

# Belief Space Planning for Autonomous Navigation while Modeling Landmark Identification

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**TECHNION**  
Israel Institute  
of Technology

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# Introduction - Applications

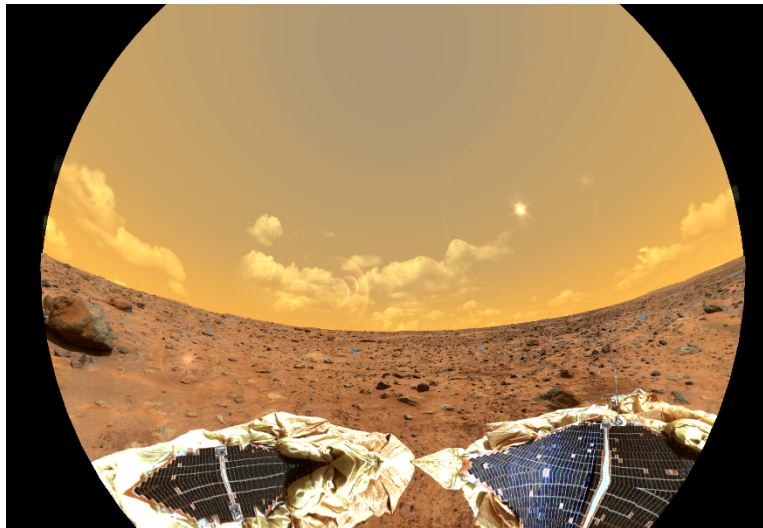
## Autonomous navigation in unknown environment

Under sea exploration



[listverse.com]

Space exploration



[Nasa.gov]

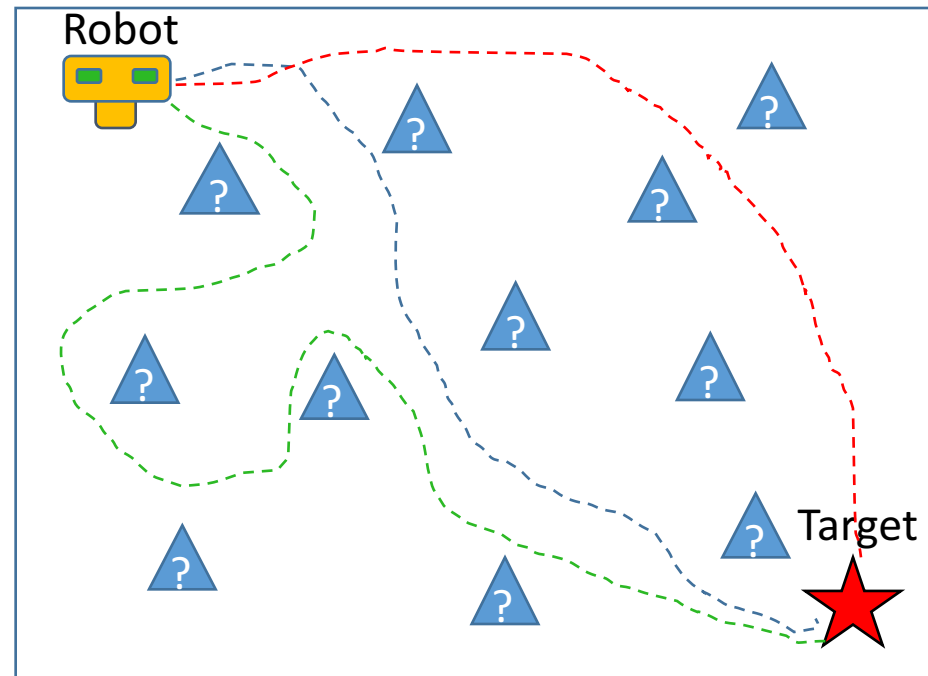
Navigation in GPS-deprived environments



[Nasa.gov]

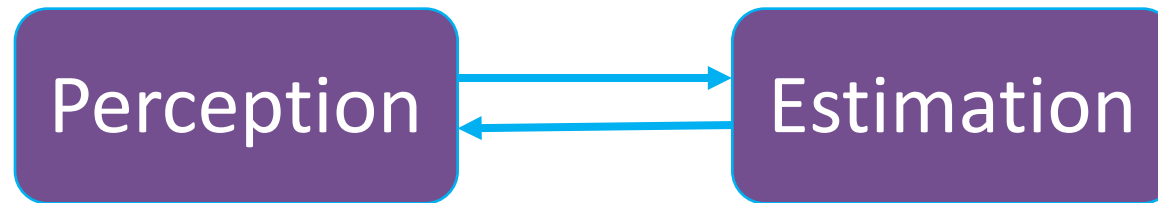
# Introduction - Problem

- Autonomous navigation in unknown environment
- Planning a suitable control strategy to accomplish a given task
- Reaching a goal with highest estimation accuracy

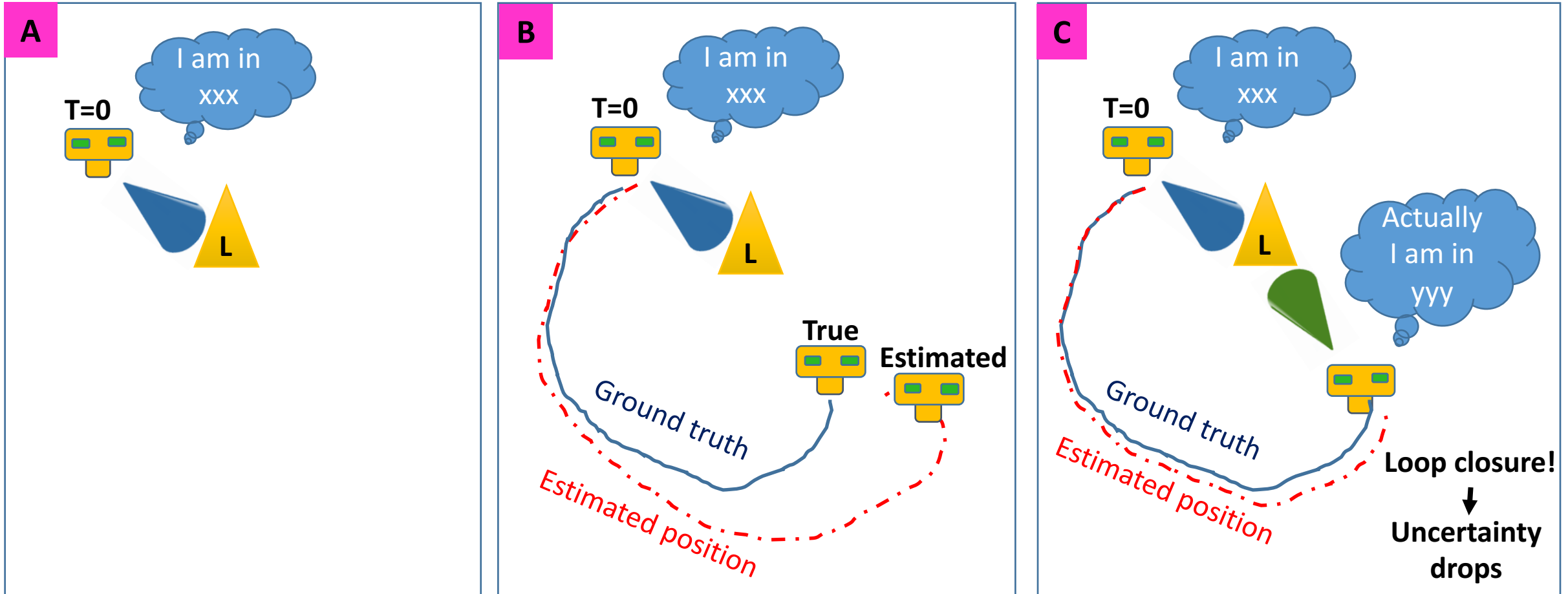


# Introduction - SLAM

- **SLAM - simultaneous localization and mapping**
- Based on sensor observations, the robot :
  - Infers its own state
  - Creates a model of the environment

