Cooperative Multi-Robot Belief Space Planning for Visual-Inertial Navigation and Online Sensor Calibration

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Overview

- Introduction
- Related work
- Contribution
- Single Robot Approach
- Multi-robot Approach
- Conclusion

Introduction – Autonomous Navigation

Autonomous cars
(google)

Military

Mines/Tunnels Exploration

Highly Accurate Navigation

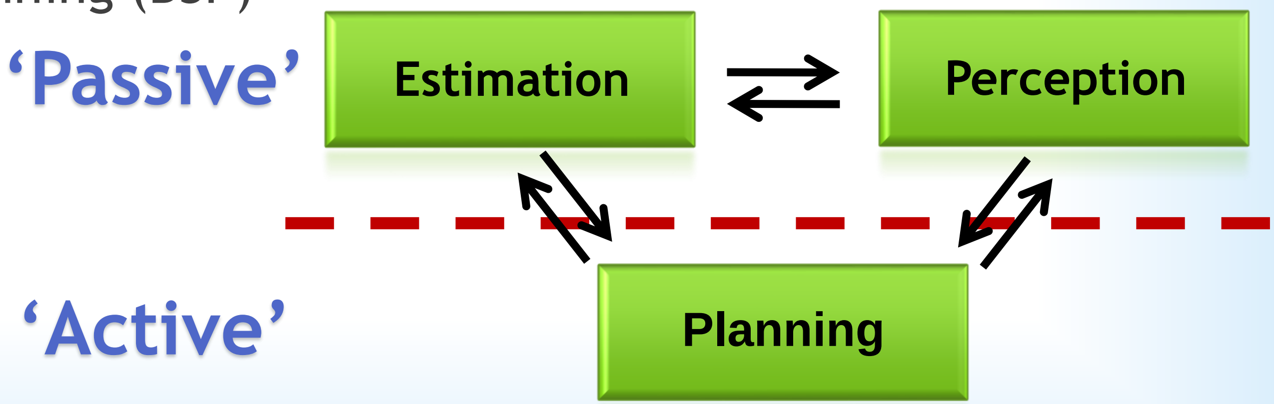
Indoor Operation

Space Exploration

Sub-marine exploration

Introduction - SLAM & Planning

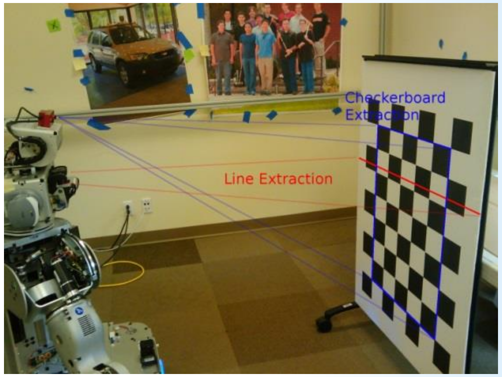
- Navigation and mapping in **unknown, GPS-deprived, environment**
- Key components for autonomous operation include:
  - **Estimation and Perception** - Where am I? What is the environment?
    - Simultaneous localization and mapping (**SLAM**)
  - **Planning** - What is the best action to do next?
    - Traditional planning approaches
    - Belief Space Planning (BSP)



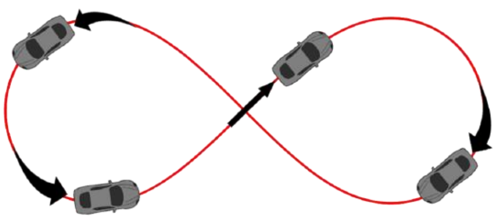
Introduction – Autonomous Navigation

Inertial sensor
offline calibration

Sensor Calibration

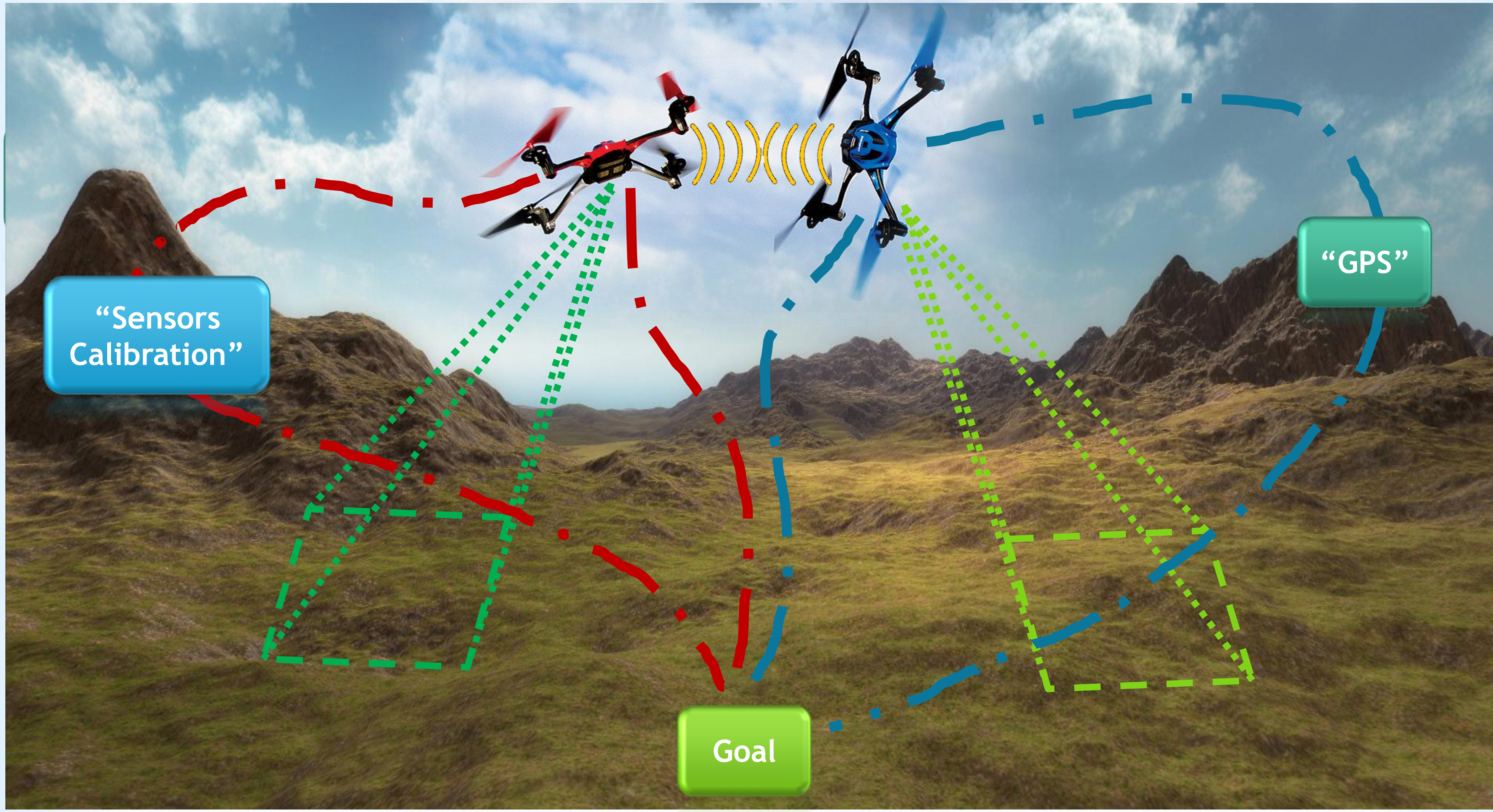


Camera
offline calibration



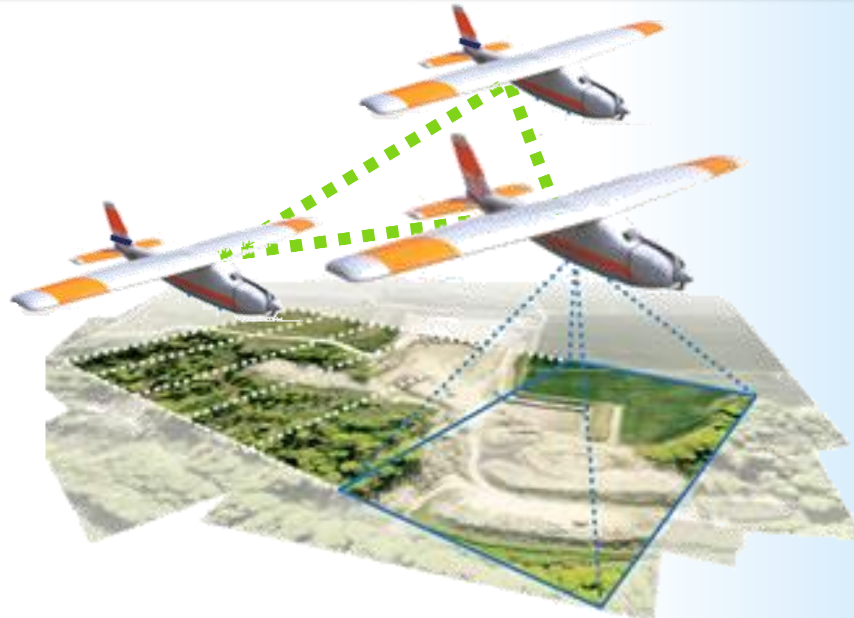
Inertial Sensor
Online calibration
(pre-determined maneuvers)

Introduction - Scenario



Introduction - Scenario

- Aerial cooperative multi-robot
- Sensors:
  - Inertial Measurements Unit (IMU)
  - Monocular downward-looking camera
- Vision inertial navigation system (VINS)
- Partially unknown environment
- GPS-deprived environment
- Centralized architecture



Planning optimal trajectories for cooperative robots to achieve online IMU calibration and accurate navigation

Related Work

- **Online Calibration**

  (V. Indelamn, 2012), (J. Maye, 2016)

  - **SLAM** considering IMU and extrinsic parameters calibration
  - Without planning

- **Belief Space Planning (BSP)**

  (V. Indelman, 2013), (G. A. Hollinger, 2014) , (V. Indelman, 2015)

  - **Performance improvement** in SLAM
  - Not considering IMU measurements

- **Planning Considering Online Calibration**

  - **Extrinsic parameters calibration** (transformation between frames)

    (W. Achtelik, 2013), (D.J. Webb, 2014), (J. Maye, 2016)

