Scalable Sparsification for Efficient Decision Making Under Uncertainty in High Dimensional State Spaces

IROS 2017

KHEN ELIMELECH ROBOTICS AND AUTONOMOUS SYSTEMS PROGRAM
VADIM INDELMAN DEPARTMENT OF AEROSPACE ENGINEERING

TECHNION
Israel Institute of Technology

ANPL
Autonomous Navigation and Perception Lab

Decision Making under Uncertainty

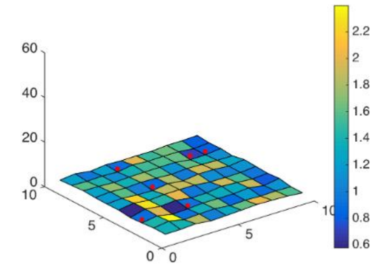
- Fundamental problem in robotics and AI

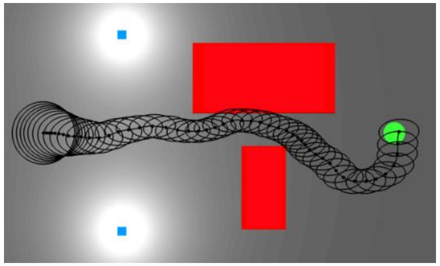
- Applications:
  - Active SLAM
  - Autonomous navigation
  - Object manipulation
  - Sensor deployment

- Treating uncertainty is <u>essential</u> for reliable and robust performance

But…

- <u>Decision making under uncertainty is computationally expensive</u>
  - Especially for high dimensional states





Our Contribution

Conventional methods

- Focus on optimizing properties of specific problems/scenarios
  [Indelman 2015], [Carlevaris-Bianco 2014]
- Naively evaluate a revenue/objective function for each candidate action
  [Kim, 2013], [Singh, 2009], [Krause, 2008]
- Sparsification is used for passive state inference
  [Mazuran 2014], [Huang 2012], [Vial 2011]

Our approach

- A general approach, focusing on the basis of the decision making
- Can be used alongside any other optimization method
- Sparsification is used only for efficient action selection – state inference stays exact!
- Extends the foundations from our recent work [ICRA 2017]

Belief Space Planning

- A belief is a stochastic state, given actions and obtained observations (POMDP):

$$b[X_k] \doteq \mathbb{P}(X_k | a_{0:k-1}, z_{0:k}) \sim \mathcal{N}^{-1}(\times, \Lambda)$$

- Updating the belief according to an action a and a future observation:

$$b[X_{k+1}] \doteq \mathbb{P}(X_{k+1} | a_{0:k-1}, z_{0:k}, a, z^a) \sim \mathcal{N}^{-1}(\times, \Lambda_{\mathrm{a}}^{+})$$

- The posterior information matrix of this future belief:

$$\Lambda_a^+ = \Lambda + A^T A$$

The collective Jacobian A encapsulates information regarding the transition and its following observation

Uncertainty and Revenue Calculation

- Measuring the uncertainty using entropy: $entropy(b) = 0.5 \cdot \ln \left[{(2\pi e)^n}/{\det(\Lambda)} \right]$
  - Calculating a determinant of the information matrix
  - $O(n^3)$ for n-dimensional belief

- Minimizing the uncertainty in future beliefs using the following revenue/reward function:

$$J(b,a) \doteq |\Lambda_a^+| = |\Lambda + A^T A|$$

- **The decision making problem is:**

$$a^* = \underset{a}{\operatorname{argmax}}\, J(b,a)$$

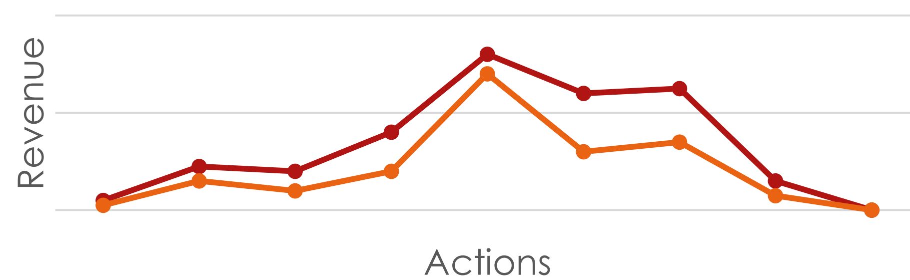
- **Do we have to explicitly calculate all future revenues?**

