iX-BSP: Belief Space Planning through Incremental Expectation
Supplementary Material

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This document provides supplementary material to the paper [2]. Therefore, it should not be considered a self-contained document, but instead regarded as an appendix of [2], and cited as:

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Throughout this report, all notations and definitions are with compliance to the ones presented in [2].

In order to address the more general and realistic scenario as presented in [1], the DA might require correction before proceeding to update the new acquired measurements. This report covers the possible scenarios of inconsistent data association and its graphical materialization - Appendix A, followed by a paradigm to update inconsistent DA from planning stage according to the actual DA attained in the consecutive inference stage - Appendix B.

Appendix A: Types of inconsistent DA

We would now discuss, without loosing generality, the actual difference between the two beliefs $b[X_{k+1|k}]$ and $b[X_{k+1|k+1}]$. As already presented in [1], in case of a consistent DA i.e. $\mathcal{M}_{k+1|k} = \mathcal{M}_{k+1|k+1}$, the difference between the two beliefs is narrowed down to the RHS vectors $d_{k+1|k}$ and $d_{k+1|k+1}$ which encapsulates the measurements $z_{k+1|k}$ and $z_{k+1|k+1}$ respectively. However, in the real world it is possible that the DA predicted in precursory planning would prove to be inconsistent to the DA attained in inference, i.e. $\diamondsuit\langle\mathcal{M}_{k+1|k} \neq \mathcal{M}_{k+1|k+1}\rangle$.

There are six possible scenarios representing the relations between DA in inference and precursory planning:

- In planning, association is assumed to either a new or existing variable, while in inference no measurement is received.
- In planning it is assumed there will be no measurement to associate to, while in inference a measurement is received and associated to either a new or existing variable.
- In planning, association is assumed to an existing variable, while in inference it is to a new variable.
- In planning, association is assumed to a new variable, while in inference it is to an existing variable.
- In planning, association is assumed to an existing variable, while in inference it is also to an existing variable (whether the same or not).
- In planning, association is assumed to a new variable, while in inference it is also to a new variable (whether the same or not).

While the first four bullets always describe inconsistent DA situations (e.g. in planning we assumed a known tree would be visible but instead we saw a new bench, or vice versa), the last two bullets may provide consistent DA situations. In case associations in planning and in inference are to the same (un)known variables we would have a consistent DA.

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While different planning paradigms might diminish occurrences of inconsistent DA, e.g. by better predicting future associations, none can avoid it completely. Methods to better predict future observations/associations will be investigated in future work, potentially leveraging Reinforcement Learning (RL) techniques. In this report we do not predict occurrences of new landmarks, hence every new landmark in inference would result in inconsistent DA.

In the following Appendix we provide a method to update inconsistent DA, regardless of a specific inconsistency scenario or a solution paradigm. This method utilizes the incremental methodologies of iSAM2 [3] in order to efficiently update the belief from the planning stage to be with consistent DA to that of the succeeding inference.

Appendix B: Updating Inconsistent DA

Inconsistent DA can be interpreted as disparate connections between variables. As discussed earlier, these connections, denoted as factors, manifest in rows of the Jacobian matrix or in factor nodes of a FG. Two FGs with different DA would thus have different graph topology. We demonstrate the inconsistent DA impact over graph topology using the example presented in Figure 1: Figure 1a represents the belief $b[X_{k+1|k}]$ from planning stage, and Figure 1b represents the belief $b[X_{k+1|k+1}]$ from the inference stage. Even-though the same elimination order is used, the inconsistent DA would also create a different topology between the resulting BTs, e.g. the resulting BTs for the aforementioned FGs are Figure 1d and Figure 1e accordingly.

Performing action $u_{k|k+1}$, provides us with new measurements $z_{k+1|k+1}$, which are gathered to the factor set $\{f_j\}_{k+1|k+1}$ (see Appendix A for factor definition). From the precursory planning stage we have the belief $b[X_{k+1|k}]$ along with the corresponding factor set $\{f_i\}_{k+1|k}$ for time $k+1$. Since we performed inference over this belief during the planning stage, we have already eliminated the FG, denoted as $\mathcal{FG}_{k+1|k}$, into a BT denoted as $\mathcal{T}_{k+1|k}$, e.g. see Figure 1a and Figure 1d, respectively.

