. Alternatively, we can sample a finite set of realizations and approximate the objective via proper averaging. Starting with β_t, we suggest considering only triplets' realizations containing the robot's current frame. This heuristic prioritizes triplets that are more likely to be observed, as those containing the current frame are more likely to be closer. For instance, in case $F_k{=}AB$, then ABC is being considered, also ABD, but not ACD. This heuristic yields $N_\beta{=}|\mathbb{L}|{-}2$ β_t's realizations, which is a significant reduction compared to all $\binom{|\mathbb{L}|}{3}$ triplets existing under the given landmark alphabet based on $\mathbb{L}$. In practice, among these, we can only consider those which are available in b_{t-1}, that is, n_β realizations, where $n_\beta{\leq}N_\beta$. Crucially, as we shall see in Sec. III-F, utilizing composition we are able to extend n_β to come closer to, and under a certain assumption equal to, N_β. Considering also a finite set of n_z measurement realizations z_t for each realization of β (see Fig. 1 for illustration), the objective (7) can be approximated as follows:

$$J(b_k,a_{k+})\approx \sum_{i=1}^{n_\beta}\frac{w^i}{n_z}\sum_{j=1}^{n_z}\mathbb{P}(z^{i,j}_{k+1}|\beta^i_{k+1},b_k,a_k)\cdot(c_1+J(b_{k+1},a_{(k+1)+})), \tag{11}$$

where $w^i{\triangleq}\frac{\tilde{w}^i}{\sum_{q=1}^{n_{\beta_k}}\tilde{w}^q}$, with $\tilde{w}^i{\triangleq}\frac{\mathbb{P}(\beta^i_{k+1}|b_k,a_k)}{\sum_{q=1}^{N_\beta}\mathbb{P}(\beta^q_{k+1}|b_k,a_k)}$. For the full derivation of Eq. (11), see [17, Sec. 3].

D. Link *Action and Transition Model*

The *Link* action allows the robot to switch between different reference frames. Even though it is considered an action, in practice, the robot does not make any actual movement when executing it. Accordingly, a *Link* action taken at time step t is denoted by a_t^{Link}, but alternatively can be written as a tuple of source and target frames, (F_{t-1},F_t). As Fig. 1 illustrates, the *Link* action adds another level of decision-making compared to the traditional BSP approaches, which use a single global frame.

We assume a probabilistic *Link* model,

$$\mathbb{P}(\mathcal{S}^{X_t}_{F_t}|\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\tau},\mathcal{H}_t), \tag{12}$$

representing the probability of the robot being located at state $\mathcal{S}^{X_t}_{F_t}$, given its state relative to the former frame, $\mathcal{S}^{X_t}_{F_{t-1}}$, the state of the triplet relates between the frames, $\mathcal{S}^{\tau}$, with $\tau{\triangleq}F_{t-1}{:}F_t\backslash F_{t-1}$, and history, which includes the action a_t^{Link}. To understand why this model is a probabilistic one, consider the following example. Suppose that, at time step t, the robot is located relative to an old frame $F_{t-1}{=}AB$ and aims to link to a new one, $F_t{=}BC$. To that end, it must translate its location from AB's terms to BC's. Given its (unknown) current metric location $\mathcal{X}^{X_t}_{AB}$ and C's metric location in terms of the same frame, $\mathcal{X}^{C}_{AB}$ (i.e., $\tau{=}AB{:}C$), the robot can easily infer $\mathcal{X}^{X_t}_{BC}$, using a simple geometric transformation. Fig. 3 demonstrates two sets of metric realizations as described above. However, since the *Link* model considers only the corresponding qualitative locations rather than the actual metric ones, the robot must account for many possible metric hypotheses. For this reason, it can only infer a distribution over the resulting state $\mathcal{S}^{X_t}_{F_t}$. We further elaborate on how this model can be calculated in [17, Sec. 4].