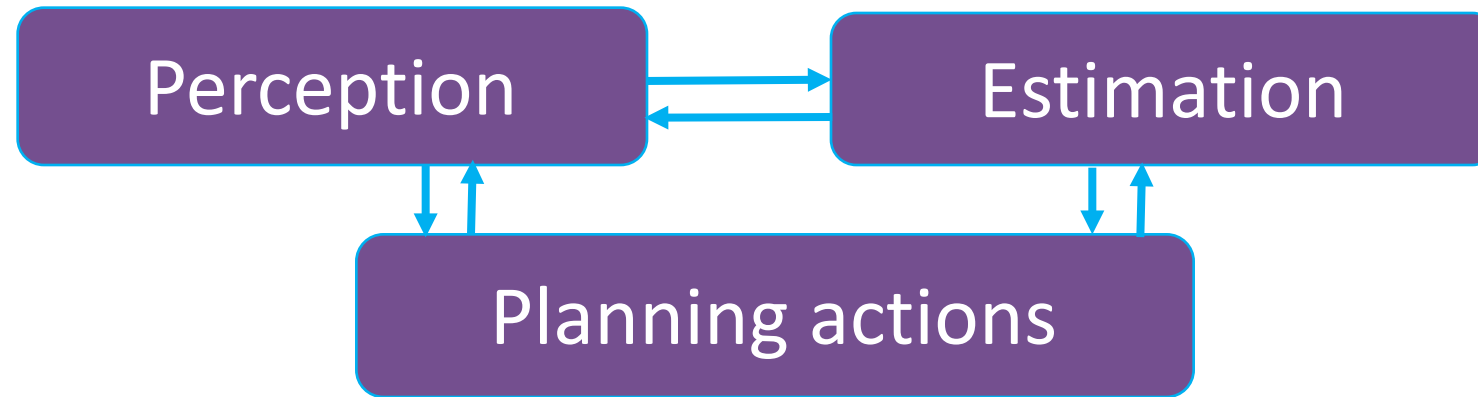


# SLAM – Loop closure



# Introduction - Belief space planning

**Belief space planning (BSP) - Planning actions while taking into account different sources of uncertainty**



- Optimizing an objective function, composed for an example by the objectives:
  - Minimum uncertainty
  - Path length
  - Reaching a specific goal

# Related Work

- Many approaches assume environment/map is known
- Recent work relaxes this assumption and enables operation in unknown environments
- **BSP approaches typically consider perfect ability to re-identify an object**

## In this work we:

- **Enable operation in unknown environments**
- **Not assuming perfect ability to re-identify an object**

# How is a landmark being re-identified?

- It can be challenging!
- Depends on: camera viewpoint, sensor capabilities and image processing capabilities
- Different view angles may cause the landmark to look completely different

## Same landmark – different view direction

- Looks completely different!
- Challenging to identify even for human





# Computer vision algorithms

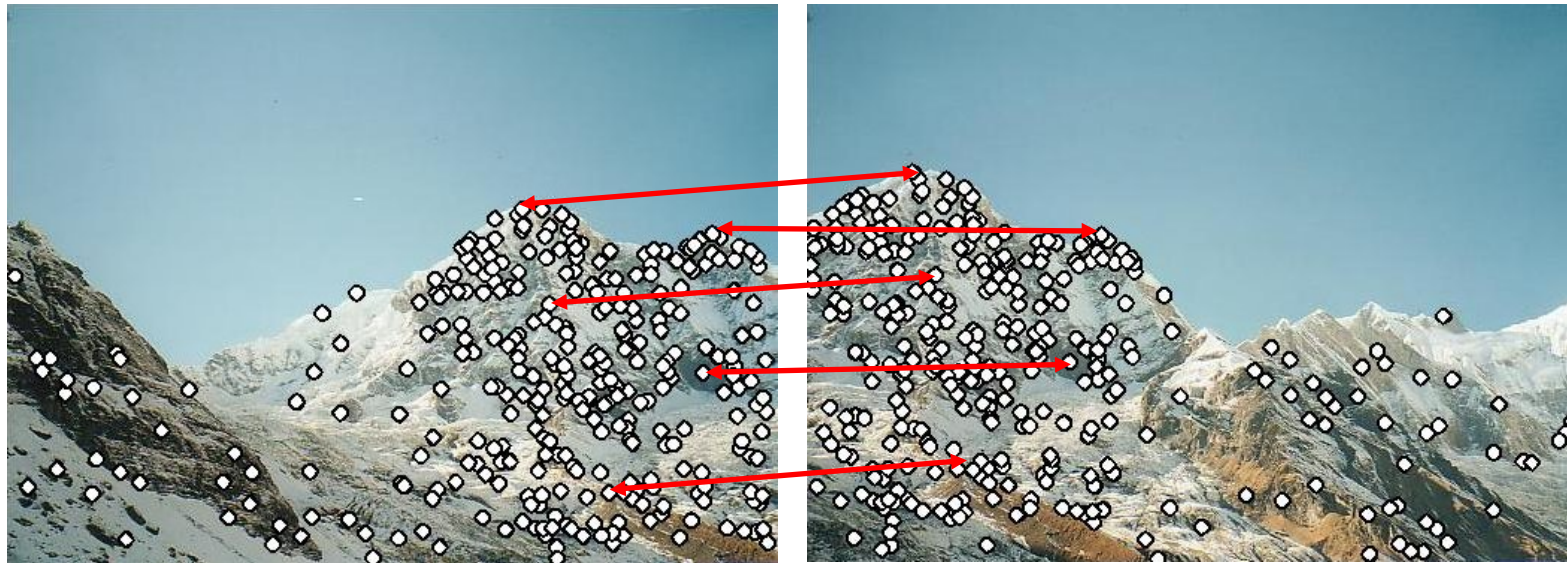
The decision on landmark identification depends on the computer vision algorithm

**For an example, SIFT algorithm**

Detects  
features in  
the image

Compares  
between  
features

Determines  
correspondence  
between features  
in two views



Images adapted from Steve Seitz and Rick Szeliski

# Computer vision algorithms

- Limited in their identification ability
- Defines the conditions in which two views of the same scene will be identified as same object
- In SIFT algorithm , an object will be identified when viewpoint direction is changing in up to  $30^\circ$  -  $40^\circ$

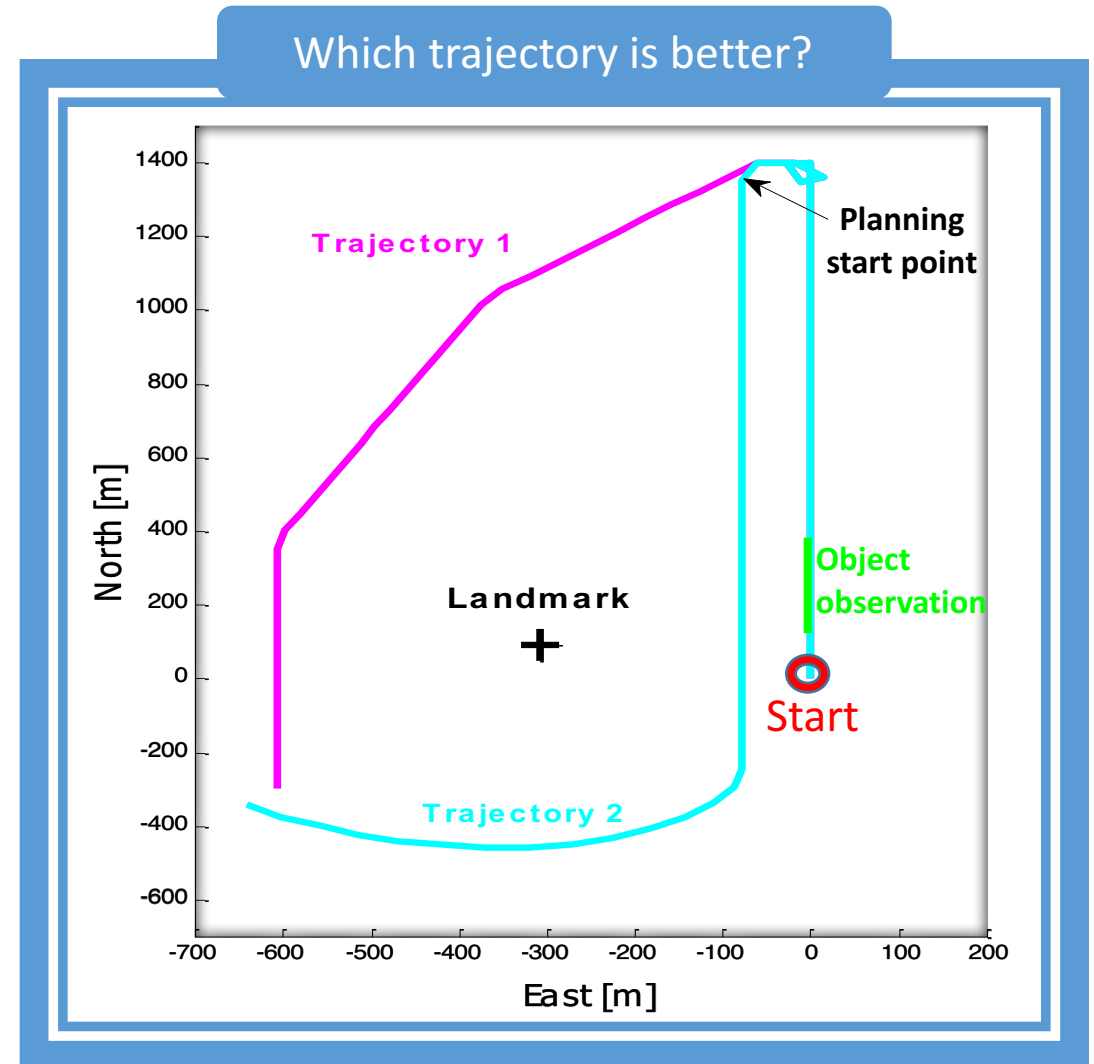
# Contribution

BSP approaches typically consider perfect ability to re-identify an object

inconsistent uncertainty prediction with reality (inference)