We would like to update both the FG $\mathcal{FG}_{k+1|k}$ and the BT $\mathcal{T}_{k+1|k}$ from the planning stage, using the new factors $\{f_j\}_{k+1|k+1}$ from the inference stage. Without loosing generality we use Figure 1 to demonstrate and explain the DA update process. Let us consider all factors of time $k+1$ from both planning $\{f_i\}_{k+1|k}$ and inference $\{f_j\}_{k+1|k+1}$. We can divide these factors into three categories:

The first category contains factors with consistent DA - Good Factors. These factors originate from only the last two DA scenarios, in which both planning and inference considered either the same existing variable or a new one. Consistent DA factors do not require our attention (other than updating the measurements in the RHS vector). Indices of consistent DA factors can be obtained by intersecting the DA from planning with that of inference:

$$\mathcal{M}_{k+1}^{\bigcap} = \mathcal{M}_{k+1|k} \bigcap \mathcal{M}_{k+1|k+1}. \tag{1}$$

The second category - Wrong Factors, contains factors from planning stage with inconsistent DA to inference, which therefore should be removed from $\mathcal{FG}_{k+1|k}$. These factors can originate from all DA scenarios excluding the second. Indices of inconsistent DA factors from planning, can be obtained by calculating the relative complement of $\mathcal{M}_{k+1|k}$ with respect to $\mathcal{M}_{k+1|k+1}$:

$$\mathcal{M}_{k+1}^{rmv} = \mathcal{M}_{k+1|k} \setminus \mathcal{M}_{k+1|k+1}. \tag{2}$$

The third category - New Factors, contains factors from the inference stage with inconsistent DA to planning; hence, these factors should be added to $\mathcal{FG}_{k+1|k}$. These factors can originate from all DA scenarios excluding the first. Indices of inconsistent DA factors from inference, can be obtained by calculating the relative complement of $\mathcal{M}_{k+1|k+1}$ with respect to $\mathcal{M}_{k+1|k}$:

$$\mathcal{M}_{k+1}^{add} = \mathcal{M}_{k+1|k+1} \setminus \mathcal{M}_{k+1|k}. \tag{3}$$

We now use our example from Figure 1 to illustrate these different categories:

- The first category - Good Factors, contains all factors from time $k+1$ that appear both in Figure 1a and 1b, i.e. the motion model factor between x_k to x_{k+1}.

- The second category - Wrong Factors, contains all factors that appear only in Figure 1a, i.e. the star marked factor in Figure 1a. In this case the inconsistent DA is to an existing variable, landmark l_j was considered to be observed in planning but is not seen in the succeeding inference.

- The third category - New Factors, contains all factors that appear only in Figure 1b, i.e. the star marked factors in Figure 1b. In this case the inconsistent DA is both to an existing and a new variable. Instead of landmark l_j that was considered to be observed in planning, a different existing landmark l_i has been seen, along with a new landmark l_r.