Action Consistency [ICRA 2017]

Definition: Two beliefs b, b_s are action consistent, if the following applies:

$$J(b,a_i) < J(b,a_j) \Leftrightarrow J(b_s,a_i) < J(b_s,a_j)$$
$$J(b,a_i) = J(b,a_j) \Leftrightarrow J(b_s,a_i) = J(b_s,a_j)$$

- Observations:
  - The relation between values is kept
  - No meaning for the actual values
  - Action selection is the the same



The Method: Using a **sparse** and **action consistent** approximation of Λ

Performance Improvement

Keeping Exact Results

$$J(b,a) \doteq |\Lambda_a^{+}| = |\Lambda + A^T A|$$

This Work… Extended Analysis

- Examining more **general approximations** for improved performance,
  or when action consistency cannot be proven

- A sub optimal action selection can occur

- Setting **bounds over the induced error** is critical to ensure safe operation and provide guaranteed results

Bounding the Error

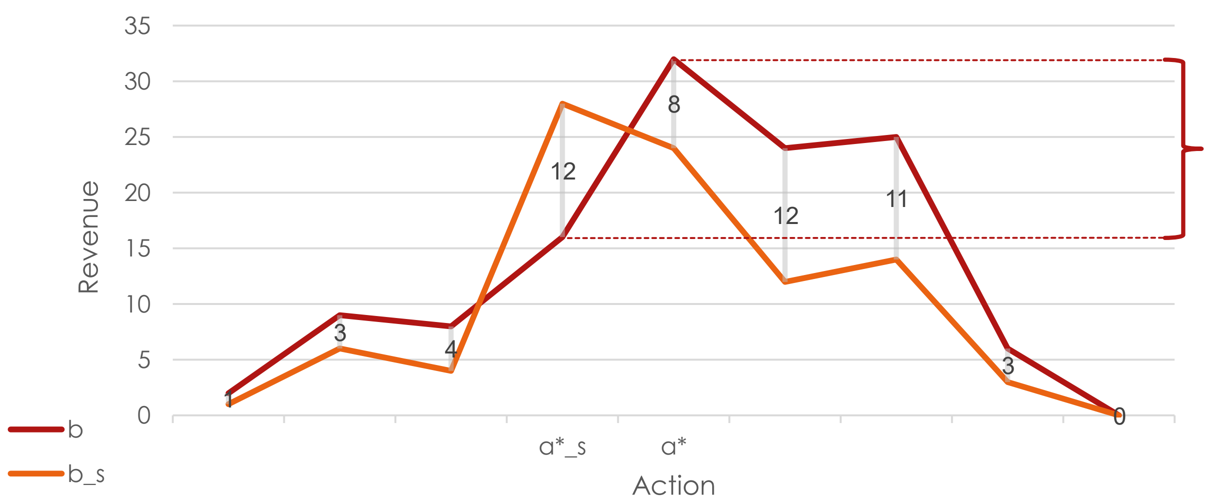
- **An intuitive "metric" between states, in the context of decision making**

Definition: The **revenue offset of an action** a is:

$$\gamma(b, b_s, a) \doteq |J(b,a) - J(b_s,a)|$$

The **revenue offset between the two states** is:

$$\gamma(b, b_s) \doteq \max_a \gamma(b, b_s, a)$$

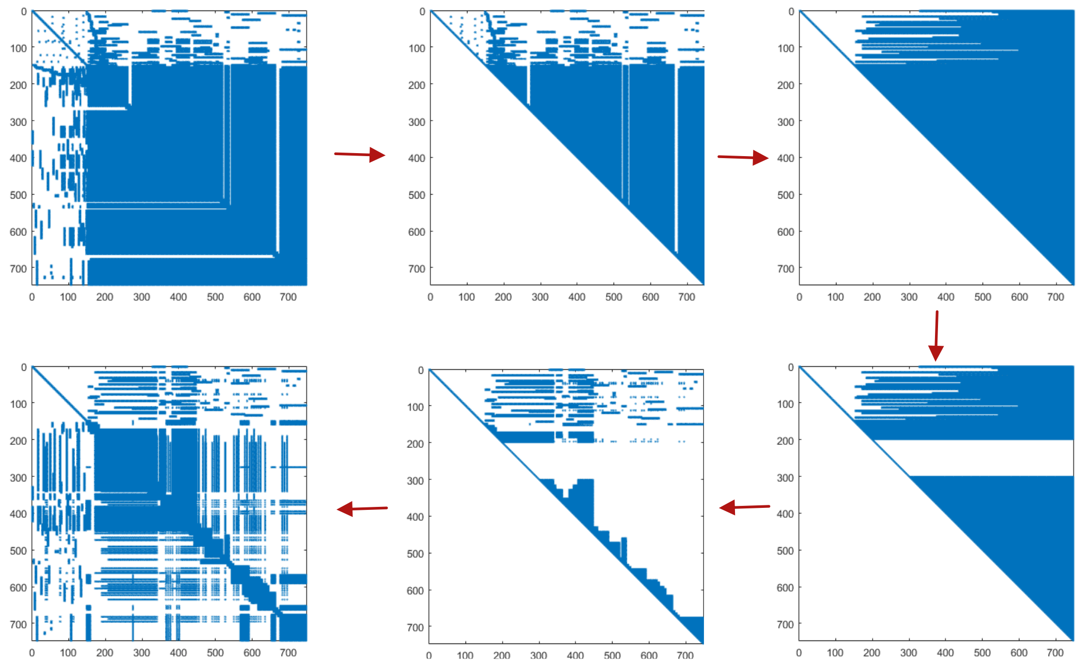


$$error \leq 2 \cdot \gamma(b, b_s)$$

K. Elimelech and V. Indelman
Scalable Sparsification for Efficient Decision Making under Uncertainty in High Dimensional State Spaces

Generating a Sparse Approx.

1. Information Matrix 2. Root 3. Inverse

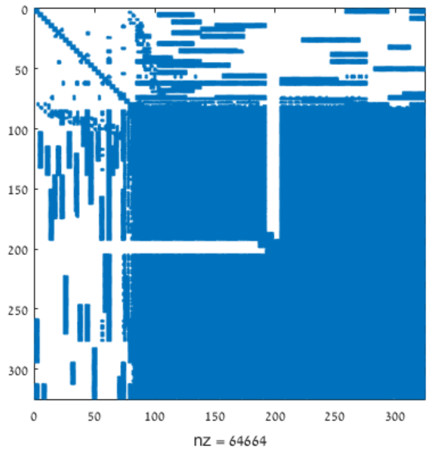


6. Sparse Information 5. Inverse Back 4. Remove Rows

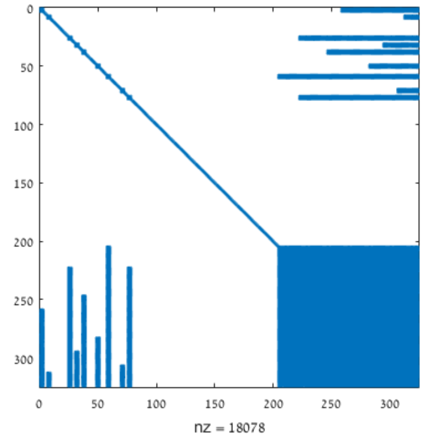
Scalable degree of sparsification is dependent on a selection of rows

* Matrices of a SLAM problem. The state vector holds all previous poses and observed landmarks.

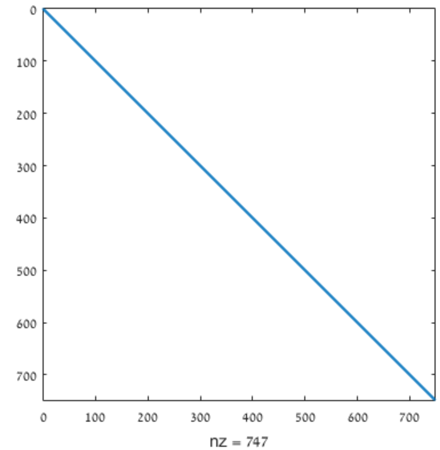
Scalability



Original Information Matrix



Uninvolved variables



All variables

Action Consistent Approx.