  - **IMU calibration** assuming GPS availability

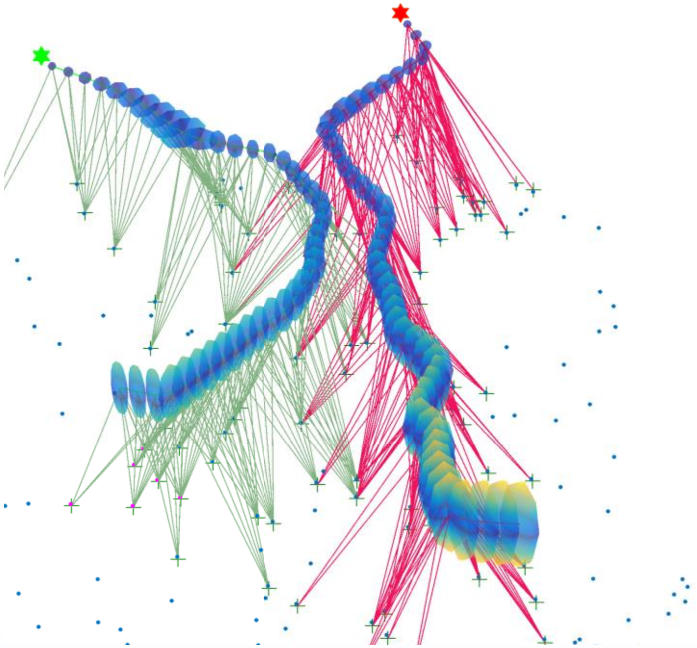
    (K. Hausman, 2016)

Related Work

- **Multi-robot Belief Space Planning**

  (A. Kim, 2014, IJRR) (V. Indelman, 2015, IJRR) (V. Indelman, 2015, ISRR)

  - Operation in **unknown environments**
  - Cooperation via **mutual landmark observations** or **observing other robots**



(V. Indelman, 2015, ISRR)

Contributions

- Incorporating ***online sensor calibration into belief space planning (BSP)*** for ***visual-inertial navigation systems***
  - Allows to consider, within planning, reduction of navigation estimation uncertainty and reduction of the uncertainty evolution rate
- Approach for ***cooperative multi-robot BSP using future indirect constraints*** given past correlation between robots
  - Introduce concept of “expendable” robots used for updating other robots
- Incorporate recently developed concept of ***IMU pre-integration into BSP***

Problem Formulation - Notations

Robot's Nav. State

$X_k^r \doteq \{x_0^r, \ldots, x_k^r\}$

IMU Sensors Calibration

$C_k^r \doteq \{c_0^r, \ldots, c_k^r\}$

Perceived Environment

$L_k \doteq \{l_0, \ldots, l_n\}$

Random Variables

$C_k \doteq \left\{ C_k^r \right\}_{r=1}^{R}$

$X_k \doteq \left\{ X_k^r \right\}_{r=1}^{R}$

Joint State

$\Theta_k \doteq \{X_k, C_k, L_k\}$

Control Actions

$U_{0:k-1}^r \doteq \{u_0^r, \ldots, u_{k-1}^r\}$

Sensors Measurements

$Z_{1:k}^r \doteq \{z_1^r, \ldots, z_k^r\}$

Given

Single Robot

Probabilistic Formulation - Inference

- Joint probability distribution function (pdf)

$$b\left(\Theta_k\right) = p\left(\Theta_k \mid Z_{1:k}, U_{0:k-1}\right)$$

$$\propto priors \cdot \prod_{i=1}^{k} \underbrace{p\left(x_i \mid x_{i-1}, c_{i-1}, z_{i-1}^{IMU}\right)}_{\text{Motion Model}} \cdot \underbrace{p\left(c_i \mid c_{i-1}\right)}_{\text{Calibration Model}} \underbrace{p\left(z_i \mid \Theta_i^o\right)}_{\text{Observation Model}}$$

Probabilistic Formulation – Models Definition

$$p\left(x_i \mid x_{i-1}, c_{i-1}, z_{i-1}^{IMU}\right)$$

Motion Model:

$$x_i \doteq [\, p_i \quad v_i \quad q_i \,]$$

$$x_i = f\left(x_{i-1}, c_{i-1}, z_{i-1}^{IMU}\right) + w_{i-1}$$

$$Gaussian\ \ Noise: w_{i-1} \sim N\left(0, \Sigma_w\right)$$

Strapdown Equations:

$$f\left(x_{i-1}, c_{i-1}, z_{i-1}^{IMU}\right) \propto \begin{cases} \dot{p}_i = v_i \\ \dot{v}_i = C^T\left(q_i\right)\left(a_i^m - b_i - n^a\right) - g \\ \dot{q}_i = \frac{1}{2}\Omega\left(\omega_i^m - d_i - n^g\right) q_i \end{cases}$$

Probabilistic Formulation - Models Definition

$$p\left(c_i \mid c_{i-1}\right)$$

Calibration Model:

$$c_i \doteq [d_i \quad b_i]$$

$$c_i = g\left(c_{i-1}\right) + e_{i-1}$$

$$Gaussian\ \ Noise: \ e_{i-1} \sim N\left(0, \Sigma_e\right)$$

Random constant model:

$$c_i = c_{i-1} + e_{i-1}$$

Probabilistic Formulation – Models Definition

$$p\left(z_i \mid \Theta_i^o\right)$$

Single Observation Model:

$$z_{i,j} = h\left(x_i, l_j\right) + v_i$$

$$Gaussian\ \ Noise: v_i \sim N\left(0, \Sigma_v\right)$$

$$h\left(x_i, l_j\right) - Pinhole\ \ Camera\ \ Model$$

General Observation Model:

$$p\left(z_k \mid \Theta_k^o\right) = \prod_{l_j \in \Theta_k^o} p\left(z_{k,j} \mid x_k, l_j\right)$$

$\Theta_k^o \subseteq \Theta_k$: Landmarks observed from x_k

Probabilistic Formulation - Models Definition

$$p\left(x_i \mid x_{i-1}, c_{i-1}, z_{i-1}^{IMU}\right)$$

Motion Model

$$x_i \doteq [p_i \quad v_i \quad q_i]$$

$$x_i = f\left(x_{i-1}, c_{i-1}, z_{i-1}^{IMU}\right) + w_{i-1}$$

$$w_{i-1} \sim N\left(0, \Sigma_w\right)$$

$$p\left(c_i \mid c_{i-1}\right)$$

Calibration Model

$$c_i \doteq [d_i \quad b_i]$$

$$c_i = g\left(c_{i-1}\right) + e_{i-1} = c_{i-1} + e_{i-1}$$

$$e_{i-1} \sim N\left(0, \Sigma_e\right)$$

$$p\left(z_i \mid \Theta_i^o\right)$$

Observation Model

$$z_{i,j} = h\left(x_i, l_j\right) + v_i$$

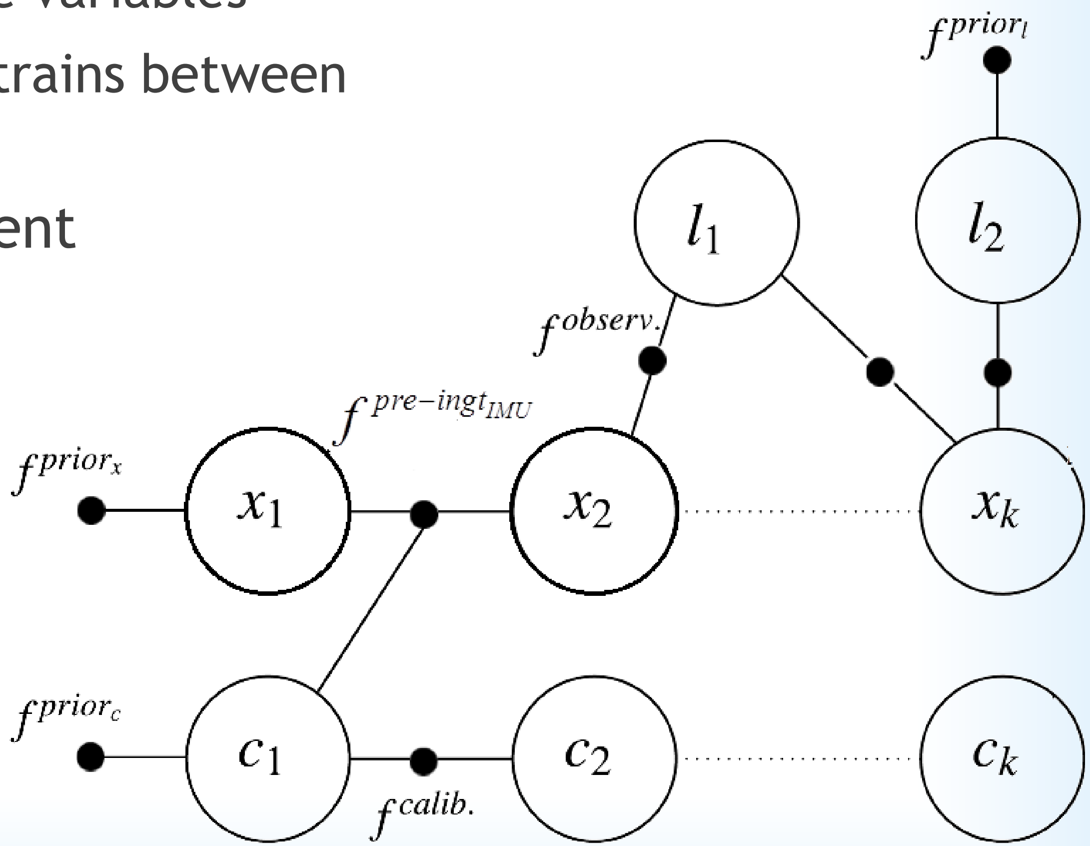
$$v_i \sim N\left(0, \Sigma_v\right)$$

$$p\left(z_k \mid \Theta_k^o\right) = \prod_{l_j \in \Theta_k^o} p\left(z_{k,j} \mid x_k, l_j\right)$$

$$\Theta_k^o \subseteq \Theta_k$$

Factor Graph Representation

- Factorization of a joint pdf in terms of process and measurement models
  - ***Vertices*** represent the variables
  - ***Nodes*** represent constrains between variables, the factors
- Computationally efficient probabilistic inference