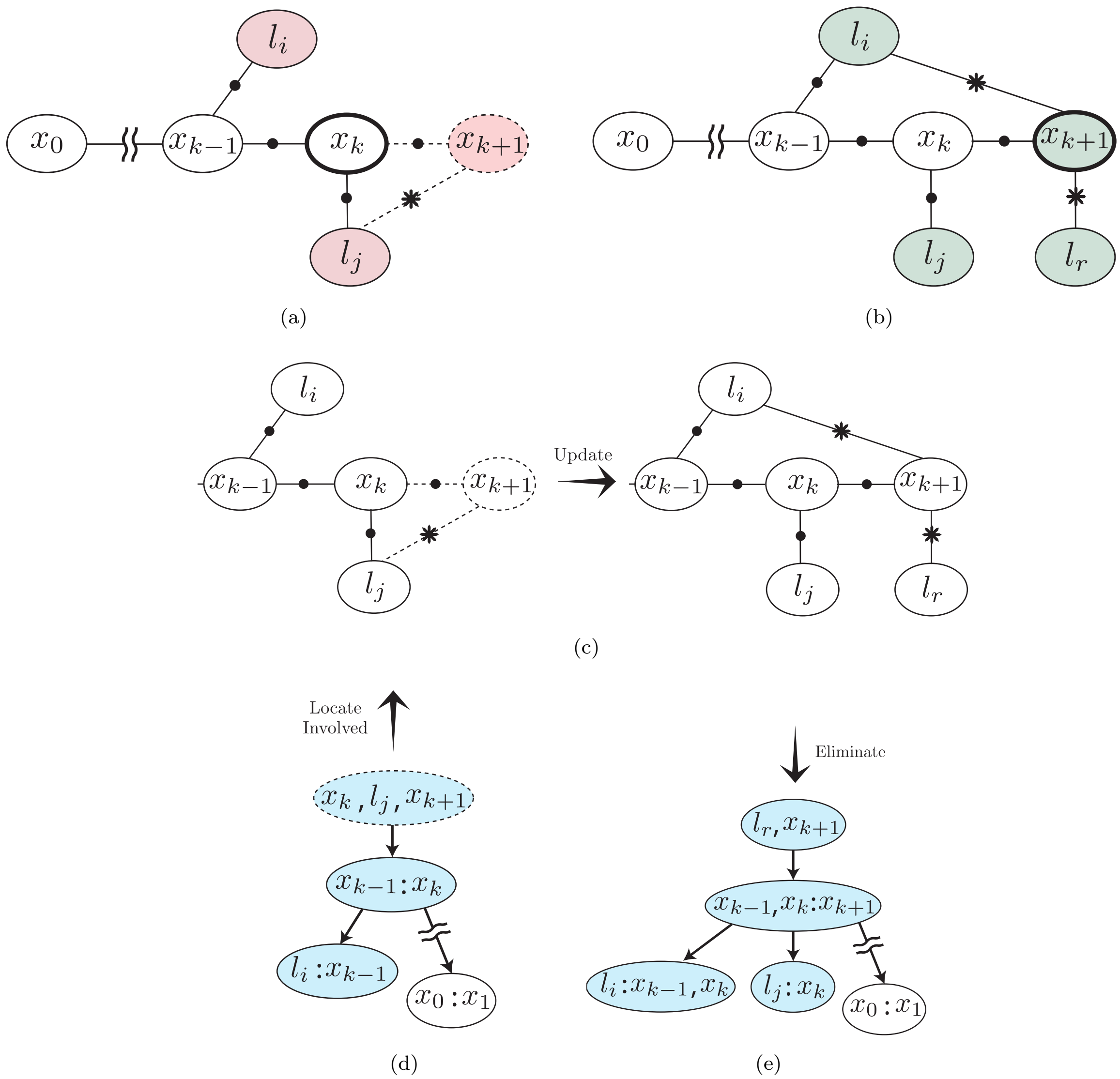

Figure 1: The process of incremental DA update, following on `iSAM2` methodologies. (a) and (b) show factor graphs for $b[X_{k+1|k}]$ and $b[X_{k+1|k+1}]$, respectively, which differ due to incorrect association considered in the planning phase - l_j was predicted to be observed within planning, while in practice l_i and l_r were observed at time instant $k+1$. In (a), current-time robot pose is bolded, horizon factors and states are dotted. Involved variables from DA comparison are marked in red in (a) and green in (b). The belief $b[X_{k+1|k}]$, represented by a Bayes tree shown in (d), is divided in two: sub Bayes tree containing all involved variables and parent cliques up to the root (marked in blue) and the rest of the Bayes tree in white. The former sub Bayes tree is re-eliminated by (i) forming the corresponding portion of the factor graph, as shown in the left figure of (c); (ii) removing incorrect DA and adding correct DA factors, which yields the factor graph shown in the right figure of (c); (iii) re-eliminating that factor graph into a sub Bayes tree, marked blue in (e), and re-attaching the rest of the Bayes tree. While the obtained Bayes tree now has a correct DA, it is conditioned on (potentially) incorrect measurement values for consistent-DA factors, which therefore need to be updated (as detailed in [1]), to recover the posterior belief $b[X_{k+1|k+1}]$.