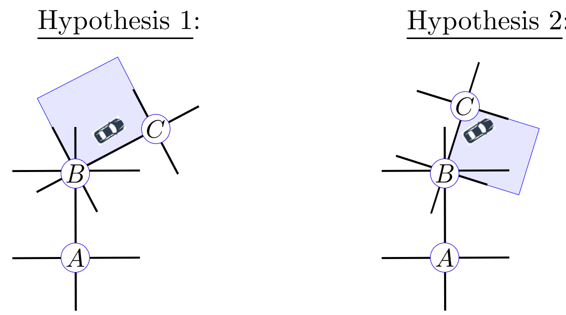


Fig. 3: Illustration of two possible hypotheses for $\mathcal{X}^{C}_{AB}$. While both hypotheses yield the same qualitative state $\mathcal{S}^{C}_{AB}$, they yield different states for $\mathcal{S}^{X_t}_{BC}$.

E. Belief Update Step

This section focuses on updating the qualitative belief defined in Eq. (1) a single step into the future. Formally, consider the belief from time step $t{-}1{\in}\{k,\ldots,k{+}l{-}1\}$, b_{t-1}, candidate action, $a_{t-1}{\triangleq}\{a^{\mathrm{q}}_{t-1},a^{\mathit{Link}}_{t-1}\}$, a measurement, z_t, and a corresponding data association, β_t. We aim to infer the belief at time step t, b_t, via a proper Bayesian update rule:

$$b_t = \psi(b_{t-1},a_{t-1},z_t,\beta_t). \tag{13}$$

We start by marginalizing over the next robot's position relative to the current frame, $\mathcal{S}^{X_t}_{F_{t-1}}$:

$$b_t{=}\sum_{\mathcal{S}^{X_t}_{F_{t-1}}}\mathbb{P}(\mathcal{S}^{X_{1:t}},\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_t}|\mathcal{H}_t). \tag{14}$$

We continue by breaking the inner term using chain rule:

$$\mathbb{P}(\mathcal{S}^{X_{1:t}},\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_t}|\mathcal{H}_t){=} \\ \mathbb{P}(\mathcal{S}^{X_t}_{F_t}|\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\mathcal{H}_t)\mathbb{P}(\mathcal{S}^{X_{1:t-1}},\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_t}|\mathcal{H}_t). \tag{15}$$

The left term, obtained after omitting all triplets' states from $\mathcal{S}^{\mathcal{M}_t}$ except for β_t's, is the *Link* Model stated in (12). We continue developing the right term via Bayes rule over z_t and β_t taken from $\mathcal{H}_t$ while omitting irrelevant information:

$$\mathbb{P}(\mathcal{S}^{X_{1:t-1}},\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_t}|\mathcal{H}_t) = \\ \eta_t\mathbb{P}(z_t|\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\beta_t,\mathcal{H}_t^-)\mathbb{P}(\beta_t|\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\mathcal{S}^{F_{t-1}},\mathcal{H}_t^-) \\ \mathbb{P}(\mathcal{S}^{X_t}_{F_{t-1}}|\mathcal{S}^{X_{t-1}}_{F_{t-1}},a^{\mathrm{q}}_{t-1})\mathbb{P}(\mathcal{S}^{X_{1:t-1}},\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_t}|\mathcal{H}_t^-), \tag{16}$$

where $\eta_t{\triangleq}\mathbb{P}(z_t,\beta_t|\mathcal{H}_t^-)$ is a normalization term, $\mathbb{P}(z_t|\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\beta_t,\mathcal{H}_t^-)$ and $\mathbb{P}(\beta_t|\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\mathcal{S}^{F_t},\mathcal{H}_t^-)$ are the measurement and association models discussed in Section III-C, respectively, and finally, $\mathbb{P}(\mathcal{S}^{X_t}_{F_{t-1}}|\mathcal{S}^{X_{t-1}}_{F_{t-1}},a^{\mathrm{q}}_{t-1})$ is the qualitative transition model (5). We continue by applying another chain rule over the remaining term in (16):

$$\mathbb{P}(\mathcal{S}^{X_{1:t-1}},\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_t}|\mathcal{H}_t^-){=} \\ \mathbb{P}(\mathcal{S}^{F_{t-1}}|\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_{t-1}},\mathcal{H}_t^-)\mathbb{P}(\mathcal{S}^{X_{1:t-1}},\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_{t-1}}|\mathcal{H}_t^-). \tag{17}$$