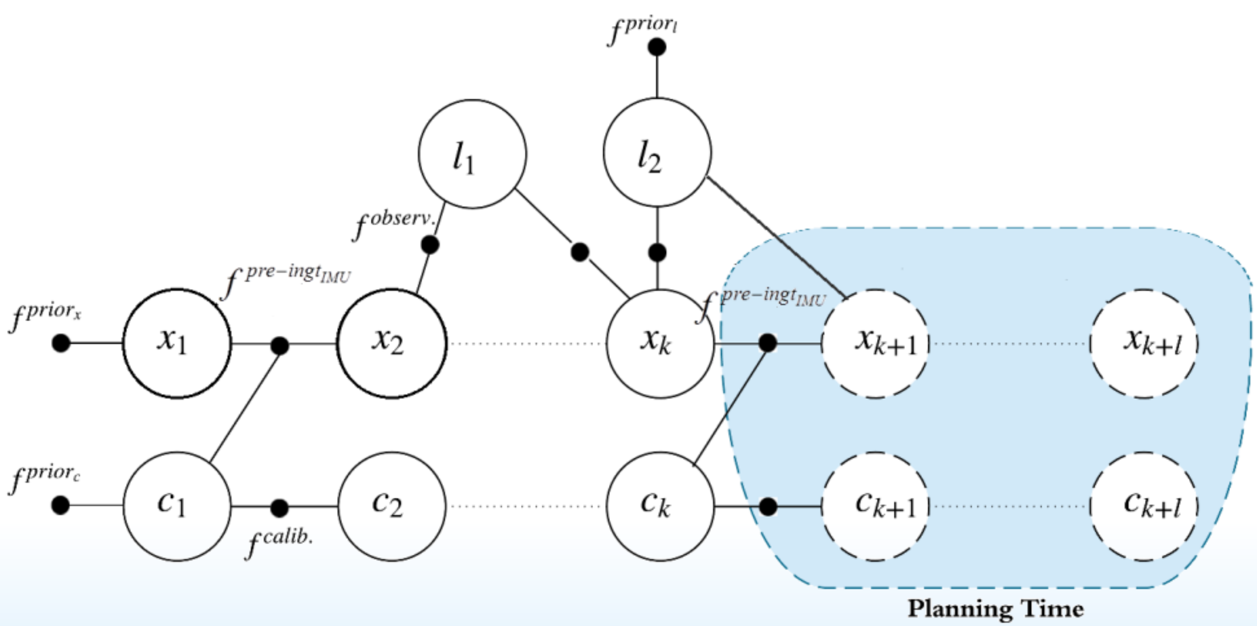


Belief Space Planning (BSP)

- The belief at the l^th look ahead time is defined as:

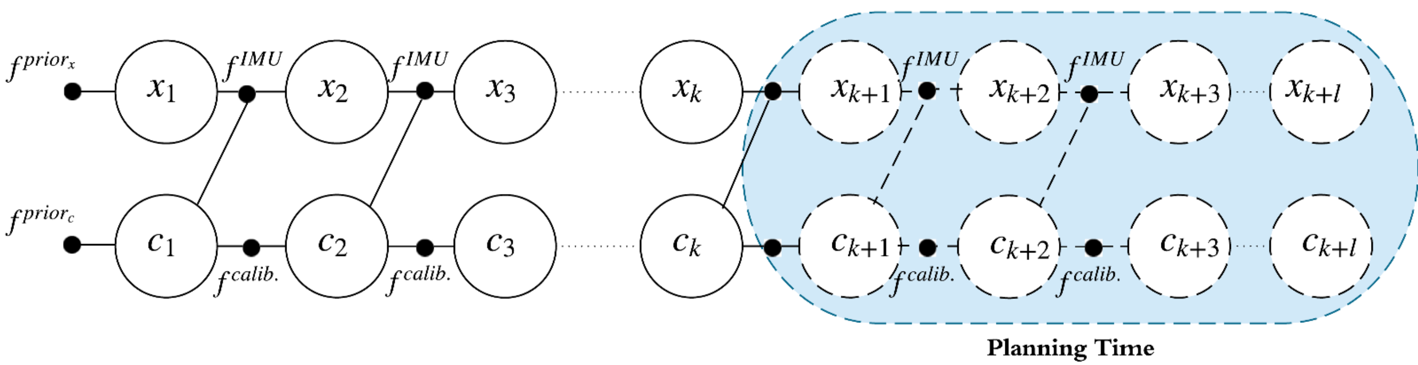
$$b\left(\Theta_{k+l}\right) \doteq p\left(\Theta_{k+l} \mid Z_{0:k+l}, U_{0:k+l-1}\right)$$
$$= \eta b\left(\Theta_{k+l-1}\right) p\left(x_{k+l} \mid x_{k+l-1}, u_{k+l-1}, c_{k+l-1}\right) \boxed{p\left(c_{k+l} \mid c_{k+l-1}\right)} p\left(z_{k+l} \mid \Theta^{o}_{k+l}\right)$$

- Maximum a posteriori (MAP) inference: $b\left(\Theta_{k+l}\right) = \mathcal{N}\left(\Theta^{\star}_{k+l}, \Sigma_{k+l}\right)$



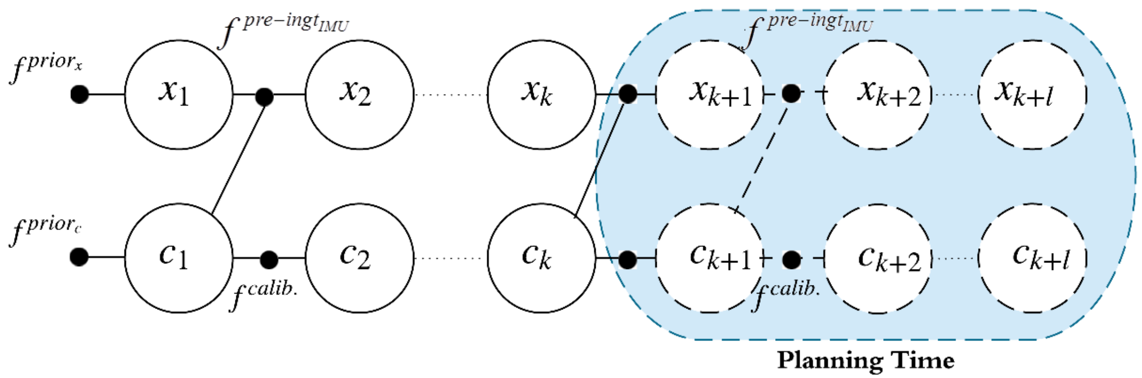
Factor Graph - Pre-Integrated IMU Factors

- **Challenge**:
  - High rate IMU measurements
  - Updating the graph at high rate
  - High complexity
  - No real time performance in inference, limits planning horizon in BSP



Factor Graph - Pre-Integrated IMU Factors

- **Solution:**
  - Integrate IMU measurements into a **single factor**
  - Add corresponding factor at the frequency of other sensors (e.g. camera)
  - Previous works use this concept within inference only
  - Additionally **use this concept within BSP** to increase planning horizon



[T. Lupton, 2012]
[V. Indelman, 2013]

Active Online Calibration Approach

- The objective function over a planning horizon of L steps:

$$J_k\left(b\left(\Theta_{k+L}\right), U_{k:k+L-1}\right) \doteq \sum_{l=0}^{L-1} \mathbb{E}\left(cf_l\left(b\left(\Theta_{k+l}\right), u_{k+l}\right)\right) + \mathbb{E}\left(cf_L\left(b\left(\Theta_{k+L}\right)\right)\right)$$

- Optimal control:

$$U^{\star}_{k:k+L-1} = \underset{U_{k:k+L-1}}{\operatorname{argmin}}\, J_k\left(b\left(\Theta_{k+L}\right), U_{k:k+L-1}\right)$$

- Choice of cost function

$$cf\left(b\left(\Theta_{k+l}\right), u_{k+l}\right) = \underbrace{\left\| X^{\star}_{k+l} - X^{Goal} \right\|_{M_\Theta}}_{\text{penalizes reaching the goal}} + \underbrace{\left\| \zeta\left(u_{k+l}\right) \right\|_{M_u}}_{\text{penalizes control actions}} + \boxed{\underbrace{tr\left(M_\Sigma \Sigma_{k+l} M_\Sigma^T\right)}_{\text{penalizes joint state uncertainty}} \triangleq cf^{\Sigma}\left(M_\Sigma, \Sigma_{k+l}\right)}$$

Cost function

Calculating The Optimal Control

- **Optimal control:**

$$U^{\star}_{k:k+L-1} = \underset{U_{k:k+L-1}}{\operatorname{argmin}} J_k\left(b\left(\Theta_{k+L}\right), U_{k:k+L-1}\right)$$

- **Discrete Methods:**
  - Choose best path from a set of candidate paths
  - Sampling methods (e.g. RRT or PRM)

- **Continuous Methods:**
  - Direct optimization or Gradient descent methods

Results – Assumptions

- Matlab simulation using GTSAM library
- Synthetic IMU measurements and camera observations
- Assuming data association is solved
- Assuming **heading angle control** only
- A priori-known regions with different uncertainty levels

IMU Calibration - Observability Aspects

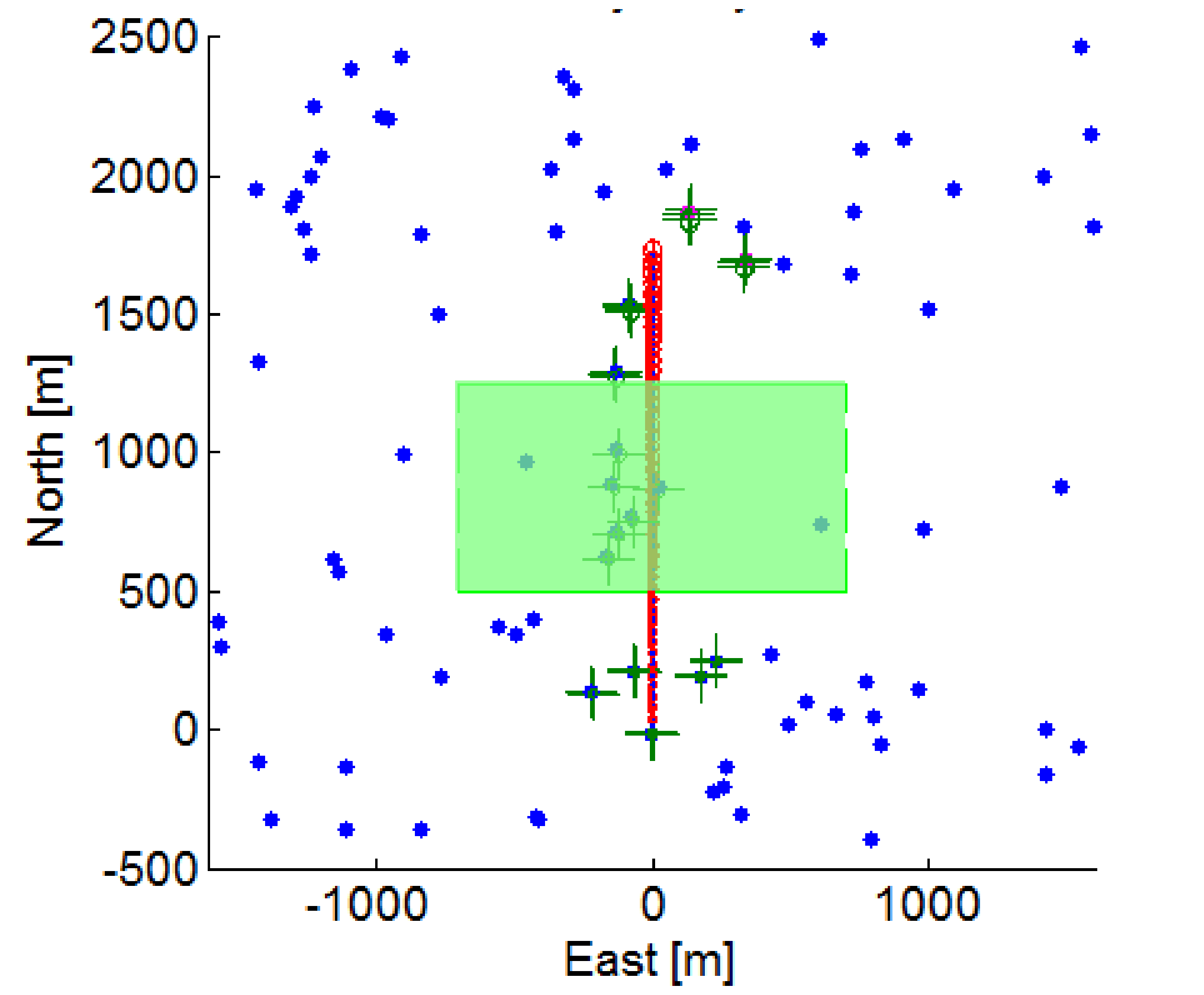
- **Theorem:** Full observability requires the camera-IMU platform to undergo rotation about at **least two IMU axes** and acceleration along **two IMU axes**