Once the three aforementioned categories are determined, we use iSAM2 methodologies, presented in [3], to incrementally update $\mathcal{FG}_{k+1|k}$ and $\mathcal{T}_{k+1|k}$, see Alg. 1. The involved factors are denoted by all factors from planning needed to be removed (Wrong Factors), and all factors from inference needed to be added (New Factors),

$$\{f_r\}^{rmv}_{k+1} = \prod_{r \in \mathcal{M}^{rmv}_{k+1}} f_r \quad , \quad \{f_s\}^{add}_{k+1} = \prod_{s \in \mathcal{M}^{add}_{k+1}} f_s. \tag{4}$$

The involved variables, denoted by $\{X\}_{k+1}^{inv}$, are all variables related to the factor set $\{f_r\}_{k+1}^{rmv}$ and the factor set

Algorithm 1 - Data Association Update

1: **function** UPDATEDA($\mathcal{FG}_{k+1|k}$, $\mathcal{M}_{k+1|k}$, $\mathcal{FG}_{k+1|k+1}$, $\mathcal{M}_{k+1|k+1}$)

2: $\mathcal{M}_{k+1}^{rmv} \leftarrow \mathcal{M}_{k+1|k} \setminus \mathcal{M}_{k+1|k+1}$ ▷ indices of factors required to be removed

3: $\mathcal{M}_{k+1}^{add} \leftarrow \mathcal{M}_{k+1|k+1} \setminus \mathcal{M}_{k+1|k}$ ▷ indices of factors required to be added

4: $\{f_r\}_{k+1}^{rmv} \leftarrow \prod_{r \in \mathcal{M}_{k+1}^{rmv}} \{f_r\}_{k+1}$ ▷ factors required to be removed

5: $\{f_s\}_{k+1}^{add} \leftarrow \prod_{s \in \mathcal{M}_{k+1}^{add}} \{f_s\}_{k+1}$ ▷ factors required to be added

6: $\{X\}_{k+1}^{inv} \leftarrow Variables(\{f_r\}_{k+1}^{rmv}) \bigcup Variables(\{f_s\}_{k+1}^{add})$ ▷ get involved variables

7: $\mathcal{T}_{k+1}^{inv} \leftarrow \mathcal{T}_{k+1|k}^{\{X\}_{k+1}^{inv}}$ ▷ get corresponding sub-BT

8: $\{X\}_{k+1}^{inv\star} \xleftarrow{\text{get all variables}} \mathcal{T}_{k+1}^{inv}$ ▷ update involved variables

9: $\mathcal{FG}_{k+1}^{inv} \leftarrow \mathcal{FG}_{k+1|k}^{\{X\}_{k+1}^{inv\star}}$ ▷ get corresponding sub-FG

10: $\mathcal{FG}_{k+1}^{upd} \leftarrow [\mathcal{FG}_{k+1}^{inv} \setminus \{f_r\}_{k+1}^{rmv}] \bigcup \{f_s\}_{k+1}^{add}$ ▷ Update the sub Factor Graph

11: $\mathcal{T}_{k+1}^{upd} \xleftarrow{\text{eliminate}} \mathcal{FG}_{k+1}^{upd}$ ▷ re-eliminate the updated sub-FG into BT

12: $\mathcal{FG}_{k+1|k}^{upd} \leftarrow [\mathcal{FG}_{k+1|k} \setminus \mathcal{FG}_{k+1}^{inv}] \bigcup \mathcal{FG}_{k+1}^{upd}$ ▷ Update the Factor Graph

13: $\mathcal{T}_{k+1|k}^{upd} \leftarrow [\mathcal{T}_{k+1|k} \setminus \mathcal{T}_{k+1}^{inv}] \bigcup \mathcal{T}_{k+1}^{upd}$ ▷ Update the Bayes Tree

14: **return** $\mathcal{FG}_{k+1|k}^{upd}$, $\mathcal{T}_{k+1|k}^{upd}$.