In the above, we consider the new set of frames' scales at time step t, $\mathcal{S}^{\mathbb{F}_t}$, as the former set from time step $t{-}1$, $\mathcal{S}^{\mathbb{F}_{t-1}}$ unified with the currently considered scale, $\mathcal{S}^{F_{t-1}}$, i.e.,

$\mathcal{S}^{\mathbb{F}_t}{=}\mathcal{S}^{\mathbb{F}_{t-1}}{\cup}\mathcal{S}^{F_{t-1}}$. The term $\mathbb{P}(\mathcal{S}^{F_{t-1}}|\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_{t-1}},\mathcal{H}_t^-)$ is the posterior over the relevant frame scale, $\mathcal{S}^{F_{t-1}}$, given the map, $\mathcal{S}^{\mathcal{M}_t}$, available frames' scales, $\mathcal{S}^{F_{t-1}}$, and history, $\mathcal{H}_t^-$. To further develop this term, we assume that the global scale of AB, $\mathcal{S}^{AB}$, is known. This assumption is fairly reasonable, as it requires prior knowledge of only one frame scale. Via this scale, we can evaluate the scale of $F_{t-1}{=}\{L_1,L_2\}$, using the states $\mathcal{S}^{L_1}_{AB}$ and $\mathcal{S}^{L_2}_{AB}$, which locate F_{t-1}'s landmarks relative to AB. Mathematically, we approximate the above-mentioned term by neglecting unnecessary information:

$$\begin{aligned}&\mathbb{P}(\mathcal{S}^{F_{t-1}}|\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_{t-1}},\mathcal{H}_t^-)\approx \\ &\quad\mathbb{P}(\mathcal{S}^{F_{t-1}}|F_{t-1},\mathcal{S}^{L_1}_{AB},\mathcal{S}^{L_2}_{AB},\mathcal{H}_t^{ABL_1},\mathcal{H}_t^{ABL_2},\mathcal{S}^{AB}),\end{aligned} \tag{18}$$

where the triplet states $\mathcal{S}^{L_1}_{AB}$,$\mathcal{S}^{L_2}_{AB}$ were taken from $\mathcal{S}^{\mathcal{M}_t}$, the frame F_{t-1} and the relevant local histories $\mathcal{H}_t^{ABL_1}$,$\mathcal{H}_t^{ABL_2}$ were taken from $\mathcal{H}_t^-$, and finally, $\mathcal{S}^{AB}$ is the known global scale taken from $\mathcal{S}^{F_{t-1}}$. We further develop this term in [17, Sec. 5]. In Sec. III-F, we explain how to compose $\mathcal{S}^{L_1}_{AB}$ and $\mathcal{S}^{L_2}_{AB}$ in case they are not part of the current belief.

In total, we get the following update rule:

$$\begin{aligned}b_t{=}&\sum_{\mathcal{S}^{X_t}_{F_{t-1}}}\mathbb{P}(\mathcal{S}^{X_t}_{F_t}|\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\mathcal{H}_t)\eta_t\cdot \\ &\mathbb{P}(z_t|\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\mathcal{S}^{F_t},\beta_t,\mathcal{H}_t^-)\mathbb{P}(\beta_t|\mathcal{S}^{X_t}_{F_{t-1}},\mathcal{S}^{\tau_{\beta_t}},\mathcal{S}^{F_t},\mathcal{H}_t^-)\cdot \\ &\mathbb{P}(\mathcal{S}^{X_t}_{F_{t-1}}|\mathcal{S}^{X_{t-1}}_{F_{t-1}},a^{\mathrm{q}}_{t-1})\mathbb{P}(\mathcal{S}^{F_{t-1}}|\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_{t-1}},\mathcal{H}_t^-)b_{t-1}.\end{aligned} \tag{19}$$

The above update step describes the operator $\psi(\cdot)$ from Eq. (13) explicitly. We apply it to update the belief between consecutive time steps when constructing the tree, as illustrated in Fig. 1a.

F. Incorporating Compositions

So far, we have described our qualitative BSP approach in its base form, without utilizing compositions at all. In this section, we present our *second key contribution* and show how compositions can be integrated within our algorithm to further improve planning in two ways. Firstly, it allows us to deal with a broader range of scenarios, i.e., in some cases, a plan can be found *only* via compositions. Secondly, using compositions, we can find better plans, i.e., ones with a lower objective. In this section, we provide theoretical justification for the above. Also, we formally show how to incorporate compositions within our algorithm.