Incorrect planning and path choosing

**Correct identification of landmarks is critical**



# Contribution

Develop a viewpoint  
aware BSP approach

Modeling object  
re-identification

Considering both SLAM  
and Planning aspects

- **Focus on object re-identification from different viewpoint when the object is known**

# Concept – Modeling Object Re-Identification

LOS (Line of sight) = Straight line between the robot's camera and observed scene

We define Cone of identification

- In it, the landmark can be identified using image processing algorithms





# Concept – Modeling Object Re-Identification

Preserving all LOS from the past  
→ LOS are calculated using  
information from estimation

Calculating LOS for a future  
view point

Check if current LOS is inside a  
cone of identification from the  
past

Yes

Landmark is  
recognized

No

Landmark  
isn't  
recognized



# Formulation -SLAM

$x_i$  - Robot state at time  $i$

$u_i$  - Control action applied at time  $i$

$z_{i,j}$  - measurement of the  $j$ th landmark at time  $i$

$l_j$  - Coordinates of landmark  $j$

Notations

- The motion model is :

$$x_{i+1} = f(x_i, u_i) + w_i \quad w_i : N(0, \Sigma_w) \quad p(x_{i+1} | x_i, u_i)$$

- The observation model is :

$$z_{i,j} = h(x_i, l_j) + v_{i,j} \quad v_{i,j} : N(0, \Sigma_v) \quad p(z_{i,j} | x_i, l_j)$$

# Formulation -SLAM

$X_k$  — All robot and world states until time  $k$   
 $Z_k$  — All available observations at time  $k$   
 $u_k$  — Control action at time  $k$

Notations

- The problem to be solved in the SLAM part:

$$p(X_k | Z_{0:k}, u_{0:k-1})$$

Joint state vector  $X_k$  is  $\{ \underbrace{x_0, \dots, x_k}_{\text{Past \& current robot states}}, \underbrace{L_k}_{\text{Mapped environment}} \}$

We use maximum a posteriori (MAP) estimation in order to estimate  $X_k^*$

$$p(X_k | Z_{0:k}, u_{0:k-1}) \sim N(X_k^*, \Sigma_k) \quad X_k^* = \arg \max_{X_k} (p(X_k | Z_{0:k}, u_{0:k-1}))$$



# Formulation - SLAM

$X_k$  – All robot and world states until time  $k$   
 $Z_k$  – All available observations at time  $k$   
 $u_k$  – Control action at time  $k$   
 $n_i$  – Number of observations at time  $i$   
 $l_j$  – Coordinates of landmark  $j$

**Notations**

- Mathematical development will lead to:

$$b(X_k) \propto p(X_k | Z_k, u_{0:k-1}) = \text{priors} \cdot \prod_{i=1}^k \left[ \underbrace{p(x_i | x_{i-1}, u_{i-1})}_{\text{Motion model}} \prod_{j=1}^{n_i} \underbrace{p(z_{i,j} | x_i, l_j)}_{\text{Measurement model}} \right]$$

Data Association and landmark identification

# Formulation

## Belief Space Planning

$X_k$  – All robot and world states until time  $k$   
 $Z_k$  – All available observations at time  $k$   
 $u_k$  – Control action at time  $k$   
 $l_j$  – Coordinates of landmark  $j$

Notations

$$b(X_{k+l}) \propto p(\underbrace{X_{k+l}}_{\text{Joint state at the } l\text{-th look ahead step}} \mid \underbrace{Z_{0:k}, u_{0:k-1}}_{\text{Past controls \& measurements}}, \underbrace{Z_{k+1:k+l}, u_{k:k+l-1}}_{\text{Controls \& measurements at the first } l \text{ look-ahead steps}})$$

- This belief is represented by a Gaussian:

$$b(X_{k+l}) : N(X_{k+l}^*, \Sigma_{k+l})$$

# Formulation

## Belief Space Planning

$X_k$  – All robot and world states until time  $k$   
 $Z_k$  – All available observations at time  $k$   
 $u_k$  – Control action at time  $k$   
 $L$  – Number of planning steps

**Notations**

We want to find the planning actions

Optimizing an objective function:

$$J(u_{k:k+L-1}) @ \underset{Z_{k+1:k+L}}{\mathbb{E}} \left\{ \sum_{l=0}^{L-1} c_l \left( b(X_{k+l}), u_{k+l} \right) \right\}$$

Expectation on all  
future unknown  
measurements

$Z_{k+1:k+L}$

Sum of all  
L future  
planning  
steps

General cost function depends  
on the belief  $b(X_{k+l})$  and on  
the control action  $u_{k+l}$



Composed, for example, by:

- minimum uncertainty
- path length
- reaching a specific goal

# Formulation

## Belief Space Planning

$X_k$  – All robot ( $x_{1:k}$ ) until time k and world states ( $l_{1:j}$ )  
 $Z_k$  – All available observations at time k  
 $u_k$  – Control action at time k  
 $l_j$  – Coordinates of landmark j  
 $n_i$  – Number of observations at time i  
 $H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$  Past measurements and controls

Notations

Existing BSP approaches are solving the problem while considering ideal data association and ideal ability of object re-identification

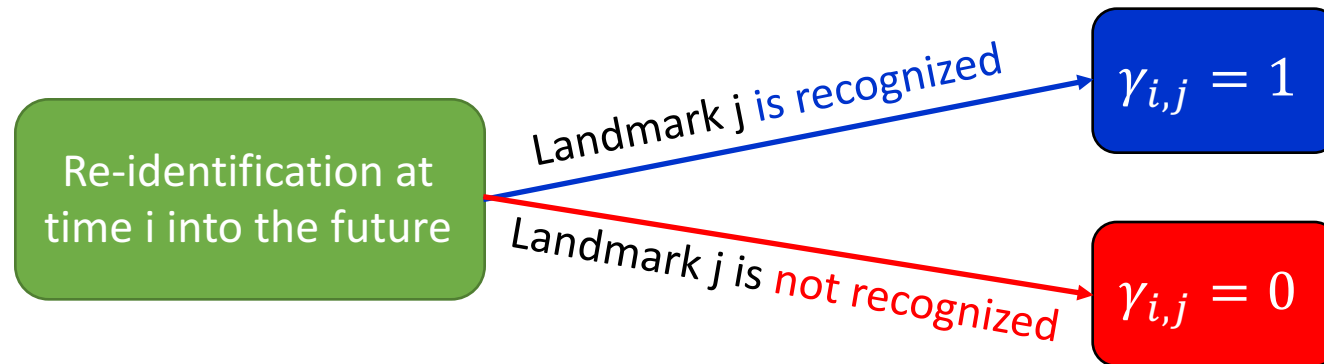
In this case, develop of the belief space leads to:

$$p(X_{k+l} | H_{k+l}) \propto \underbrace{p(X_k | H_k)}_{\text{Inference until planning time k (SLAM)}} \cdot \underbrace{\prod_{i=1}^l p(x_{k+i} | x_{k+i-1}, u_{k+i-1})}_{\text{Motion model for future states from planning time k}} \cdot \underbrace{\prod_{j=1}^{n_i} p(z_{k+i,j} | x_{k+i}, l_j)}_{\text{Measurement model for future measurements from planning time k Assuming ideal data association}}$$

# Formulation

## Belief Space Planning

In reality – re-identification is not perfect  $\longrightarrow$  Define a binary random variable  $\gamma_{i,j}$



$$\Gamma_i \mathbf{B} \left\{ \gamma_{i,j} \right\}_{j=1}^{n_i} \quad n_i \text{ is the number of possible observations at time } i$$

# Formulation

## Belief Space Planning

$X_k$  – All robot ( $x_{1:k}$ ) until time k and world states ( $l_{1:j}$ )  
 $Z_k$  – All available observations at time k  
 $u_k$  – Control action at time k  
 $l_j$  – Coordinates of landmark j  
 $H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$  Past measurements and controls  
 $\gamma_{i,j}$  – Event of acquiring measurement j at time i  
 $\Gamma_i \triangleq \{\gamma_{i,j}\}_{j=1}^{n_i}$