[Achtelik13icra]

- **Conclusion:** Heading angle control is not sufficient for full IMU calibration

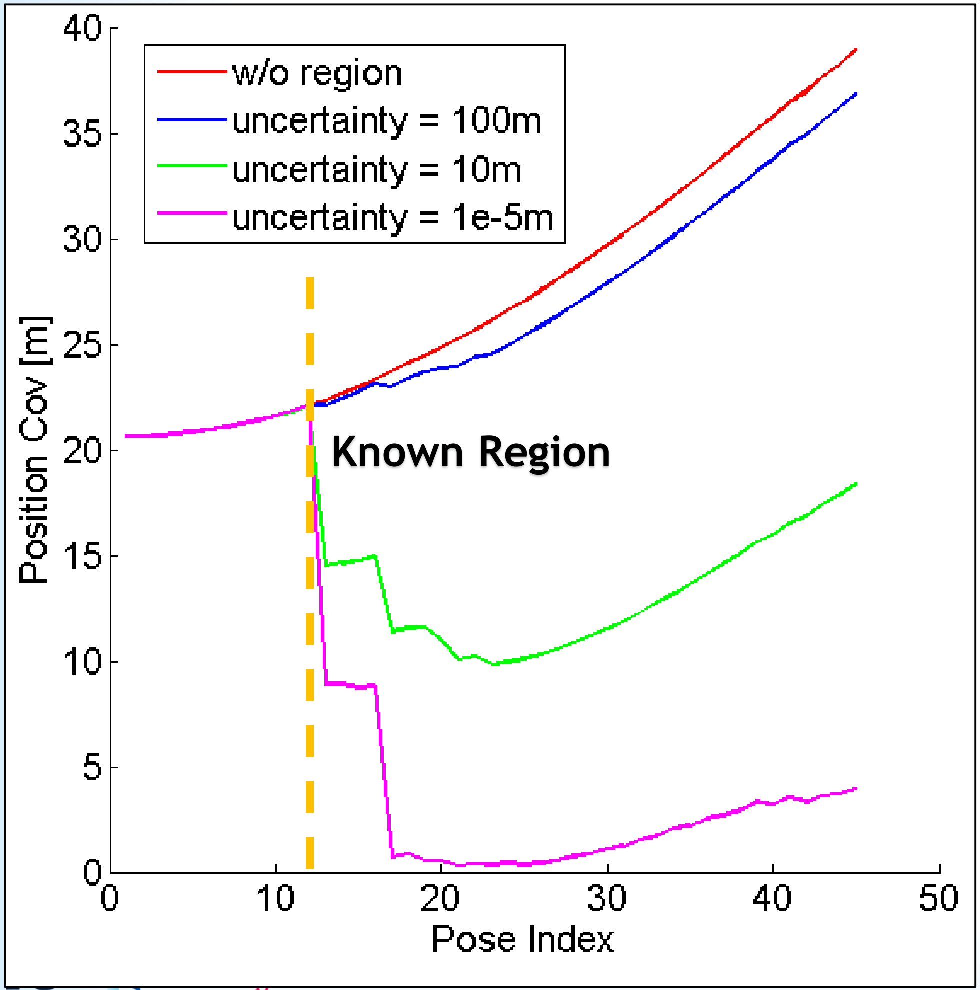
- **Alternatives:** Using a priori known regions with different levels of uncertainty to calibrate accelerometers only

Results - Influence of Known Regions

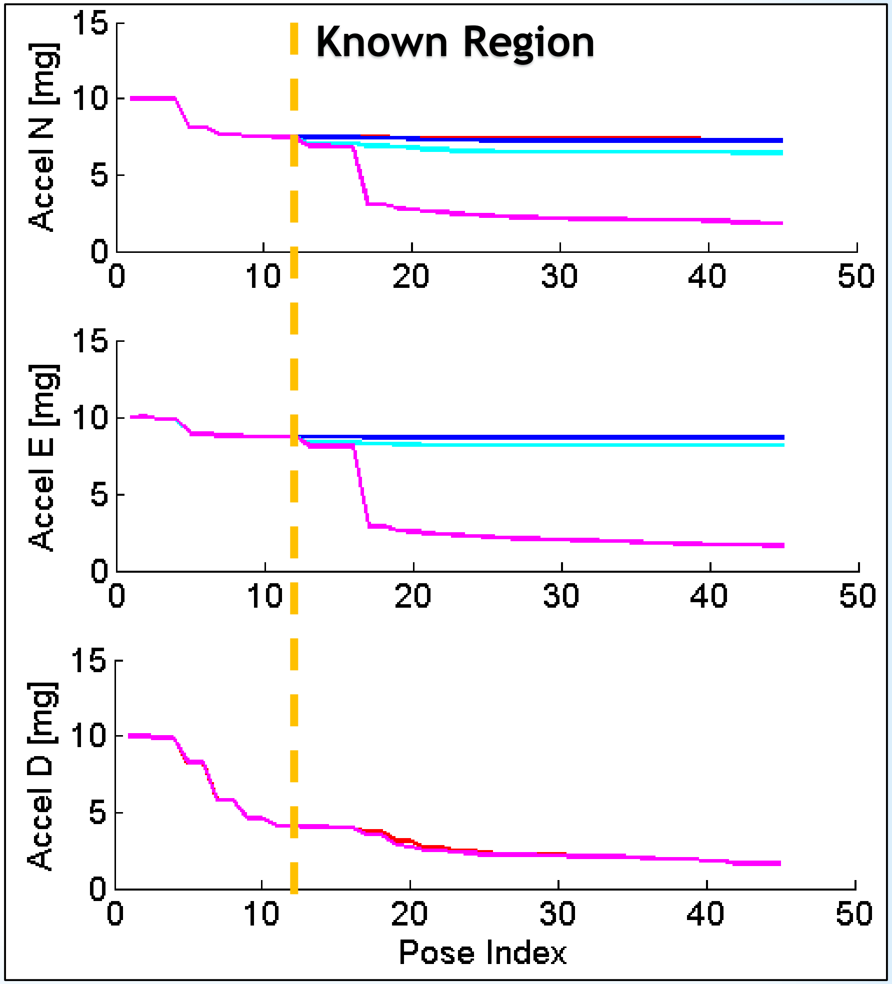


Results - Influence of Known Regions

Position Covariance



Accel. Calib. Covariance



Results – Scenario

- Environment
  - Randomly scattered unknown landmarks
  - Goal location within area without landmarks at all ("dark corridor")
  - A priori-known regions with different uncertainty levels

- Discrete method – generate candidate paths
  - Shortest path to goal
  - Shortest paths to clusters of mapped/known landmarks

Results - Compared Approaches

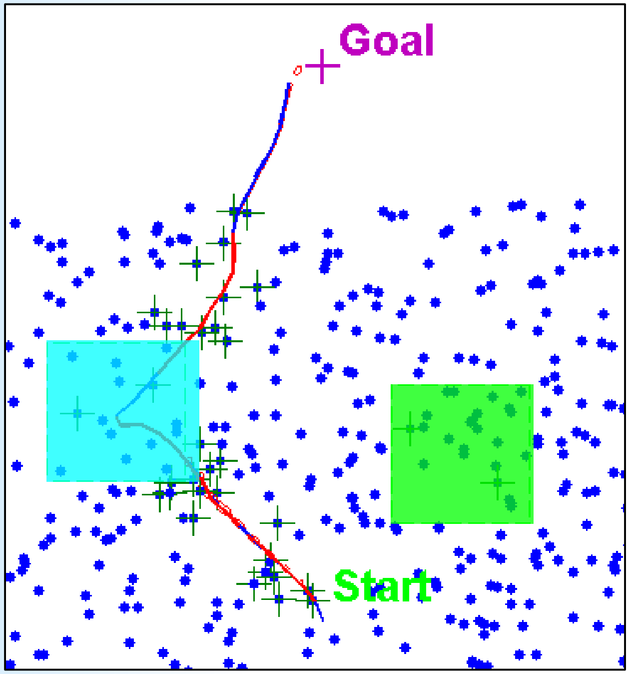
- Approach 1 - 'BSP-Calib' *(our approach)*
  - Incorporate sensor calibration states within the belief
  - "Trade-off" uncertainty cost function

$$cf^{\Sigma^{TO}} \doteq tr\left(M_{\Sigma_c}\Sigma_{k+l}M_{\Sigma_c}^{T}\right)+tr\left(M_{\Sigma_x}\Sigma_{k+l}M_{\Sigma_x}^{T}\right)$$

