Scalable Sparsification for Efficient Decision Making Under Uncertainty in High Dimensional State Spaces

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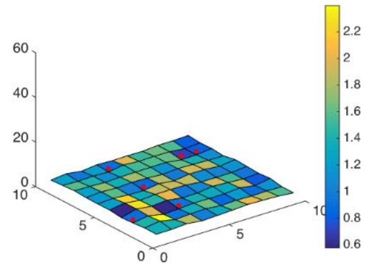
Decision Making under Uncertainty

- Fundamental problem in robotics and AI

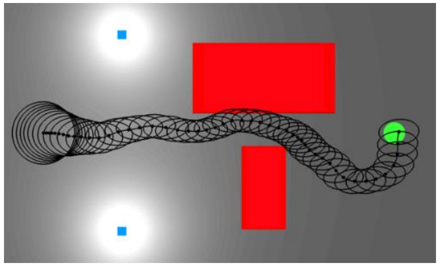
- Applications:
  - Active SLAM
  - Autonomous navigation
  - Object manipulation
  - Sensor deployment

- Treating uncertainty is <u>essential</u> for reliable and robust performance



But…

- <u>Decision making under uncertainty is computationally expensive</u>
  - Especially for high dimensional states



Our Contribution

Conventional methods

- Focus on optimizing properties of specific problems/scenarios
  [Indelman 2015], [Carlevaris-Bianco 2014]
- Naively evaluate a revenue/objective function for each candidate action
  [Kim, 2013], [Singh, 2009], [Krause, 2008]
- Sparsification is used for passive state inference
  [Mazuran 2014], [Huang 2012], [Vial 2011]

Our approach

- A general approach, focusing on the basis of the decision making
- Can be used alongside any other optimization method
- Sparsification is used only for efficient action selection – state inference stays exact!
- Extends the foundations from our recent work [ICRA 2017]

Belief Space Planning

- A belief is a stochastic state, given actions and obtained observations (POMDP):

$$b[X_k] \doteq \mathbb{P}(X_k | a_{0:k-1}, z_{0:k}) \sim \mathcal{N}^{-1}(\times, \Lambda)$$

- Updating the belief according to an action a and a future observation:

$$b[X_{k+1}] \doteq \mathbb{P}(X_{k+1} | a_{0:k-1}, z_{0:k}, a, z^a) \sim \mathcal{N}^{-1}(\times, \Lambda_{\mathrm{a}}^{+})$$

- The posterior information matrix of this future belief:

$$\Lambda_a^+ = \Lambda + A^T A$$

The collective Jacobian A encapsulates information regarding the transition and its following observation

Uncertainty and Revenue Calculation

- Measuring the uncertainty using entropy: $entropy(b) = 0.5 \cdot \ln\left[{(2\pi e)^n}/{\det(\Lambda)}\right]$
  - Calculating a determinant of the information matrix
  - $O(n^3)$ for n-dimensional belief

- Minimizing the uncertainty in future beliefs using the following revenue/reward function:

$$J(b,a) \doteq |\Lambda_a^+| = |\Lambda + A^T A|$$

- **The decision making problem is:**

$$a^* = \underset{a}{\operatorname{argmax}}\, J(b,a)$$

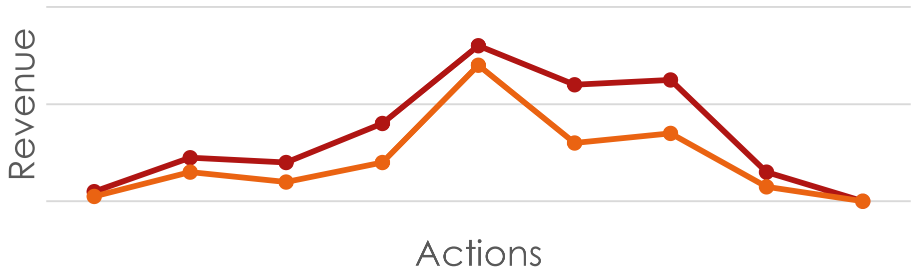
- **Do we have to explicitly calculate all future revenues?**

Action Consistency [ICRA 2017]