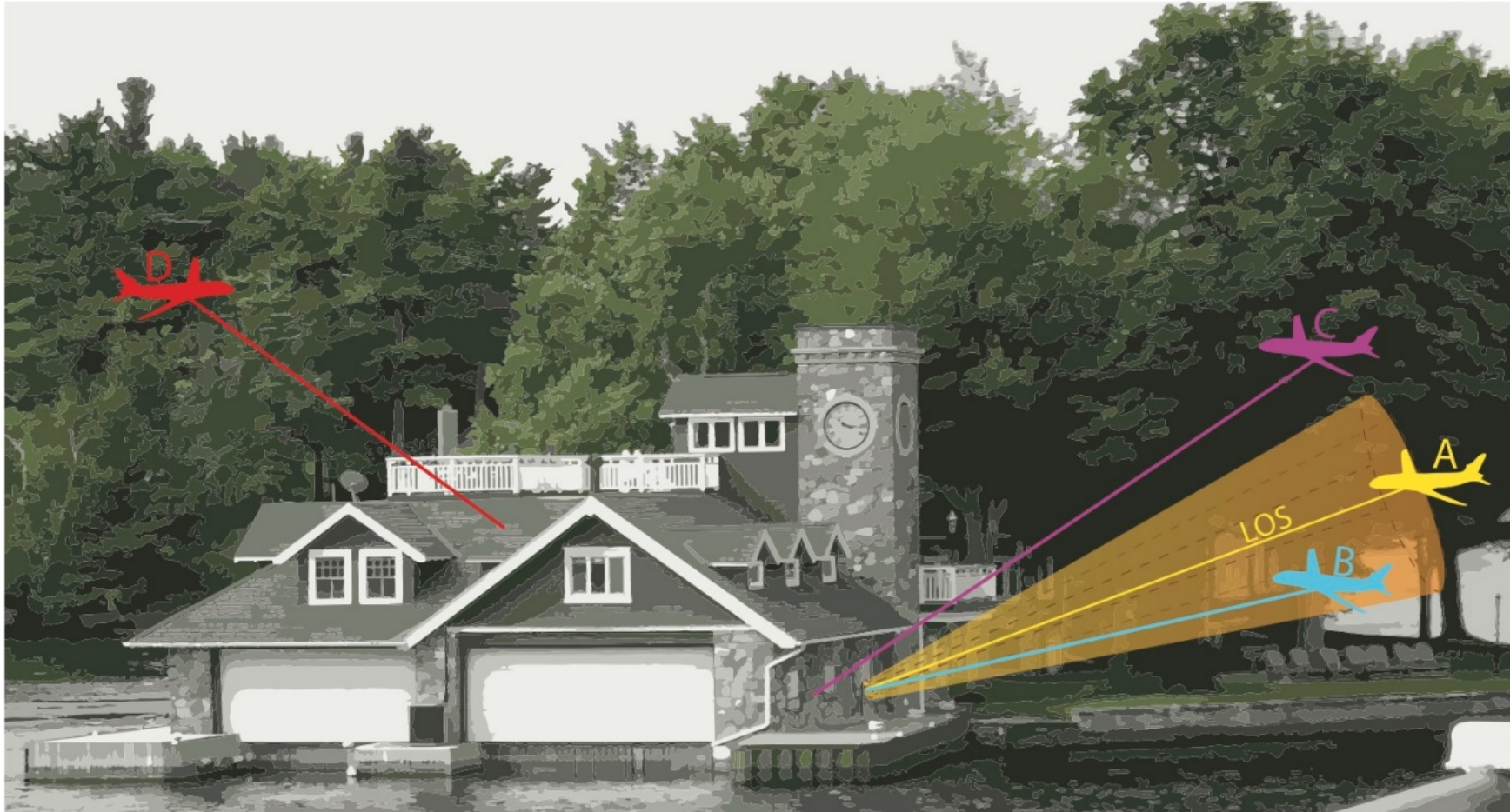
**Notations**

$$p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l}) \propto \underbrace{p(X_k | H_k)}_{\text{Inference until planning time k (SLAM)}} \cdot \underbrace{\prod_{i=1}^l p(x_{k+i} | x_{k+i-1}, u_{k+i-1})}_{\text{Motion model for future states from planning time k}} \cdot \underbrace{\prod_{j=1}^{n_i} p(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j}) p(\gamma_{k+i,j} | H_{k+i-1}, x_{k+i}, l_j)}_{\text{Measurement model for future measurements from planning time k taking into account the event of acquiring a measurement}}$$

$$p(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j}) \begin{cases} \xrightarrow{\gamma_{k+i,j} = 1} p(z_{k+i,j} | x_{k+i}, l_j) \\ \xrightarrow{\gamma_{k+i,j} = 0} \cancel{p(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j})} \end{cases}$$