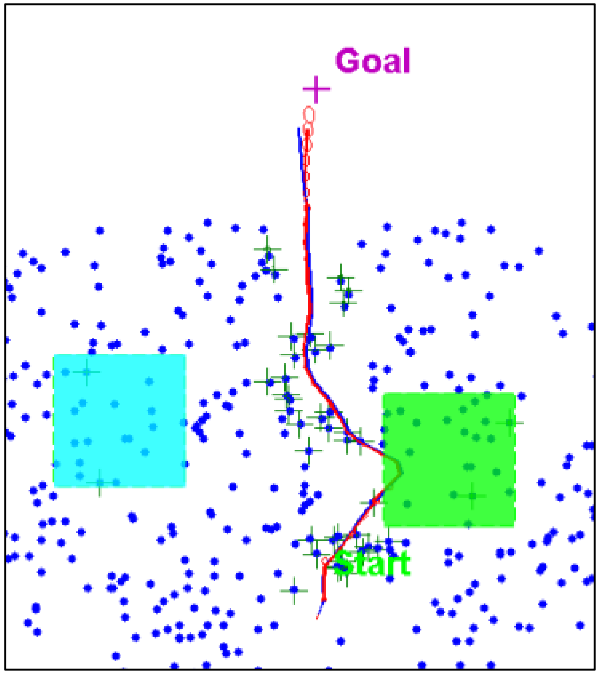
- Approach 2 - 'BSP'
  - Does not incorporate sensor calibration states within the belief
- Approach 3 - 'Shortest-Path'
  - Uncertainty cost function set to zero

Results - Compared Approaches (Trajectories)

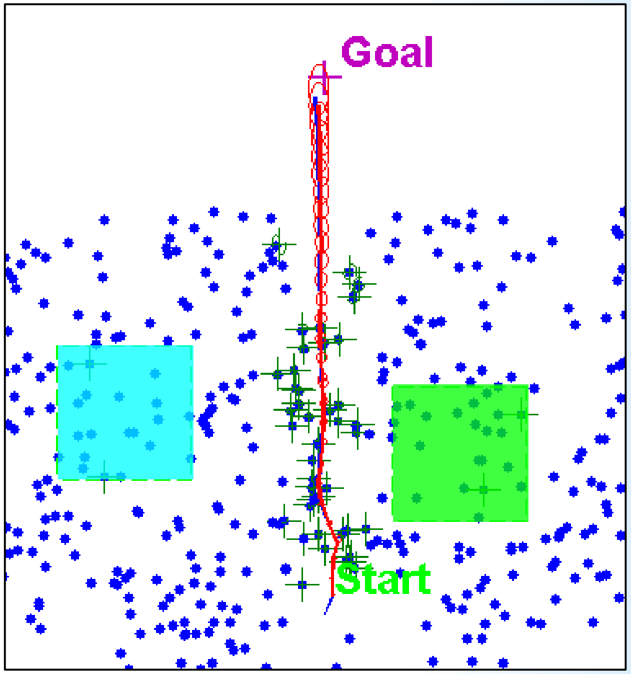
BSP-Calib



BSP



Shortest-Path



$$cf_l = \left\| E_{k+l}^{G} \Theta_{k+l}^{\star} - \Theta^{G} \right\|_{M_{\Theta}} + \left\| \zeta\left(u_{k+l}\right) \right\|_{M_u} + tr\left(M_{\Sigma_c} \Sigma_{k+l} M_{\Sigma_c}^{T} \right)$$