The composition operator, first suggested in 1992 by [11], propagates data from two source triplets, τ_1 and τ_2, to infer the third one τ_3. From a topological point of view, compositions must respect the following lemma [6]:

Lemma 1. A target triplet τ_3 can be composed using a single composition operation (or directly) based on the triplets τ_1 and τ_2, if the following hold:

1) $|\tau_1 \cap \tau_2| = 2$
2) $\tau_3 \subset \tau_1 \cup \tau_2$

For example, we can directly compose $\tau_3{=}AB{:}D$ using $\tau_1{=}AB{:}C$ and $\tau_2{=}BC{:}D$. The mutual landmarks, B and C, allow us to fix both triplets relative to the same frame and, hence, infer (compose) relationships between new triplets' combinations.

In Eq. (1), we defined the belief as a posterior distribution over, among others, the different triplets in the map. Consider the map's state at the current time step, $\mathcal{S}^{\mathcal{M}_k}$. Via compositions, the robot can augment $\mathcal{M}_k$ with new triplets. However, compositions are allowed only under specific topological conditions, formulated in Lemma 1. For example, if $\mathcal{M}_k$ is too sparse, the ability to compose new triplets might be limited or even impossible. In [16], we have formulated a sufficient topological condition attributed to the set $\mathcal{M}_k$, whose existence ensures the ability to compose any desired triplet under the considered landmark space $\mathbb{L}$. We named such a sufficiently dense set a *Composable* set under $\mathbb{L}$ (the formal definition can be found in [17, Sec. 6]). As this topological aspect is not the focus of this work, we assume that $\mathcal{M}_k$ is *Composable* under $\mathbb{L}$. However, this assumption is not a must. In Sec. III-C, we suggested a heuristic that considers a maximum of N_β triplets as possible realizations for β_t at any planning time step $t{\in}\{k{+}1,\ldots,k{+}L\}$. We stressed that, in practice, we could consider only a subset of n_β triplets out of these N_β, which are available in b_{t-1}. Since we assumed $\mathcal{M}_k$ is *Composable* under $\mathbb{L}$, we are now guaranteed that all N_β realizations can be considered in planning via compositions, as illustrated in Fig. 1b. In [17, Sec. 7], we prove that in some scenarios, a plan can be found only via compositions.

For any planning time step $t{\in}\{k{+}1,\ldots,k{+}L\}$, we shall now explain how to incorporate a new set of triplets $\mathcal{M}_t^\Delta$, which can be chosen according to the heuristic described in Sec. III-C, into the belief. We denote $\mathcal{M}_t{\triangleq}\mathcal{M}_{t-1}{\cup}\mathcal{M}_t^\Delta$ and express the belief after combining the set, given by $b^{\mathcal{M}}_{t-1}{\triangleq}\mathbb{P}(\mathcal{S}^{X_{1:t-1}},\mathcal{S}^{\mathcal{M}_t},\mathcal{S}^{\mathbb{F}_{t-1}}|\mathcal{H}_{t-1})$, in terms of the one before combining it, b_{t-1}, via chain rule and Markov assumption:

$$b^{\mathcal{M}}_{t-1}{=}\mathbb{P}(\mathcal{S}^{\mathcal{M}_t^\Delta}|\mathcal{S}^{\mathcal{M}_{t-1}},\mathcal{H}_{t-1}){\cdot}b_{t-1}. \tag{20}$$

Suppose $|\mathcal{M}_t^\Delta|{=}n_t$, the term $\mathbb{P}(\mathcal{S}^{\mathcal{M}_t^\Delta}|\mathcal{S}^{\mathcal{M}_{t-1}},\mathcal{H}_{t-1})$ can be further broken down into a product of individuals posteriors, each aims to compose a single triplet from $\mathcal{M}_t^\Delta$:

$$\mathbb{P}(\mathcal{S}^{\mathcal{M}_t^\Delta}|\mathcal{S}^{\mathcal{M}_{t-1}},\mathcal{H}_{t-1}){\approx}\prod_{i=1}^{n_t}\mathbb{P}(\mathcal{S}^{\tau_i}|\mathcal{S}^{\mathcal{M}^i_{t-1}},\mathcal{H}^i_{t-1}). \tag{21}$$