# Formulation – Belief Space Planning

## Recall - Modeling Object Re-Identification

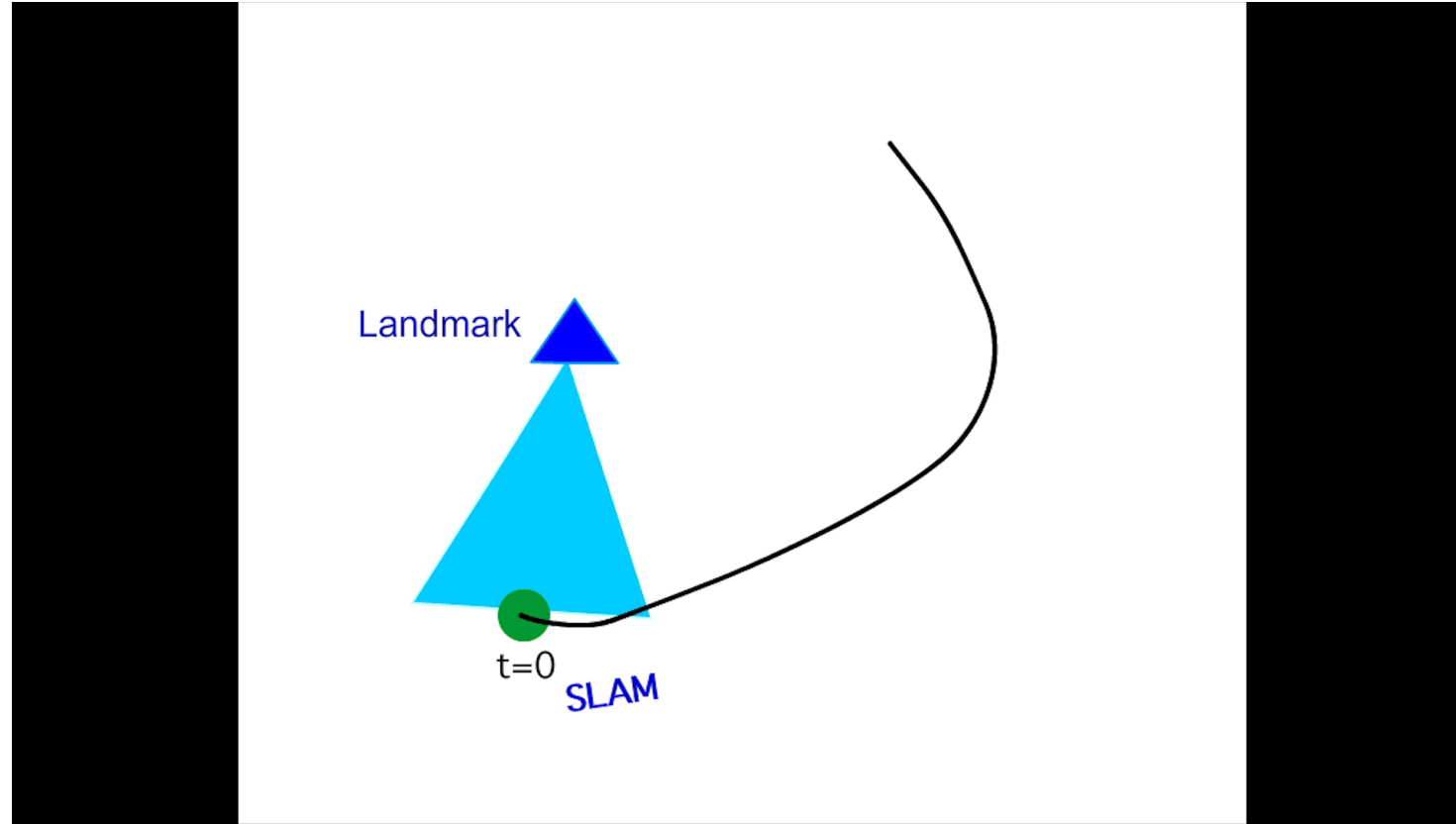


# Formulation – Belief Space Planning

$$p(\gamma_{k+i,j} \mid H_{k+i-1}, x_{k+i}, l_j)$$

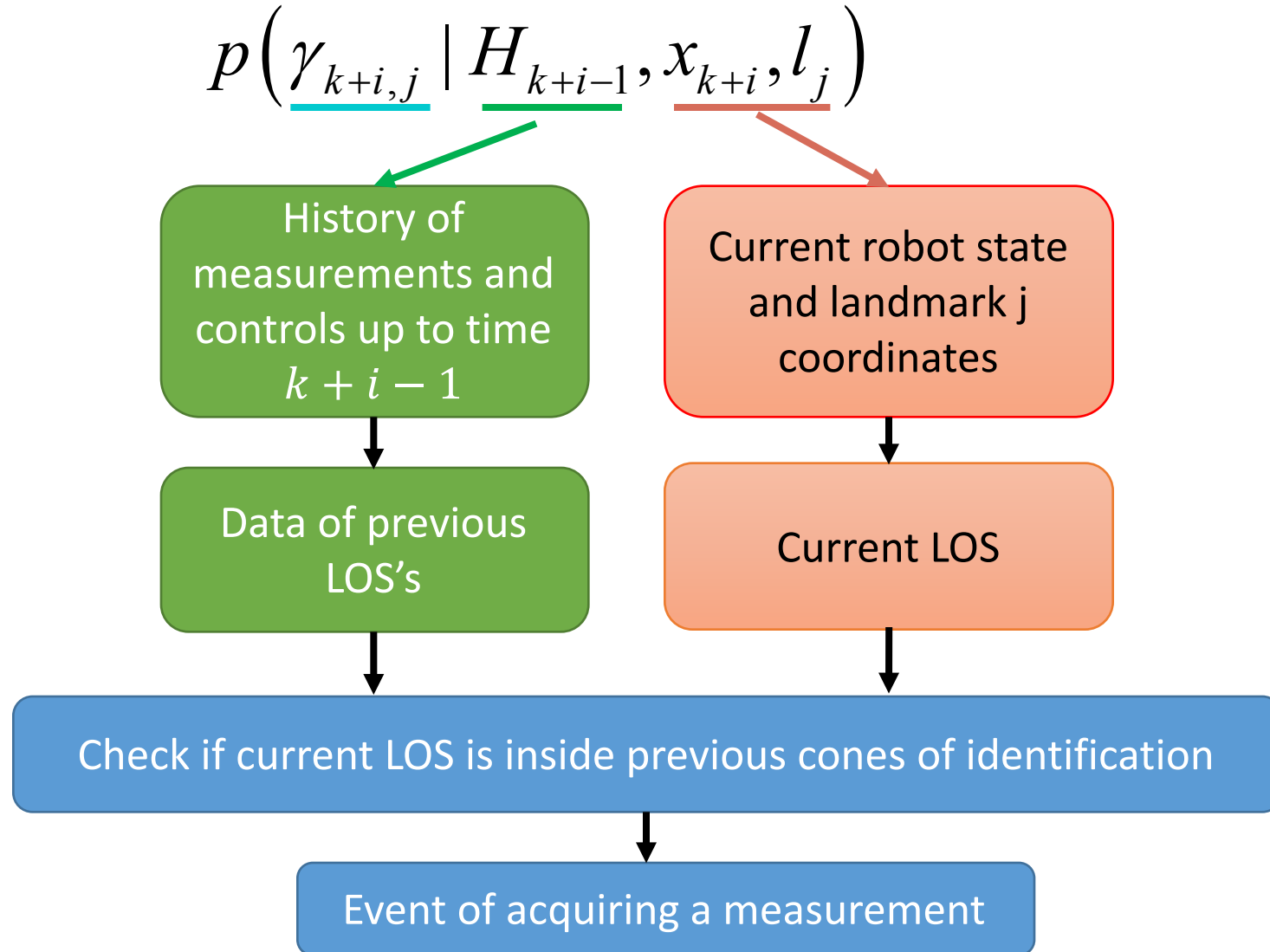
$H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$  Past measurements and controls

$H_{k+i-1} = \{H_k, u_{k:k+i-2}, z_{k+1:k+i-1}\}$





# Formulation – Belief Space Planning



# Formulation

## Belief Space Planning

$X_k$  – All robot ( $x_{1:k}$ ) until time k and world states ( $l_{1:j}$ )  
 $H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$  Past measurements and controls  
 $\gamma_{i,j}$  – Event of acquiring measurement j at time i  
 $\Gamma_i \triangleq \{\gamma_{i,j}\}_{j=1}^{n_i}$

Notations

$$p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l}) \propto p(X_k | H_k) \cdot \prod_{i=1}^l p(x_{k+i} | x_{k+i-1}, u_{k+i-1}) \cdot \prod_{j=1}^{n_i} p(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j}) p(\gamma_{k+i,j} | H_{k+i-1}, x_{k+i}, l_j)$$

The event of acquiring a measurement in the future is unknown  $\rightarrow \Gamma_i$  is a random variable

Joint probability function:

$$p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l})$$

# Formulation

## Belief Space Planning

$X_k$  – All robot ( $x_{1:k}$ ) until time  $k$  and world states ( $l_{1:j}$ )  
 $H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$  Past measurements and controls  
 $\gamma_{i,j}$  – Event of acquiring measurement  $j$  at time  $i$   
 $\Gamma_i \triangleq \{\gamma_{i,j}\}_{j=1}^{n_i}$

Notations

Recall, in order to calculate the objective function  $J$ , we are using the belief  $b(X_{k+l})$ :

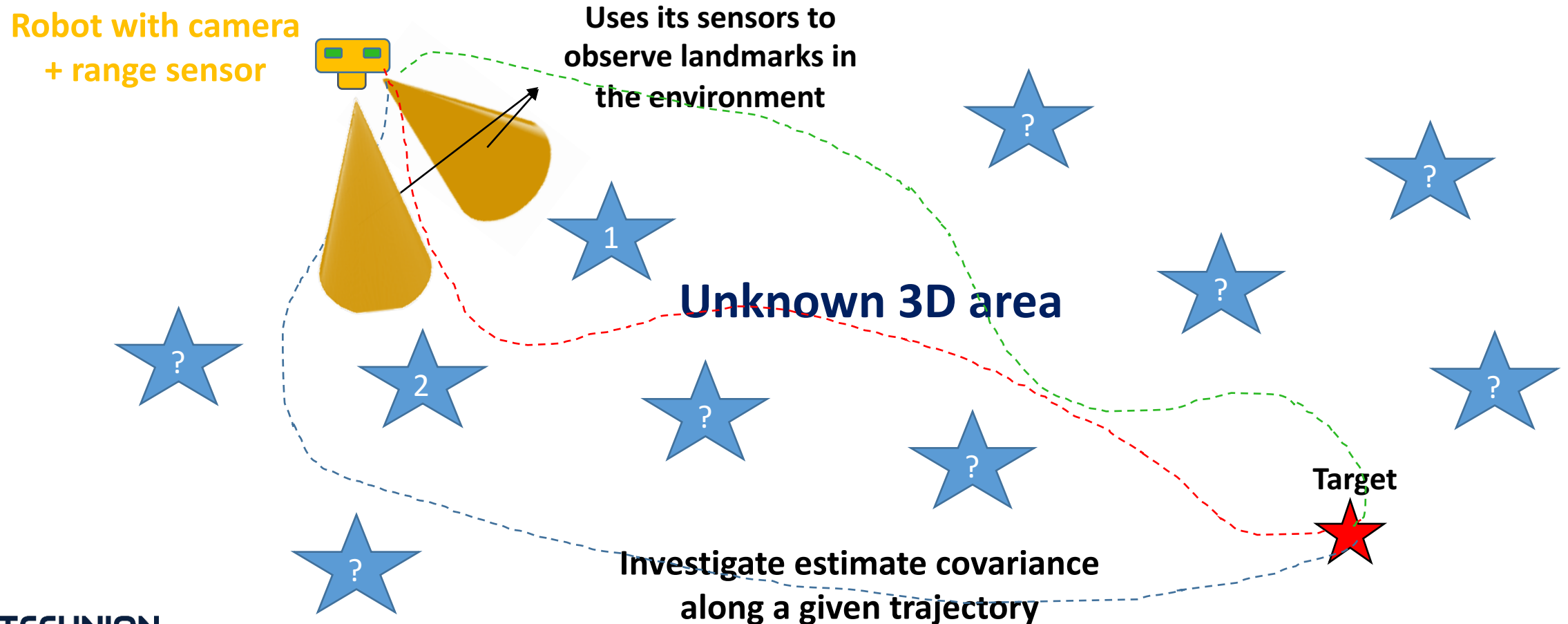
$$J(u_{k:k+L-1}) @_{Z_{k+1:k+L}} E \left\{ \sum_{l=0}^{L-1} c_l \left( b(X_{k+l}), u_{k+l} \right) \right\}$$

Therefore we do Marginalization:

$$b(X_{k+l}) = p(X_{k+l} | H_{k+l}) = \sum_{\Gamma_{k+1:k+l}} p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l})$$