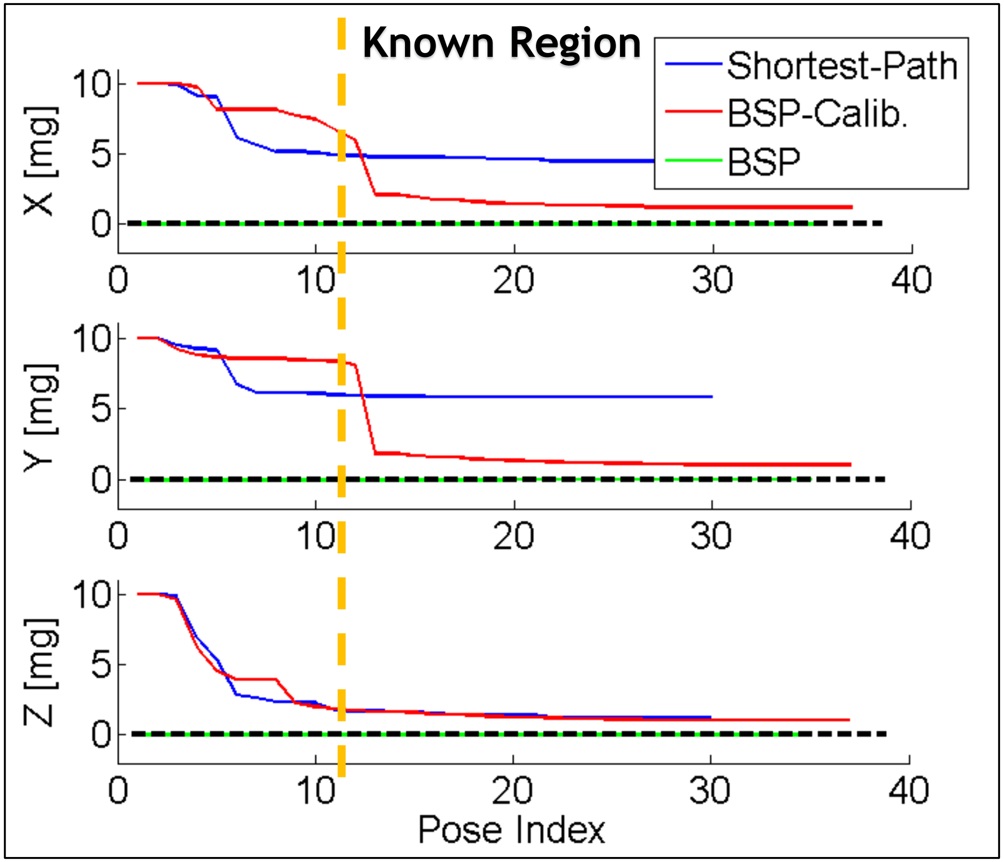
Uncertainty = 10m

Uncertainty = 1e-5m

- Covariance

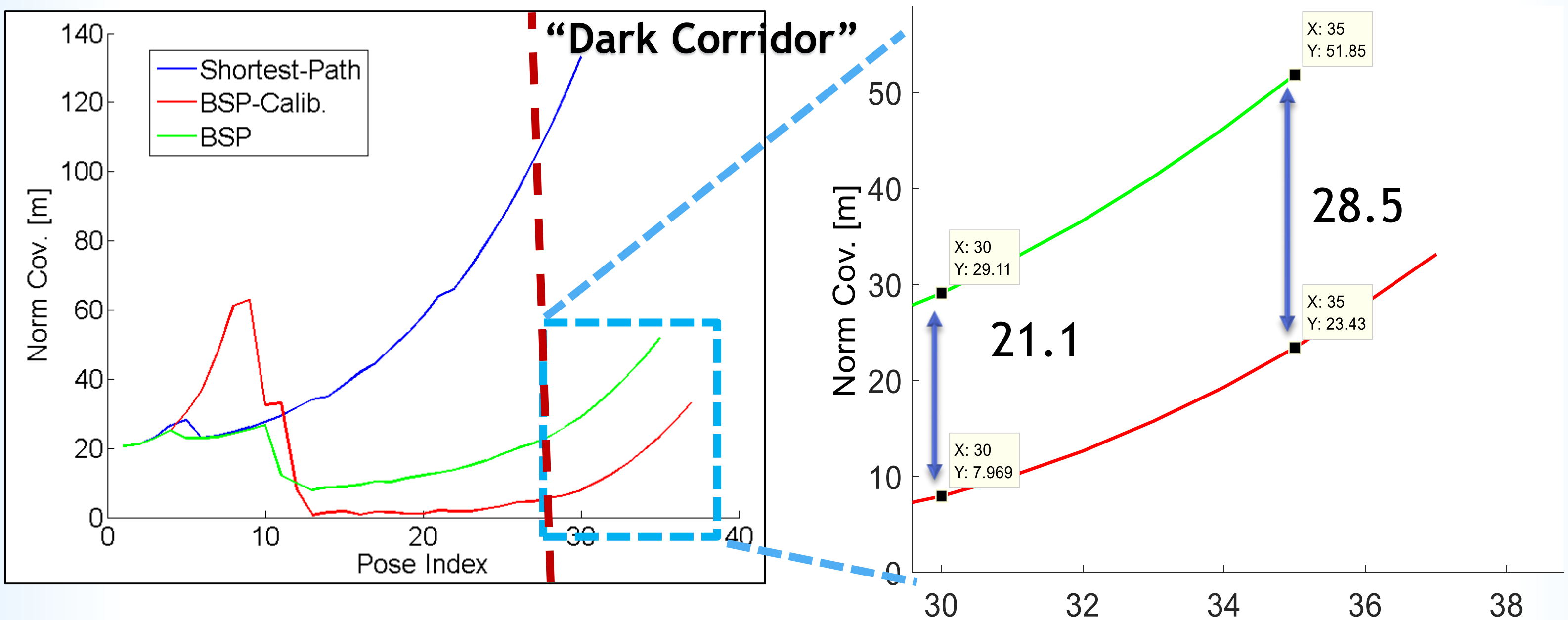
Results – Performance comparison

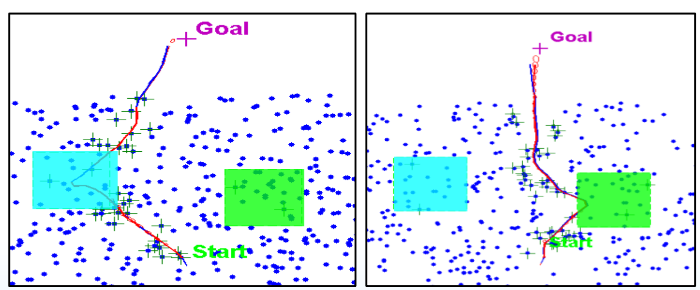
Accel. Calibration Covariance



Results - Performance comparison

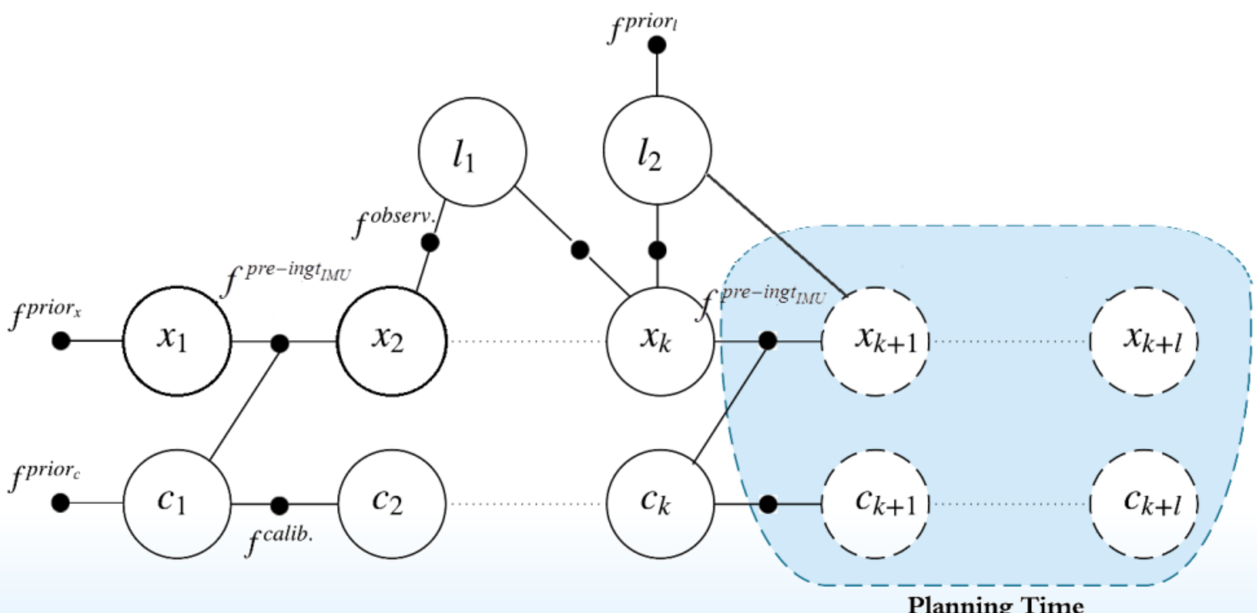
Position Covariance





Up Till Now

- Online active **self-calibration of IMU** in GPS-deprived environment using BSP
- Improved **navigation accuracy**
- Incorporated **IMU pre-integration concept within BSP** for longer planning horizons

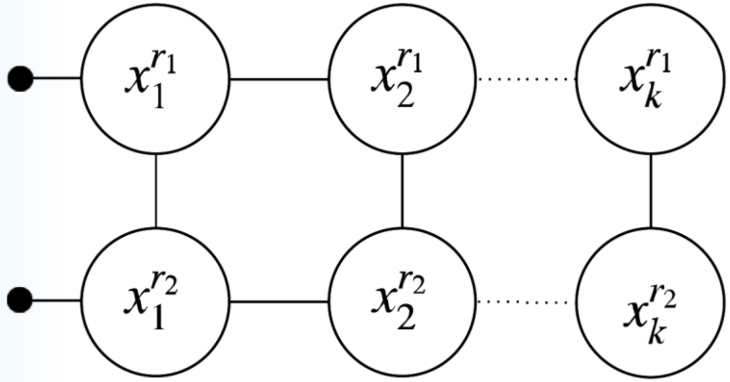


Multi-robot

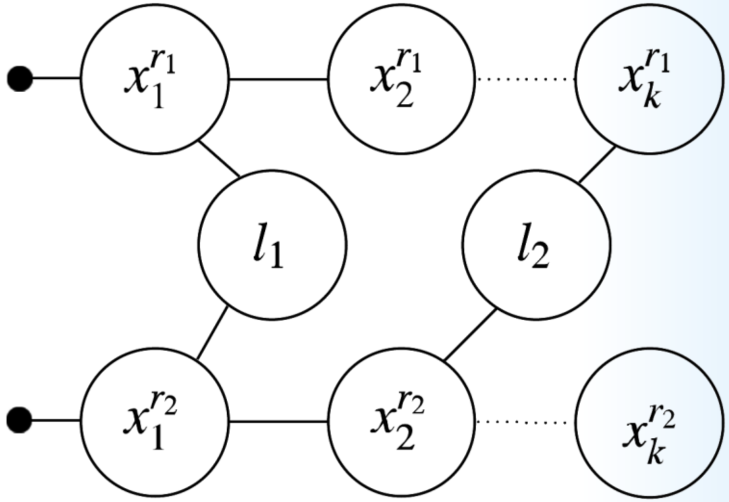
Introduction and Intuition

- **Cooperative multi-robot:**
  - Robust and faster exploration/mapping
  - Higher accuracy in a multi robot collaborative framework

Cooperation via observing other robots

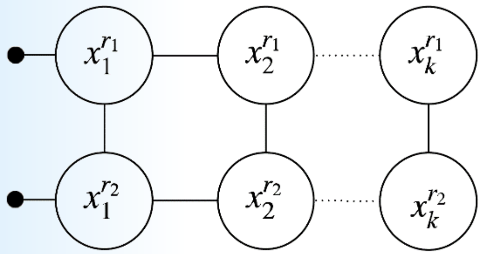


Cooperation via mutual landmark observations

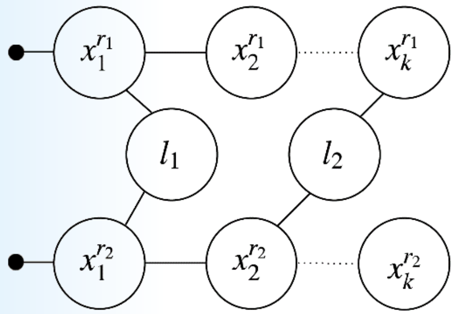


Indirect Update - Concept

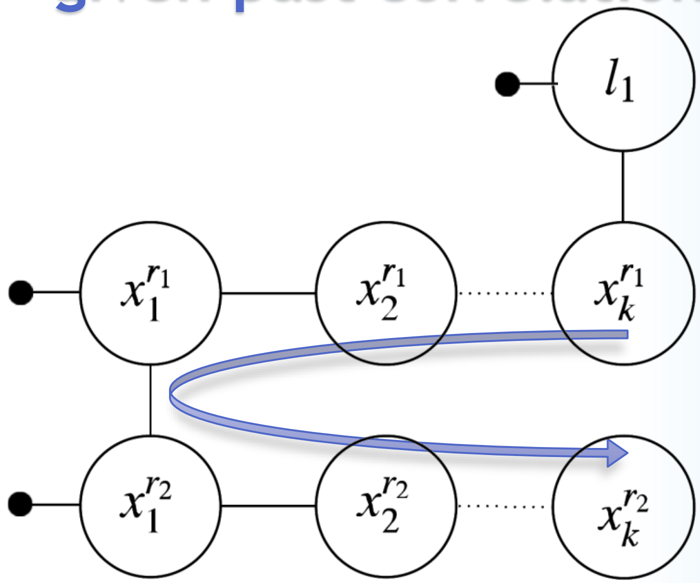
Cooperation via
observing other robots



Cooperation via
observing same landmark



Indirect cooperation
given past correlation



Given correlation,
all robots are updated when a
single robot observes a (known)
landmark

Key Observation

Given correlation,
all robots are updated when a ***single*** robot observes a (known) landmark

- Relaxing previous **cooperation constraints**
- Requires initial/past **correlation** between robots
- Given prior correlation, informative observation made by one robot, impacts also the states of other robots

Note: Study of correlation decay with time or sensitivity to correlation magnitude – not part of this work

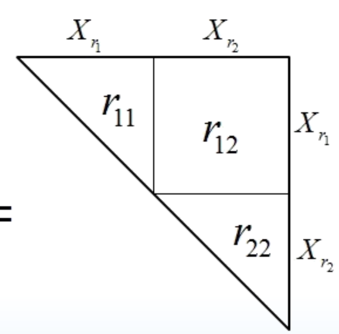
Indirect Update - Theoretical Aspects

- **Study case:**
  - Two robots, r_1 and r_2 , starting with some initial correlation
  - r_1 observes a prioiri known landmark
  - r_2 does not make any observations
- Definition of covariance matrix Σ or the information matrix I

$$I \doteq R^T R \, , \Sigma = I^{-1} \doteq \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

- **R -** The square root information matrix

$R \; =$



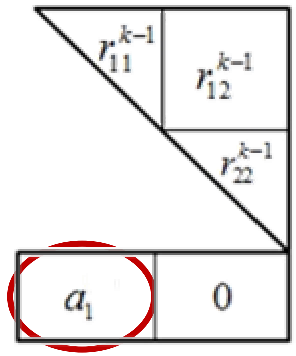
Indirect Update - Theoretical Aspects

- r_1 observes the known region at t_k

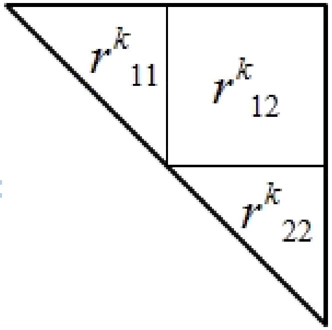
$$X \triangleq \left(x^{r_1} \quad x^{r_2}\right)^T \quad , \quad z = h\left(x^{r_1}\right) + v$$

$$A \doteq \underbrace{\nabla_X (h)}_{Jacob.} = \underbrace{\left[a_1 \quad \dots \quad a_n\right]}_{n = \#Robots} \underset{\substack{r_1 \\ observ.}}{=} \left[a_1 \quad 0\right]$$

$$\begin{bmatrix} R^{k-1} \\ R^{k-1} \\ A^k \end{bmatrix} =$$



$$R^k =$$



$$\Leftrightarrow \quad \Sigma^k = \begin{bmatrix} \Sigma_{11}^k & \Sigma_{12}^k \\ \Sigma_{21}^k & \Sigma_{22}^k \end{bmatrix}$$

Indirect Update - Theoretical Aspects

$$R^k = \begin{bmatrix} r^k_{11} & r^k_{12} \\ & r^k_{22} \end{bmatrix} \Leftrightarrow \Sigma^k = \begin{bmatrix} \Sigma^k_{11} & \Sigma^k_{12} \\ \Sigma^k_{21} & \Sigma^k_{22} \end{bmatrix}$$