Method Summary

Initial belief b

Identify uninvolved variables

Find a **sparse** approximation b_s using our suggested algorithm

Bound the potential loss using the revenue offset

Easily calculate all action revenues using b_s

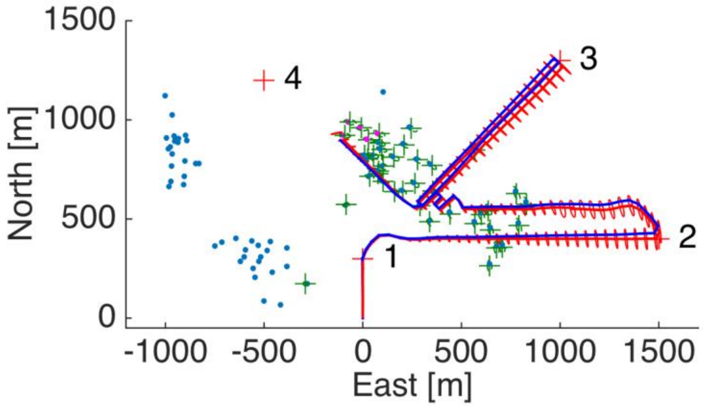
Select best candidate action

Apply action on the original belief b

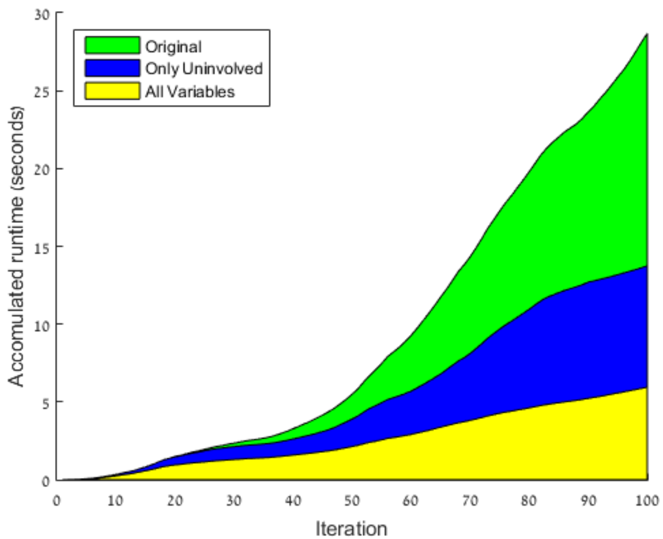
Only a <u>single sparsification</u> is required per decision, for any number of actions

SLAM Scenario

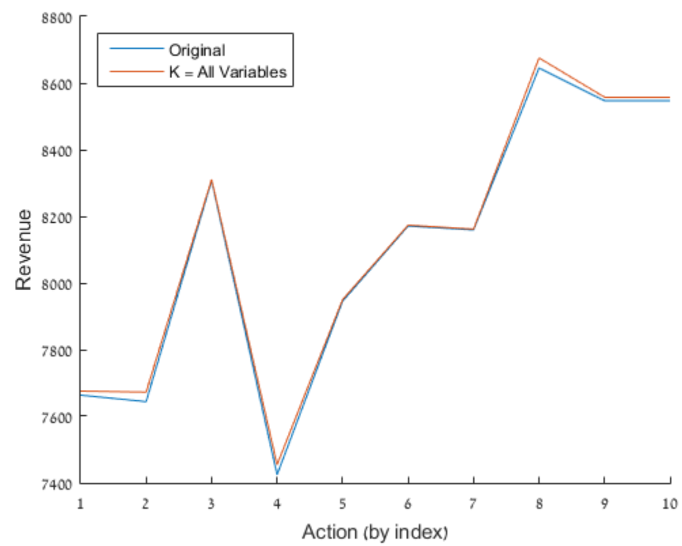
- Navigating through several predefined world points
- The state vector maintains the entire trajectory and positions of observed landmarks
- The actions refer to taking short paths to clusters around the robot
- Low uncertainty throughout the trajectory by preferring more informative actions
- **Measured the time to make each decision, for the 3 sparsification levels**



Results Comparison



Accumulation of the measured decision making time



Revenue offset compared to a fully sparsified belief

Follow up work is coming in ISRR 2017!

Find out more on:

www.khen.io

khen@campus.technion.ac.il