# Results – Simulation overview

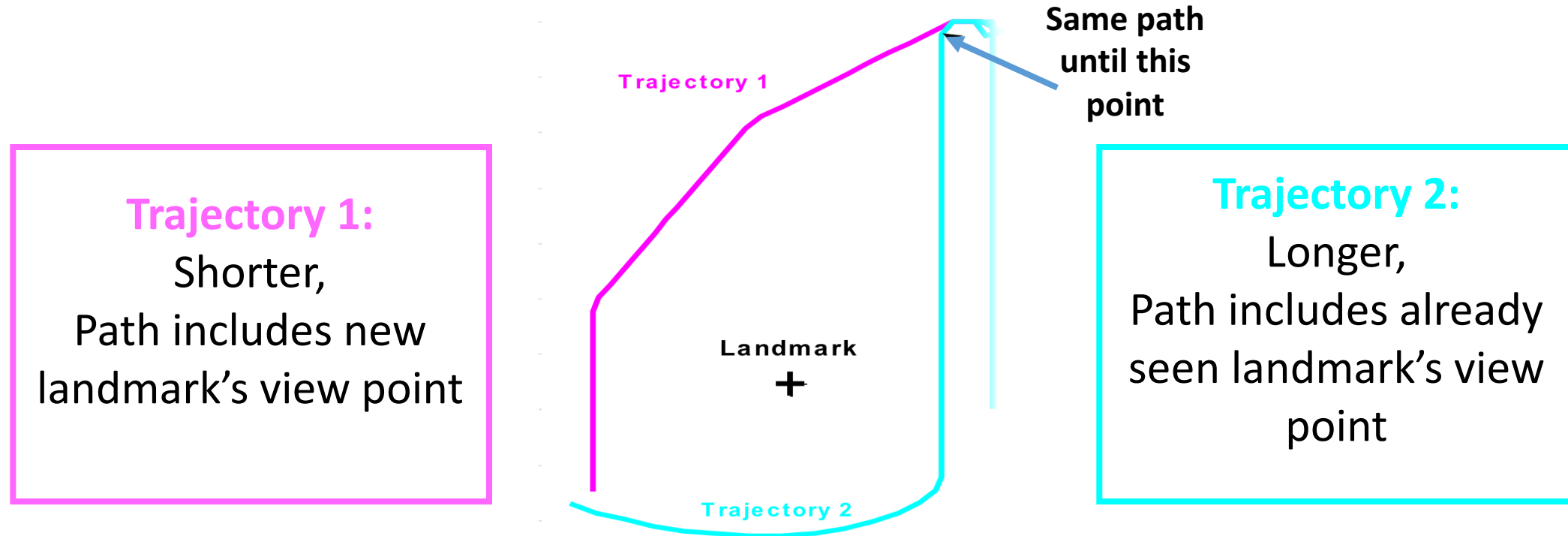
Using a simulation to check the influence of modeling object re-identification



# Results – problem definition

The objectives are:  
Minimum uncertainty  
Path length  
Reaching a specific target

- Checking two predefined trajectories that differ in:
  - Landmark's view directions
  - Trajectory length



# Results -SLAM

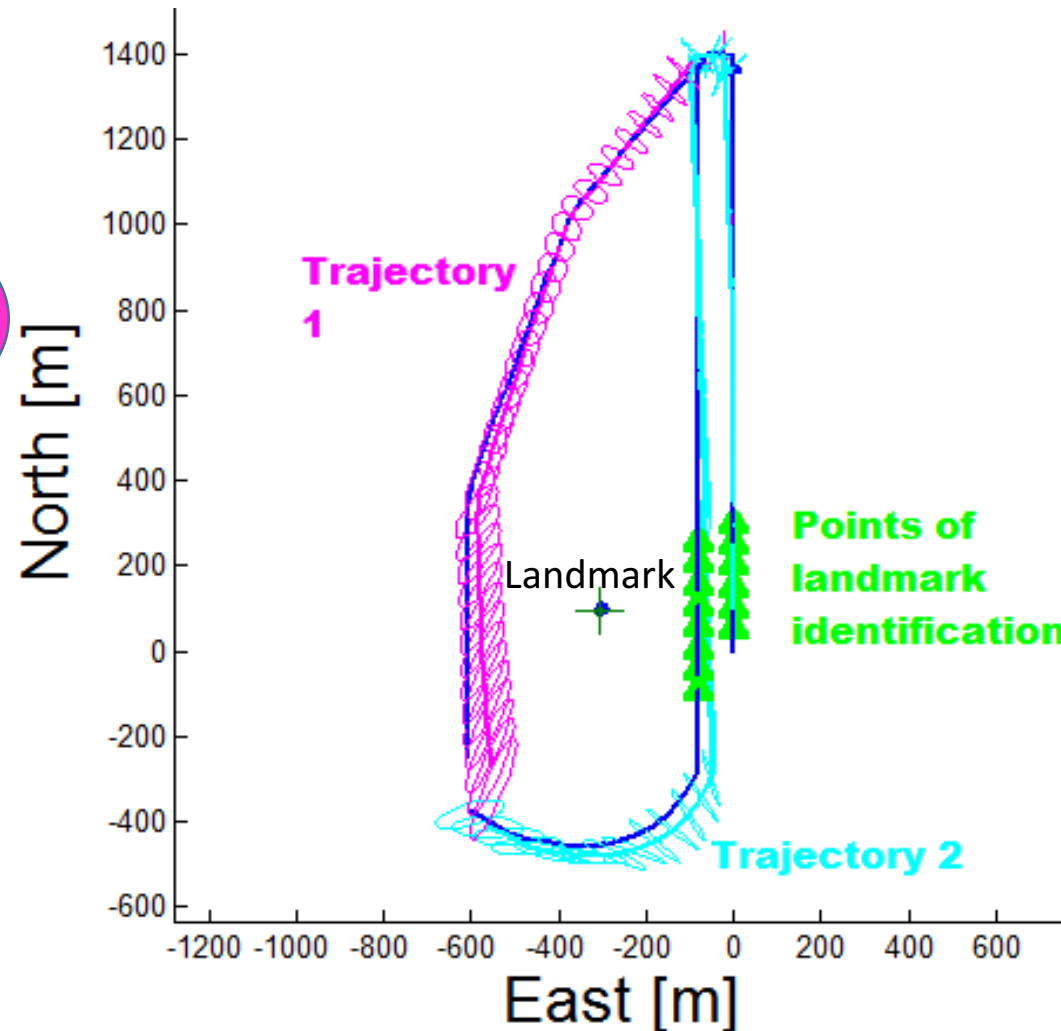
Using only SLAM , Represents true results in real world