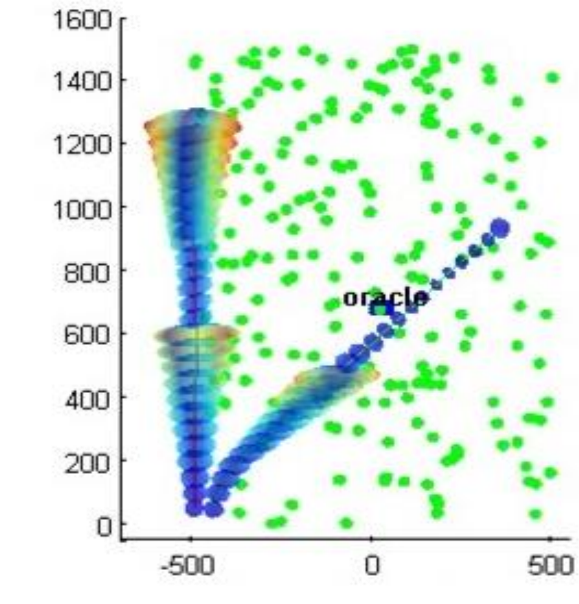
- Calculation of Σ_{22}

$$\Sigma^k_{22} = \left(r_{22}\right)^{-2} = \frac{\left(r^k_{11}\right)^2}{\left(r^{k-1}_{22}\right)^2 \left(r^k_{11}\right)^2 + \boxed{\left(-a_1 r^{k-1}_{12}\right)^2}}$$

Indirect Update – Study Case

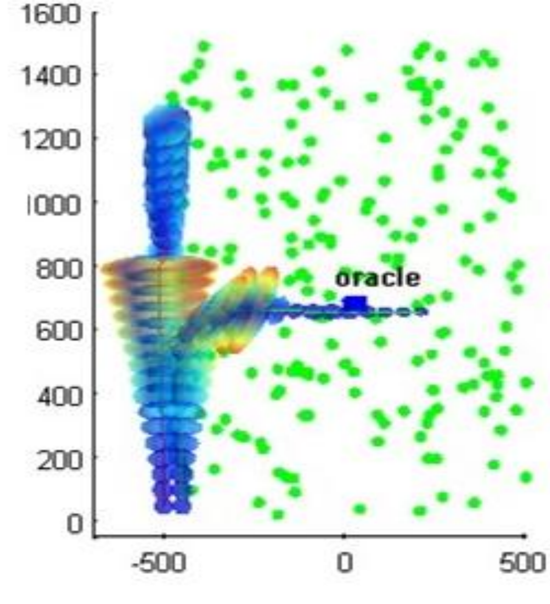
- Examined Trajectories:

"Shortest-Path"



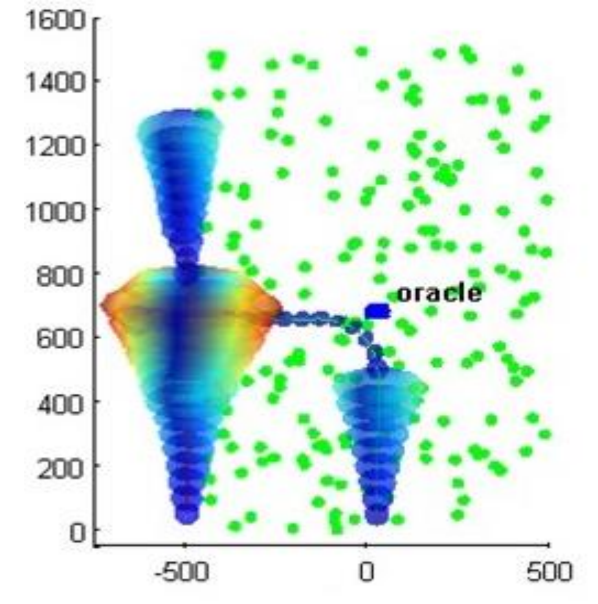
Low Prior Correlation

"Higher-Correlation"



High Prior Correlation

"Future-Correlation"

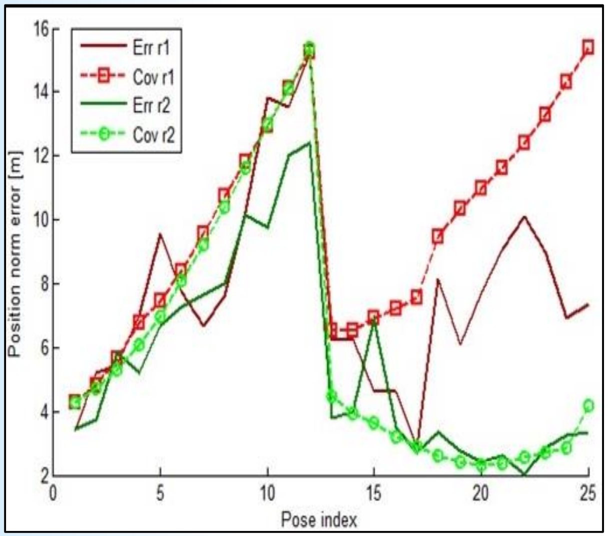


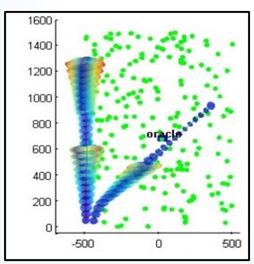
No Prior Correlation

Indirect Update – Study Case

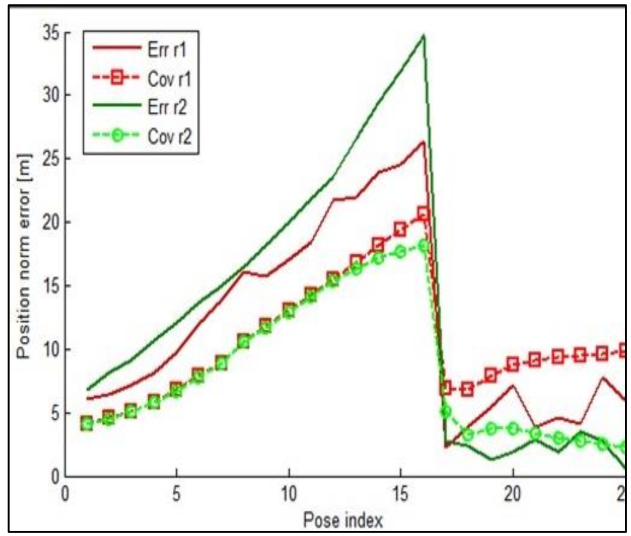
- Position Accuracy (errors and covariance):

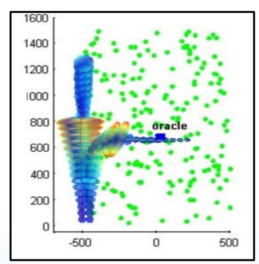
“Shortest-Path”



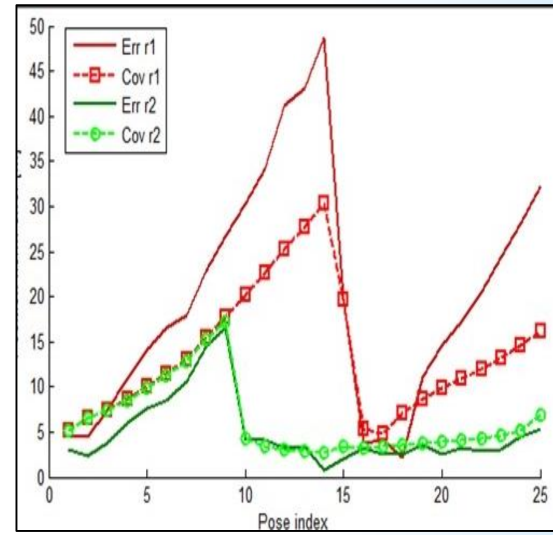


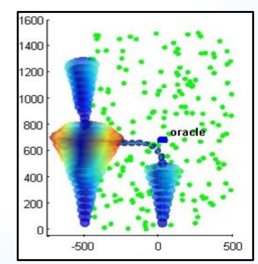
“Higher-Correlation”





“Future-Correlation”



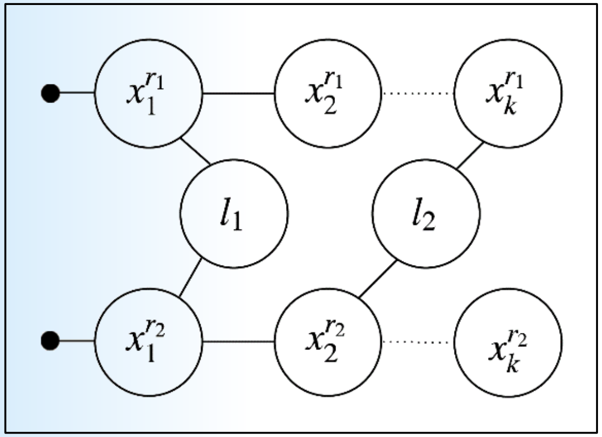


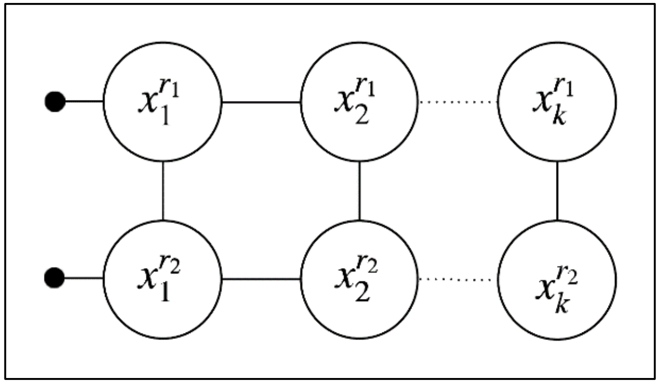
MR Belief Definition

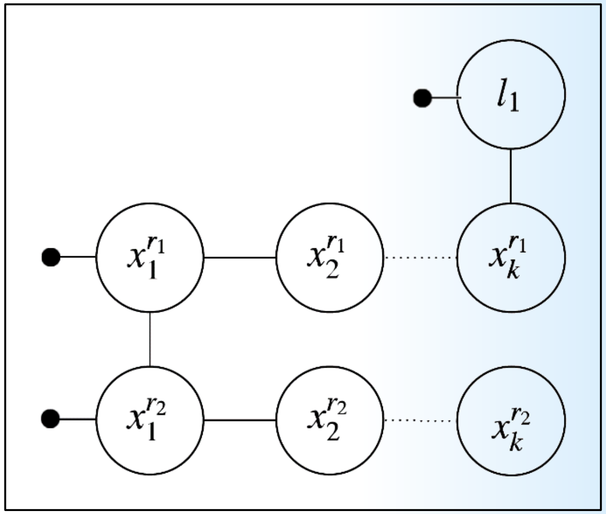
- The belief of R robots is defined as:

$$b\left(\Theta_k\right) \propto \prod_{r=1}^{R}\left\{ priors^r \prod_{i=1}^{k} p\left(x_i^r \mid x_{i-1}^r, c_{i-1}^r, z_{i-1}^{r,IMU}\right)\cdot p\left(c_i^r \mid c_{i-1}^r\right) p\left(Z_i^{r,cam.} \mid \Theta_i^{r,o}\right)\right\}$$