Definition: Two beliefs b, b_s are action consistent, if the following applies:

$$J(b,a_i) < J(b,a_j) \Leftrightarrow J(b_s,a_i) < J(b_s,a_j)$$
$$J(b,a_i) = J(b,a_j) \Leftrightarrow J(b_s,a_i) = J(b_s,a_j)$$

- Observations:
  - The relation between values is kept
  - No meaning for the actual values
  - Action selection is the the same



The Method: Using a **sparse** and **action consistent** approximation of Λ

Performance Improvement

Keeping Exact Results

$$J(b,a) \doteq |\Lambda_a^+| = |\Lambda + A^T A|$$

This Work… Extended Analysis

- Examining more **general approximations** for improved performance,
  or when action consistency cannot be proven

- A sub optimal action selection can occur

- Setting **bounds over the induced error** is critical to ensure safe operation and provide guaranteed results

Bounding the Error

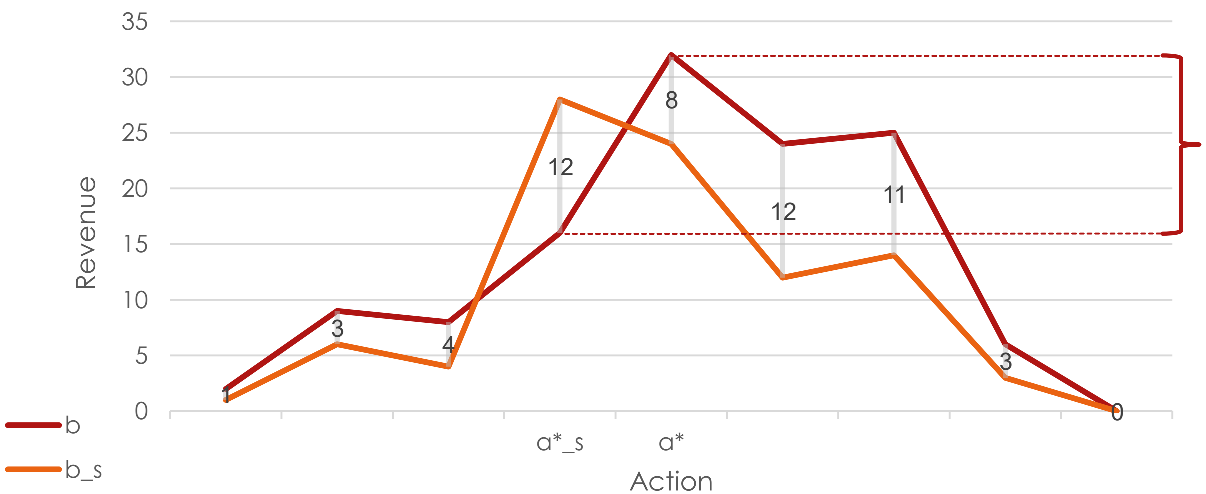
- **An intuitive "metric" between states, in the context of decision making**

Definition: The **revenue offset of an action** a is:

$$\gamma(b, b_s, a) \doteq |J(b, a) - J(b_s, a)|$$

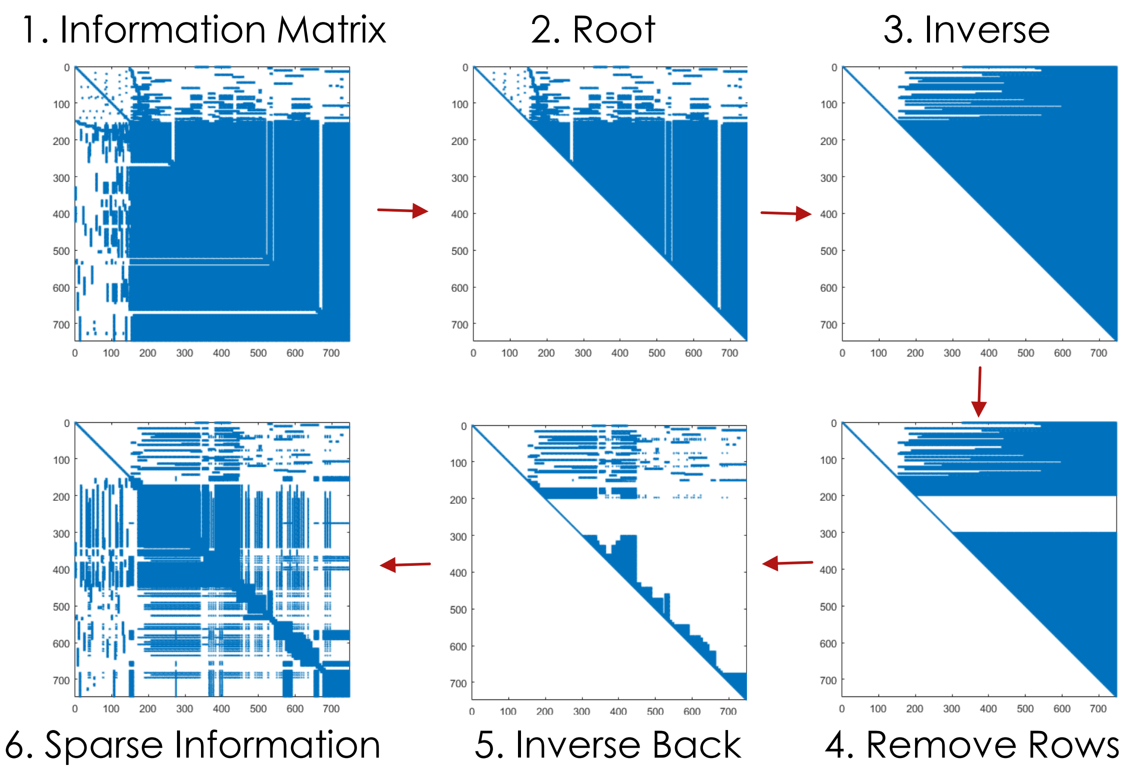
The **revenue offset between the two states** is:

$$\gamma(b, b_s) \doteq \max_{a} \gamma(b, b_s, a)$$



$$\boxed{error \leq 2 \cdot \gamma(b, b_s)}$$

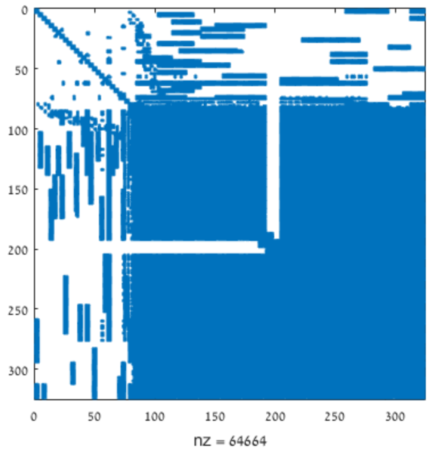
Generating a Sparse Approx.



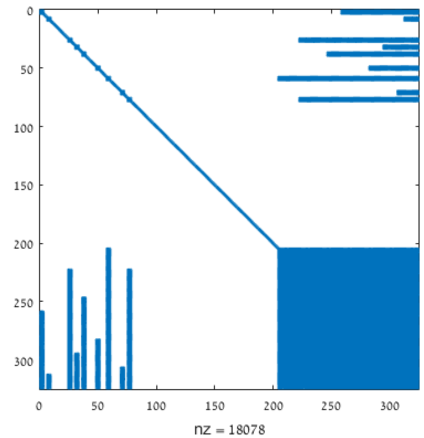
Scalable degree of sparsification is dependent on a selection of rows

* Matrices of a SLAM problem. The state vector holds all previous poses and observed landmarks.

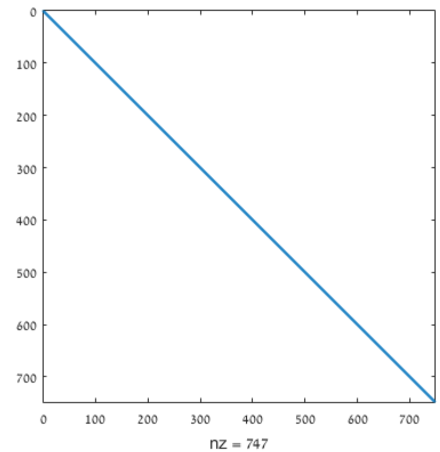
Scalability



Original Information Matrix



Uninvolved variables



All variables

Action Consistent Approx.