In the above, $\mathcal{S}^{\tau_i}$ denotes the state of the ith triplet taken from $\mathcal{S}^{\mathcal{M}_t^\Delta}$, $\mathcal{S}^{\mathcal{M}^i_{t-1}}$ denotes the minimal subset of $\mathcal{S}^{\mathcal{M}_{t-1}}$ required for the evaluation, and finally, $\mathcal{H}^i_{t-1}$ denotes the corresponding local history, where $\mathcal{S}^{\mathcal{M}_t^\Delta}{=}\bigcup_{i=1}^{n_t}\mathcal{S}^{\tau_i}$, $\mathcal{H}_{t-1}{=}\bigcup_{i=1}^{n_t}\mathcal{H}^i_{t-1}$. For instance, if $\mathcal{S}^{\tau_i}{=}\mathcal{S}^D_{AB}$ and $\mathcal{S}^{\mathcal{M}_{t-1}}{=}\{\mathcal{S}^C_{AB},\mathcal{S}^D_{BC},\mathcal{S}^E_{AD}\}$, then the subsets $\mathcal{S}^{\mathcal{M}^i_{t-1}}{=}\{\mathcal{S}^C_{AB},\mathcal{S}^D_{BC}\}$, $\mathcal{H}^i_{t-1}{=}\{\mathcal{H}^{ABC},\mathcal{H}^{BCD}\}$ are chosen as using the triplets $AB{:}C$ and $BC{:}D$, we can compose $AC{:}D$ via a single composition. We can describe the required composition using a binary tree, consisting of a root, representing the triplet to compose $AB{:}D$, and two leaves connected to it, representing the required source triplets,

		Cost 1 (# q-states)		Cost 2 (metric path length)	
		W/O Comp	W Comp	W/O Comp	W Comp
All Tests (3000)	**Plan exists**	68.4%	81.6%	68.4%	81.6%
Comparable & Different Tests (13%)	**Average executed cost**	6.45	4.97	2.76	2.32

TABLE I: The first row shows the percentage of tests (considering all 3000 tests) in which the robot was able to plan towards its goal. The second row shows the average cost among all different and comparable tests. The two main columns are divided by the different costs (Sec. III-G), where each considers two planning modes: without and with compositions.

$AB{:}C$ and $BC{:}D$. The abovementioned tree, also known as *Composition Tree*, was first introduced in [16]. Depending on the identity of the target triplet and the available source set, we can generally get a large *Composition Tree*, representing the sequence of compositions required to form the target triplet. To find the tree with the minimal number of leaves (representing the set $\mathcal{S}^{\mathcal{M}^i_{t-1}}$), we use an Ad Hoc algorithm developed in [16]. The chosen tree dictates the required sequence of compositions, where each of them is then performed using the formulation in [17, Sec. 8].

We can now incorporate compositions within the belief update step from (19) by replacing b_{t-1} with $b^{\mathcal{M}}_{t-1}$ from (20).

G. Cost Function

The objective function in (3) describes for each candidate action sequence $a_{k:k+L-1}$ its expected accumulated cost. Having access to a qualitative belief allows defining both state-dependent and belief-dependent cost functions. The cost function is chosen according to the task's nature. For instance, if we aim to find the shortest path between some initial and goal key points, then a distance-based cost can be used, whereas if we wish to reduce uncertainty, then an entropy-based cost may be a good choice.

In this paper we focus on the first type, and suggest now two alternatives for distance-based costs. The first is to evaluate the expected number of qualitative states traversals:

$$c_t(b_t,a_{t-1}) = \mathop{\mathbb{E}}_{s_1,s_2}\big[d(s_1,s_2)\big], \tag{22}$$

where $s_1 \triangleq \mathcal{S}^{X_{t-1}}_{F_{t-1}}$, $s_2 \triangleq \mathcal{S}^{X_t}_{F_{t-1}}$, and where $d(s_1,s_2)$ is a metric that returns the minimum number of qualitative states traversals required to travel from state s_1 to s_2. For a given QSR representation, this metric is a simple lookup table.