- Extendable to BSP for the lth look ahead time







Active Indirect Update Approach

- General objective function

$$J_k\left(b\left(\Theta_{k+L}\right),U_{k:k+L-1}\right)\doteq\sum_{l=0}^{L-1}\mathbb{E}\left(cf_l\left(b\left(\Theta_{k+l}\right),u_{k+l}\right)\right)+\mathbb{E}\left(cf_L\left(b\left(\Theta_{k+L}\right)\right)\right)$$

$$cf\doteq\left\{cf^i\right\}_{i=1}^{R}$$

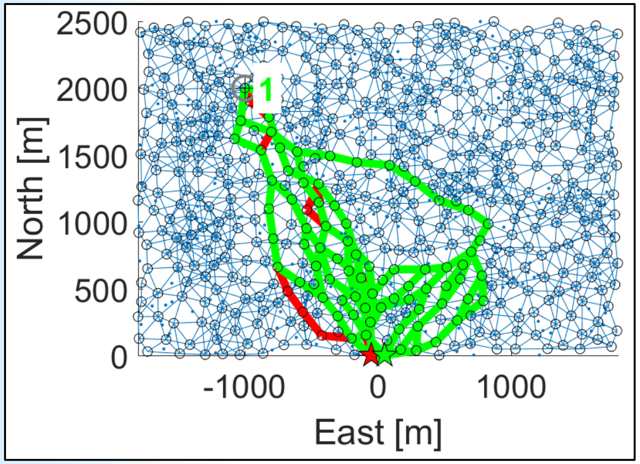
- Cost function

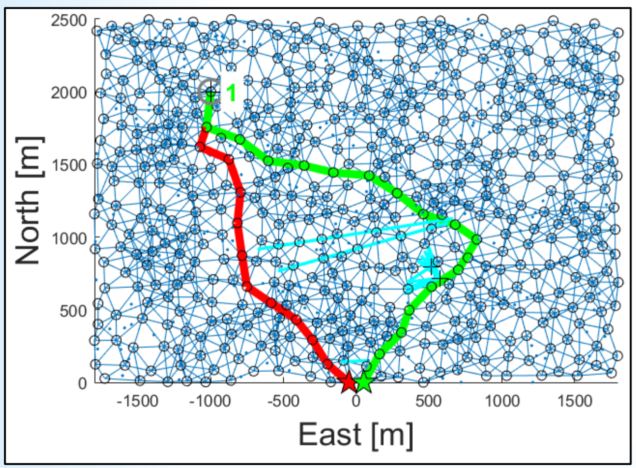
$$cf^i=\left\|\zeta\left(u_{k+l}\right)\right\|_{M_u}+\boxed{\sum_{r=1}^{R}\left\{\left\|X_{k+l}^{\star}-X^{Goal^r}\right\|_{M_\Theta^r}+cf^{\Sigma^r}\left(M_\Sigma^r,\Sigma_{k+l}^r\right)\right\}}$$

Results – Scenario and Assumptions

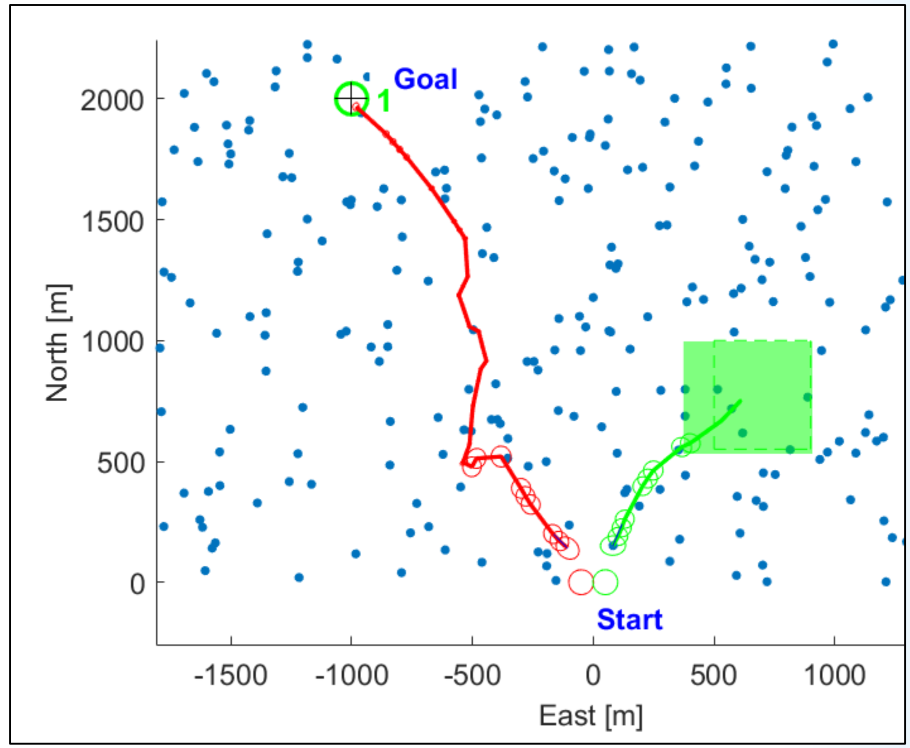
- Simulation – same as single robot
- Partial unknown environment
  - Mostly unknown environment
  - A priori-known regions with different uncertainty levels
- Discrete method – PRM
- Re-planning every 8 steps
- Initial Correlation is created by observing same landmark

Results - Trajectories





$$cf_l^1 = \left\| X_{k+l}^1 - X^{G^1} \right\|_{M_\Theta^1} \quad , \quad cf_l^2 = cf^{\Sigma^1}\left(M_\Sigma^2, \Sigma_{k+l}^1\right)$$



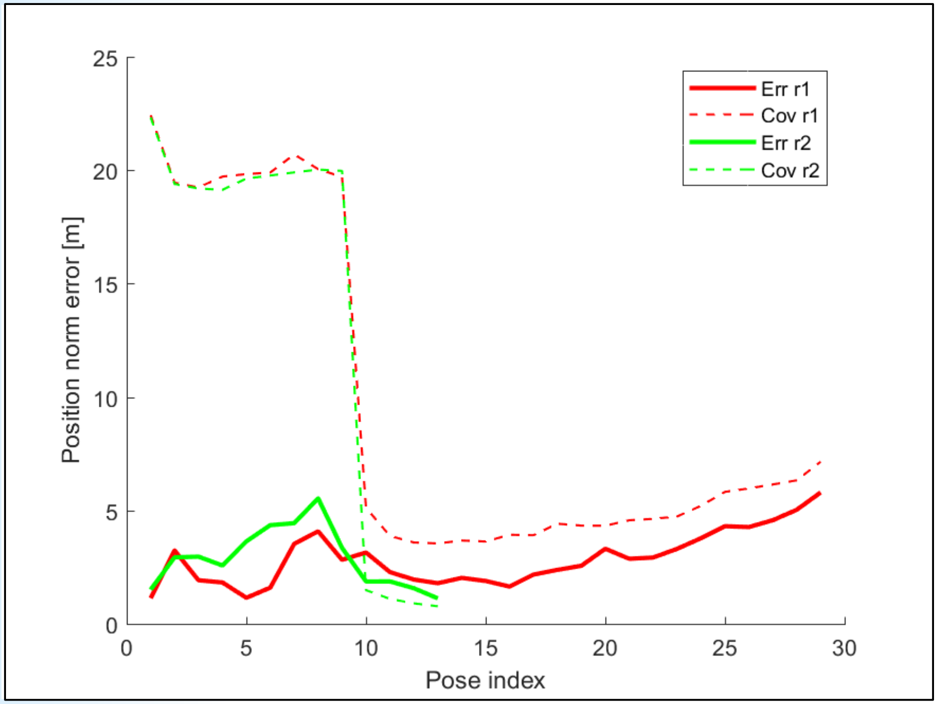
Uncertainty = 1e-5m

- Covariance

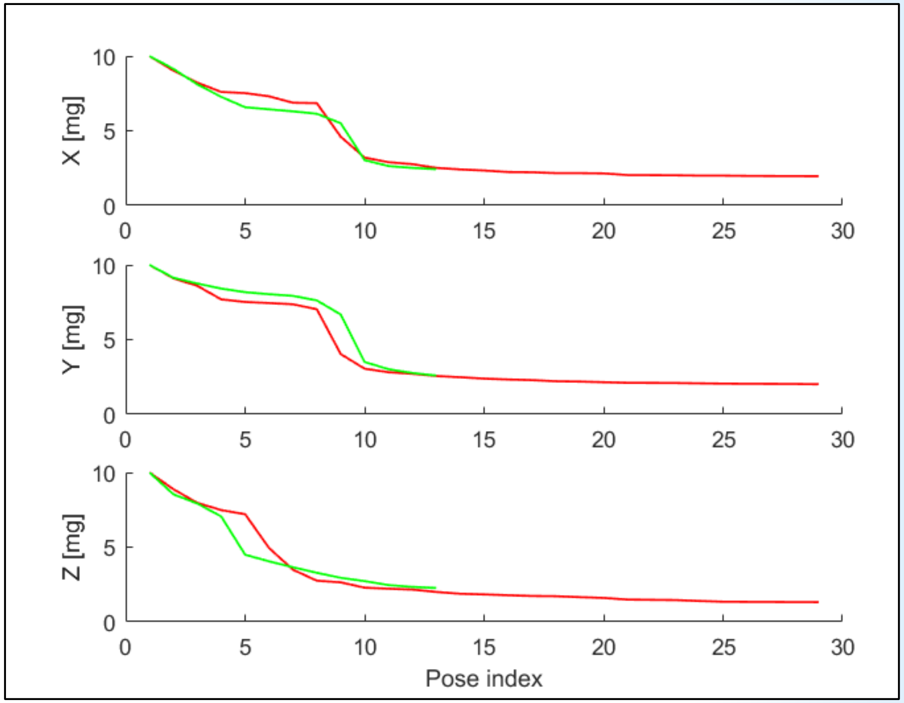
Results - Performance

$$\boxed{cf_l^1 = \left\| E_{k+l}^{G^1} \Theta_{k+l}^{\star} - \Theta^{G^1} \right\|_{M_\Theta^1}} \quad , \quad \boxed{cf_l^2 = cf^{\Sigma^1}\left(M_\Sigma^2, \Sigma_{k+l}^1 \right)}$$

Position Covariance & Error



Accel. Calibration Covariance



Conclusions

- Online **self-calibration of IMU** in GPS-deprived environment using BSP for improving **navigation accuracy**
- Cooperative multi-robot using **indirect updates within BSP**

Thank You

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