## Trajectory 1

Completely different view directions of the landmark

Landmark isn't identified

Estimate covariance keep growing

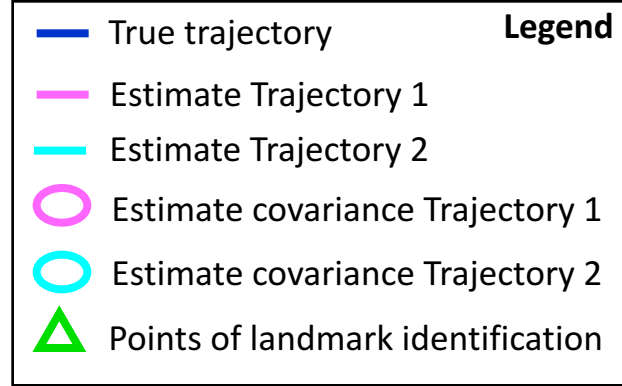


## Trajectory 2

Similar view directions of the landmark

Landmark is identified

Estimate covariance drops at identification point

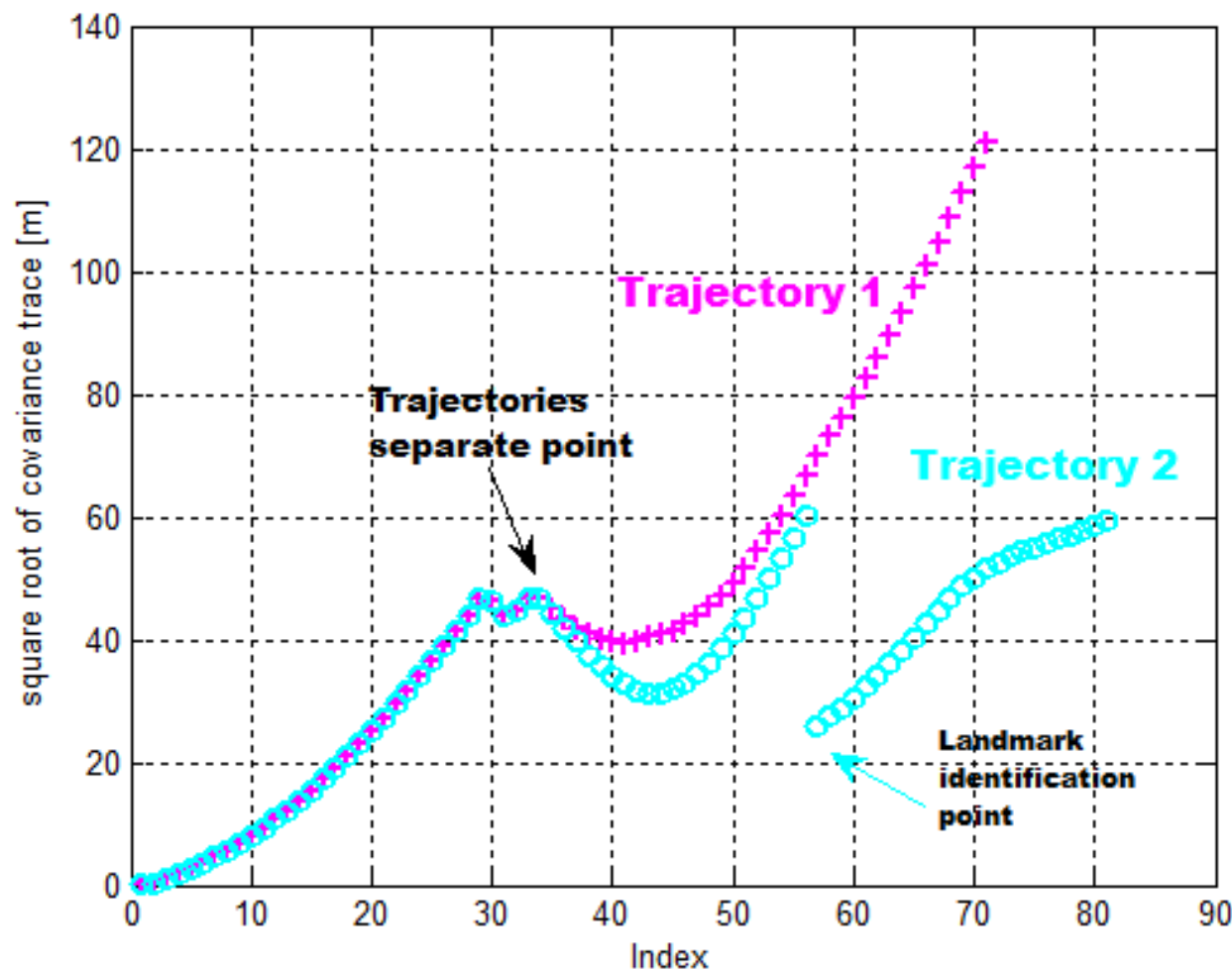


# Results -SLAM

## Trajectory 1

Landmark isn't identified

Estimate covariance keep growing



## Trajectory 2

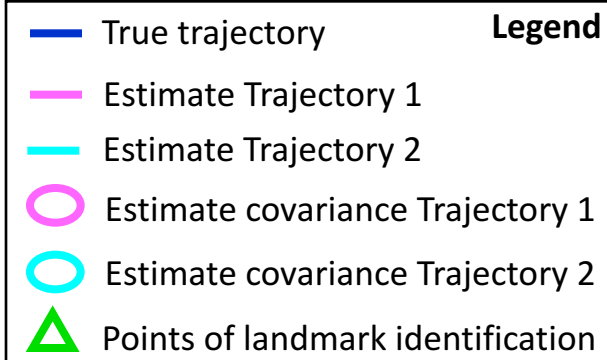
Landmark is identified

Estimate covariance drops at identification point

**Trajectory 2 has lower estimate covariance though it is longer → preferred**

# Results - Planning

## Without applying object identification



### Trajectory 1

New view directions of the landmark

Landmark is **incorrectly identified**  
(Differently than SLAM)

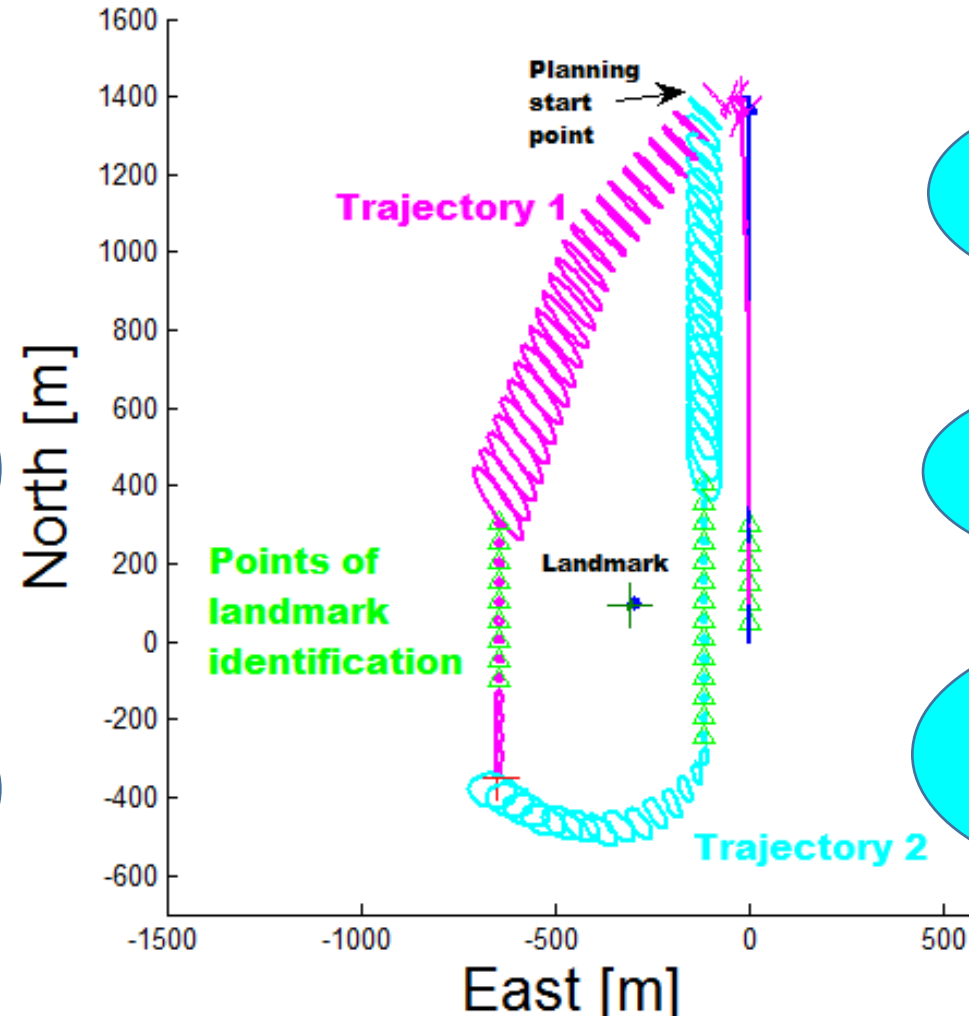
Estimate covariance **incorrectly drops**