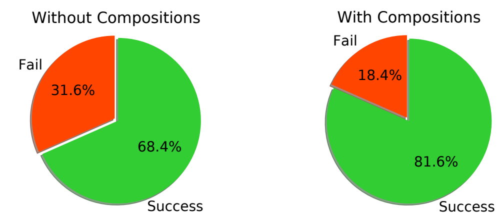
While the above cost measures distance qualitatively, there is no proven correlation to the true metric distance traveled. To bridge this gap, we propose another option of evaluating the traveled distance metrically, given the qualitative belief b_t. The idea is simple, at each time step, we evaluate the expected traveled distance in terms of the current local coordinates system, multiplied by the appropriate global scale taken from the belief. Mathematically:

$$c_t(b_t,a_{t-1}) = \mathop{\mathbb{E}}_{s}\left[\mathop{\mathbb{E}}_{x|s}\left[\left\|\mathcal{X}^{X_t}_{F_{t-1}} - \mathcal{X}^{X_{t-1}}_{F_{t-1}}\right\|_2 \cdot \mathcal{X}^{F_{t-1}}\right]\right], \tag{23}$$

where we denoted $x \triangleq \{\mathcal{X}^{X_t}_{F_{t-1}}, \mathcal{X}^{X_{t-1}}_{F_{t-1}}, \mathcal{X}^{F_{t-1}}\}$, and $s \triangleq \{\mathcal{S}^{X_t}_{F_{t-1}}, \mathcal{S}^{X_{t-1}}_{F_{t-1}}, \mathcal{S}^{F_{t-1}}\}$. Moreover, based on the Law of Total Expectation, we can further simplify the above expression and replace it with a pure metric version:

$$c_t(b_t,a_{t-1}) = \mathop{\mathbb{E}}_{x}\left[\left\|\mathcal{X}^{X_t}_{F_{t-1}} - \mathcal{X}^{X_{t-1}}_{F_{t-1}}\right\|_2 \cdot \mathcal{X}^{F_{t-1}}\right]. \tag{24}$$

To calculate this cost in practice, we must use (23) as we only have access to the metric priors conditioned on the corresponding qualitative states.



(a) **(b)**

Fig. 4: Improvement in results due to the use of compositions. (a) Without compositions, the robot succeeded in planning towards its goal in 68.4% of the tests.; (b) Via compositions, the robot improved and planned towards its goal in 81.6% of the tests.

IV. RESULTS

We evaluate our approach using a simulation developed in *Python* 3.7. We do not compare our performances to other algorithms, as there are no comparable ones in the literature. The only other existing work that considers planning under the qualitative framework, [7], assumed a deterministic framework making any comparison irrelevant. For simplicity, we consider the following: (1) Motion and Measurement models with additive Gaussian white noise. (2) Small environments with $8{-}12$ landmarks. (3) Data-Association is solved. (4) The initial belief considers a *Composable* set of triplets under $\mathbb{L}$.

We performed 3000 tests, each randomizing a different environment and choosing a target triplet randomly. We repeated each test four times, once for each cost suggested in III-G, with and without the robot using compositions. Table I summarizes the overall statistics, where Fig. 4 emphasizes the advantage of using compositions.

Two specific scenarios are demonstrated in Fig. 5, where the robot aims to reach and observe the goal triplet ABC.

In the first scenario (Fig. 5a), which considered the cost formulated in Eq. (22), the robot has succeeded in reaching and observing ABC only via compositions. Specifically, the robot's initial frame was $F_1{=}EG$, and its initial belief was over the set of source landmark triplets $\mathcal{M}_1{=}\{ABC,ABD,ABE,ABH,ACI,DEF,DEG,EGJ\}$. At time step $t{=}2$, the robot linked to a new frame, $F_2{=}AE$. This link, which was necessary to reach ABC (for more details, see [17, Sec. 7]), was feasible exclusively based on the triplet AEG, which was composed using source triplets from $\mathcal{M}_1$. Meaning, in this case, without using compositions, no feasible path would have been found.