15: **end function**

$\{f_r\}_{k+1}^{add}$ (Alg. 1, line 6), e.g. the colored variables in Figures 1a and 1b accordingly. In $\mathcal{T}_{k+1|k}$, all cliques between the ones containing $\{X\}_{k+1}^{inv}$ up to the root are marked and denoted as the involved cliques, e.g. colored cliques in Figure 1d. The involved cliques are detached and denoted by $\mathcal{T}_{k+1}^{inv} \subset \mathcal{T}_{k+1|k}$ (line 7). This sub-BT $\mathcal{T}_{k+1}^{inv}$, contains more variables than just $\{X\}_{k+1}^{inv}$. The involved variable set $\{X\}_{k+1}^{inv}$, is then updated to contain all variables from $\mathcal{T}_{k+1}^{inv}$ and denoted by $\{X\}_{k+1}^{inv\star}$ (line 8). The part of $\mathcal{FG}_{k+1|k}$, that contains all involved variables $\{X\}_{k+1}^{inv\star}$ is detached and denoted by $\mathcal{FG}_{k+1}^{inv}$ (line 9). While $\mathcal{T}_{k+1}^{inv}$ is the corresponding sub-BT to the acquired sub-FG $\mathcal{FG}_{k+1}^{inv}$.

In order to finish updating the DA, all that remains is updating the sub-FG $\mathcal{FG}_{k+1}^{inv}$ with the correct DA and re-eliminate it to get an updated BT. All factors $\{f_r\}_{k+1}^{rmv}$ are removed from $\mathcal{FG}_{k+1}^{inv}$, then all factors $\{f_r\}_{k+1}^{add}$ are added (line 10). The updated sub-FG is denoted by $\mathcal{FG}_{k+1}^{upd}$, e.g. update illustration in Figure 1c.

By re-eliminating $\mathcal{FG}_{k+1}^{upd}$, a new updated BT, denoted by $\mathcal{T}_{k+1}^{upd}$, is obtained (line 11), e.g. the colored sub-BT in Figure 1e. This BT is then re-attached back to $\mathcal{T}_{k+1|k}$ instead of $\mathcal{T}_{k+1}^{inv}$, subsequently the new BT is now with consistent DA and is denoted as $\mathcal{T}_{k+1|k}^{upd}$ (line 13). In a similar manner $\mathcal{FG}_{k+1|k}^{upd}$ is obtained by re-attaching $\mathcal{FG}_{k+1}^{upd}$ instead of $\mathcal{FG}_{k+1}^{inv}$ to $\mathcal{FG}_{k+1|k}$(line 12). At this point the DA in both the FG and the BT is fixed. For example, by completing the aforementioned steps, Figures 1a and 1d will have the same topology as Figures 1b and 1e.

After the DA update, the BT $\mathcal{T}_{k+1|k}^{upd}$ has consistent DA to that of $\mathcal{M}_{k+1|k+1}$. However, it is still not identical to $\mathcal{T}_{k+1|k+1}$ due to difference between measurement values predicted in planning to the values obtained in inference. The DA update dealt with inconsistent DA factors and their counterparts. For these factors the new measurements from inference were updated in the corresponding RHS vector values within the BT. The consistent DA factors, on the other hand, were left untouched; therefore, these factors do not contain the new measurement values from inference but measurement values from the planning stage instead. These inconsistent measurements are thus baked into the RHS vector $d_{k+1|k}$ and in the appropriate cliques of the BT $\mathcal{T}_{k+1|k}^{upd}$. In order to update the RHS vector $d_{k+1|k}$, or equivalently update the corresponding values within relevant cliques of the BT, one can use any of the methods presented in [1].

References

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