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Method Summary

Initial belief b

Identify uninvolved variables

Find a **sparse** approximation b_s using our suggested algorithm

Bound the potential loss using the revenue offset

Easily calculate all action revenues using b_s

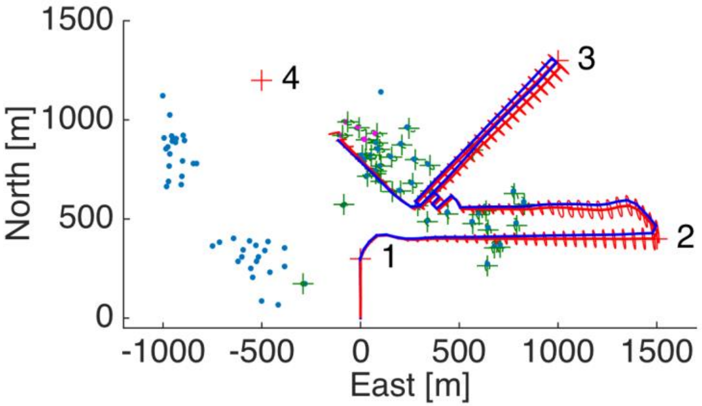
Select best candidate action

Apply action on the original belief b

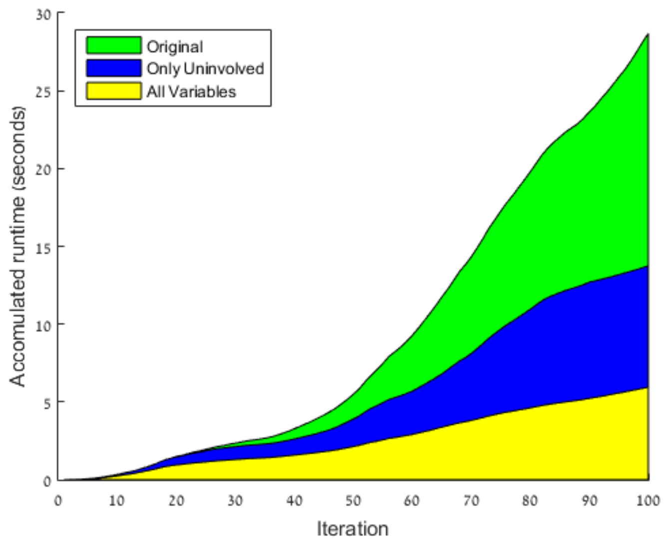
Only a <u>single sparsification</u> is required per decision, for any number of actions

SLAM Scenario

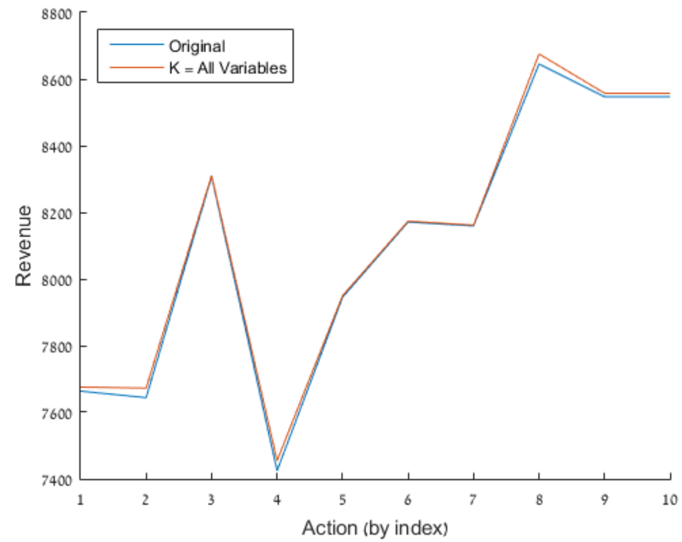
- Navigating through several predefined world points
- The state vector maintains the entire trajectory and positions of observed landmarks
- The actions refer to taking short paths to clusters around the robot
- Low uncertainty throughout the trajectory by preferring more informative actions
- **Measured the time to make each decision, for the 3 sparsification levels**



Results Comparison



Accumulation of the measured decision making time



Revenue offset compared to a fully sparsified belief

Follow up work is coming in ISRR 2017!

Find out more on:

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