In the second scenario (Fig. 5b), which considered the cost formulated in Eq. (23), the robot was able to find a plan without using composition. However, by composing the triplet AIJ, it was able to link to AJ at time step $t{=}2$ and via that link to find a shorter path towards ABC's vicinity.

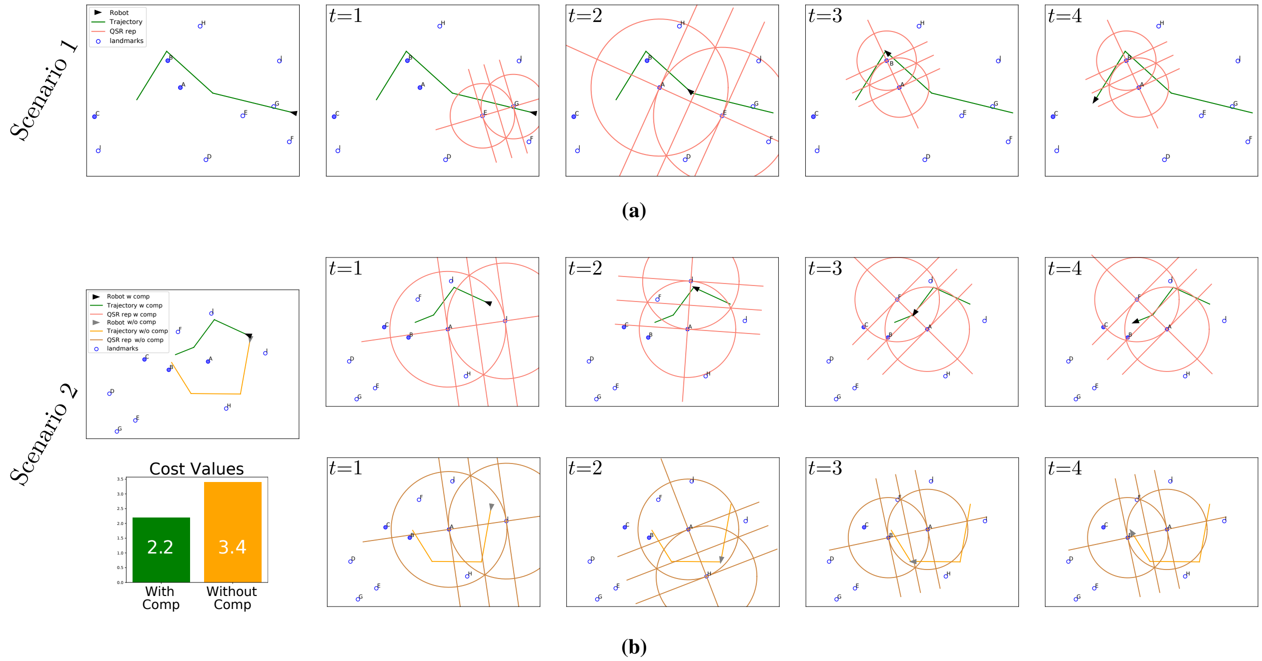


Fig. 5: Two scenarios from our simulations are presented. We show the execution of the calculated paths with replanning between consecutive time steps. For each time step, the local frame of the robot, considering the EDC partitioning, is displayed. In both scenarios, the robot aims to reach and observe the goal triplet ABC (filled blue circles). (a) In scenario 1, the robot succeeded in reaching its goal only via compositions. (b) in scenario 2, the robot was able to find a path towards its goal with (upper row) and without (bottom row) compositions. However, via compositions, the path is shorter.

V. CONCLUSIONS

In this paper, we presented a novel algorithm to address the problem of Belief Space Planning, considering a qualitative framework. Our algorithm operates in two steps. Given an initial qualitative belief and a target triplet, it first constructs a belief tree that accounts for multiple possibilities for future developments, where each corresponds to a candidate plan. This step is the main focus of this work, where the belief tree is specifically designed to support qualitative belief propagation. Then, it chooses the best plan, i.e., that minimizes a meaningful objective function. This step is a standard one. Moreover, our mechanism enables incorporating compositions to improve planning results. Finally, we suggested a novel cost function, which considers metric path length, thus being more realistic. We believe this first work on qualitative BSP opens new research opportunities to follow.

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