### Trajectory 2

Similar view directions of the landmark

Landmark is identified

Estimate covariance drops at identification point





# Results - Planning

## Without applying object identification

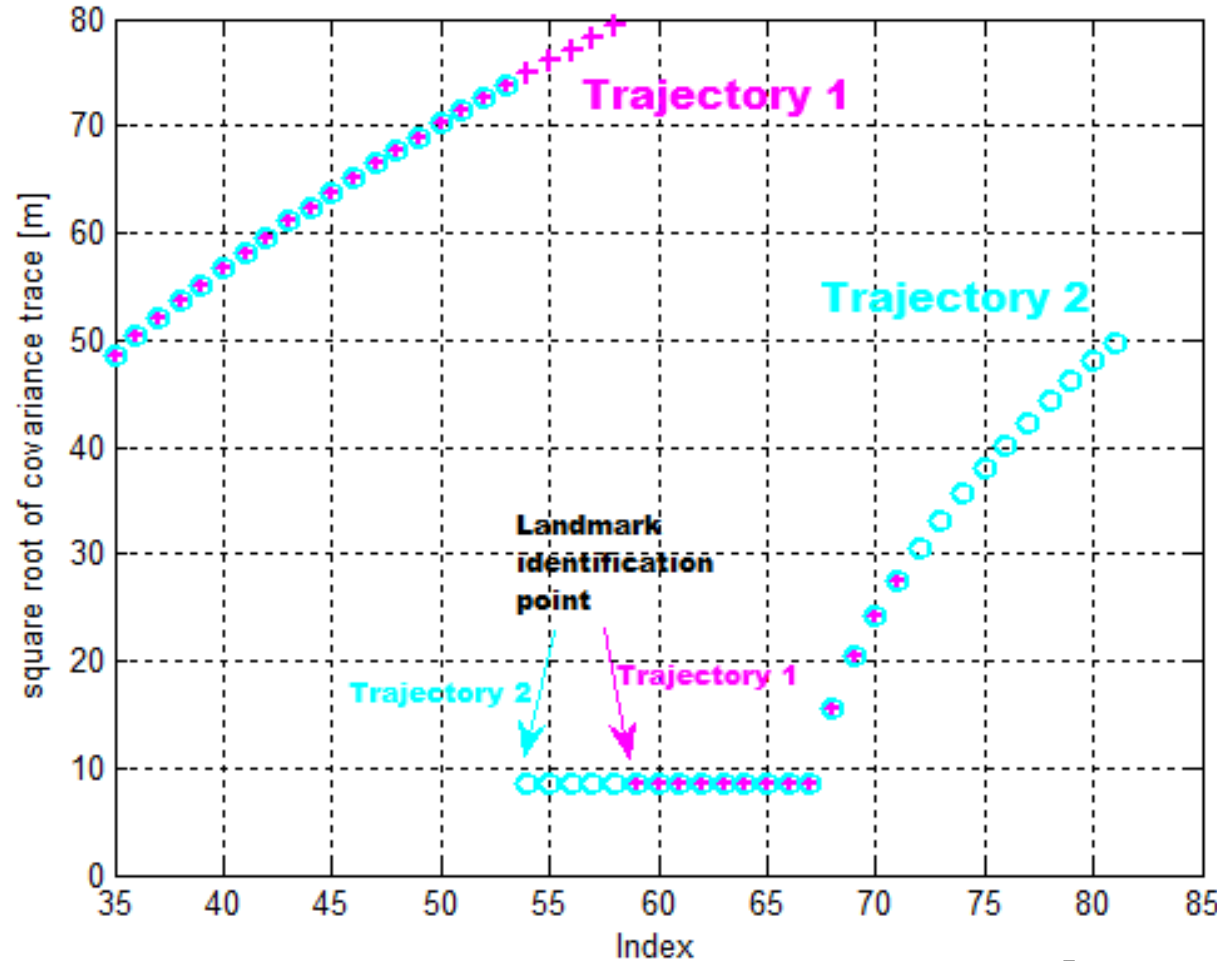
Trajectory 1

Landmark is  
incorrectly  
identified

covariance  
incorrectly drops  
at identification  
point



Inconsistent with SLAM



Trajectory 2

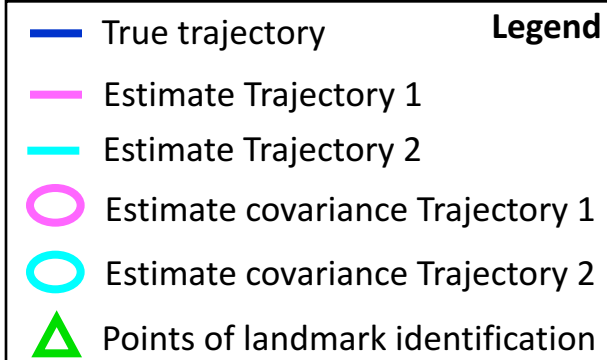
Landmark is  
identified

Estimate  
covariance drops  
at identification  
point

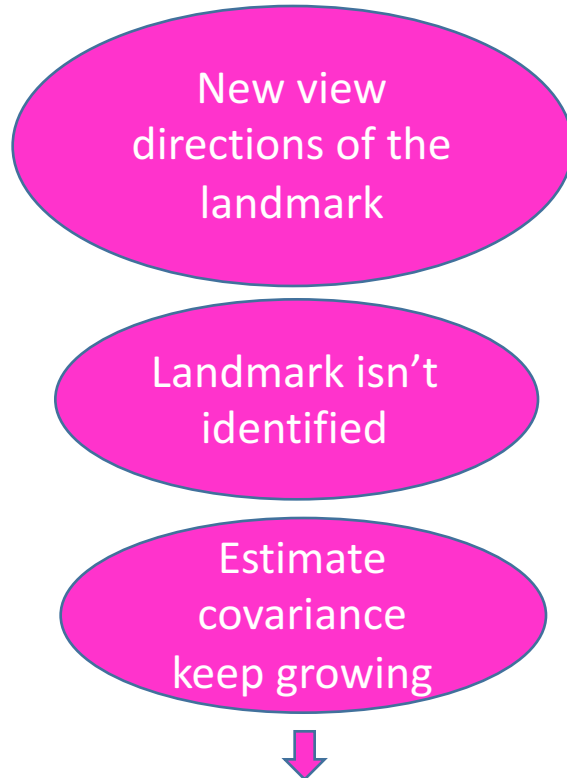
Trajectory 1 has lower estimate covariance → incorrectly preferred

# Results - Planning

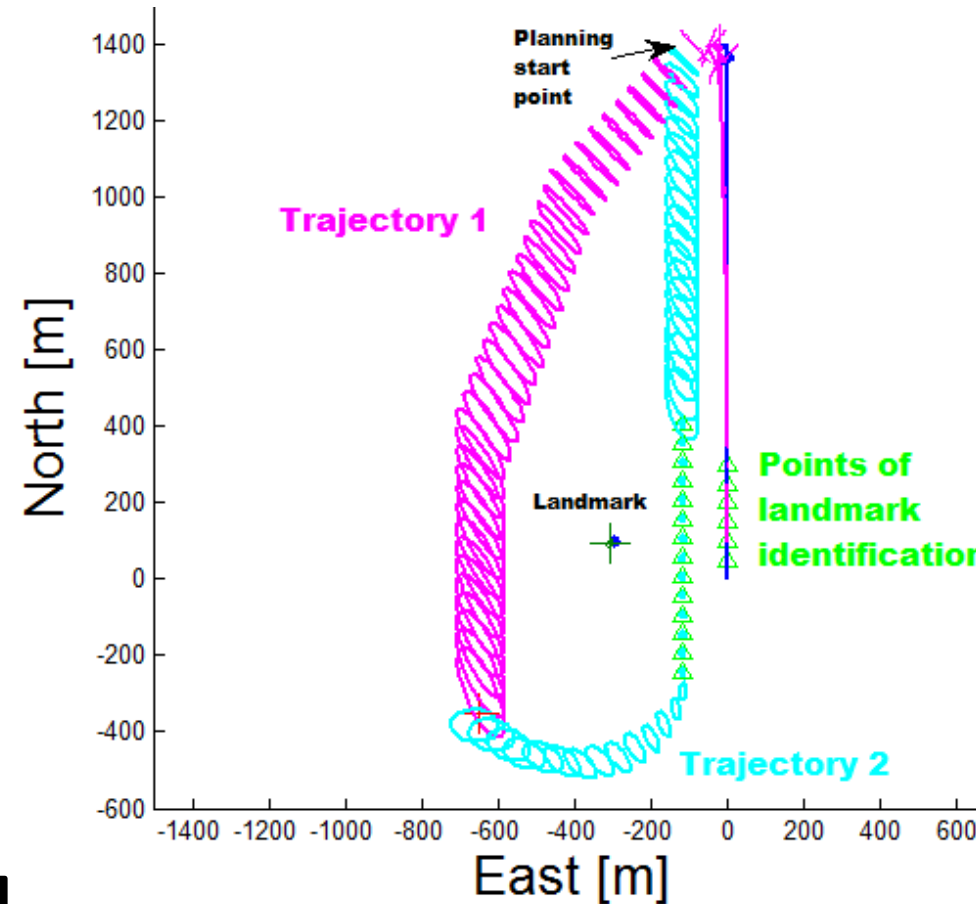
## With applying object identification



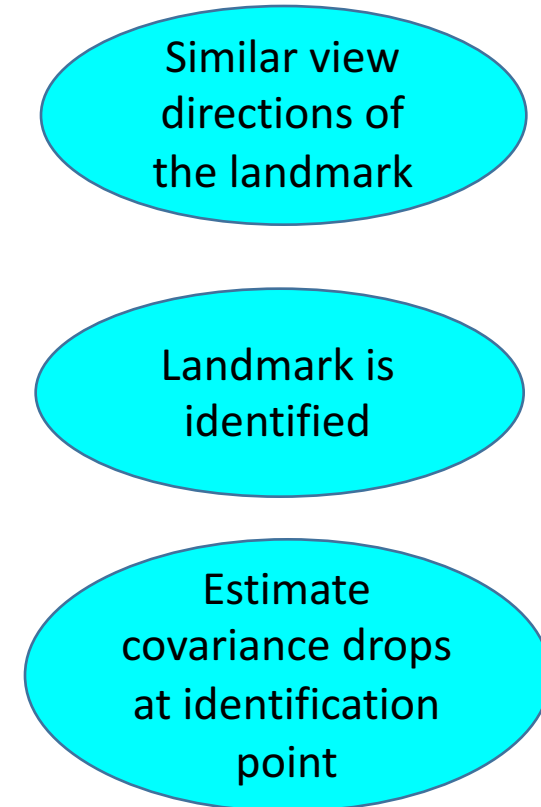
### Trajectory 1



Consistent with SLAM



### Trajectory 2



# Results - Planning

## With applying object identification

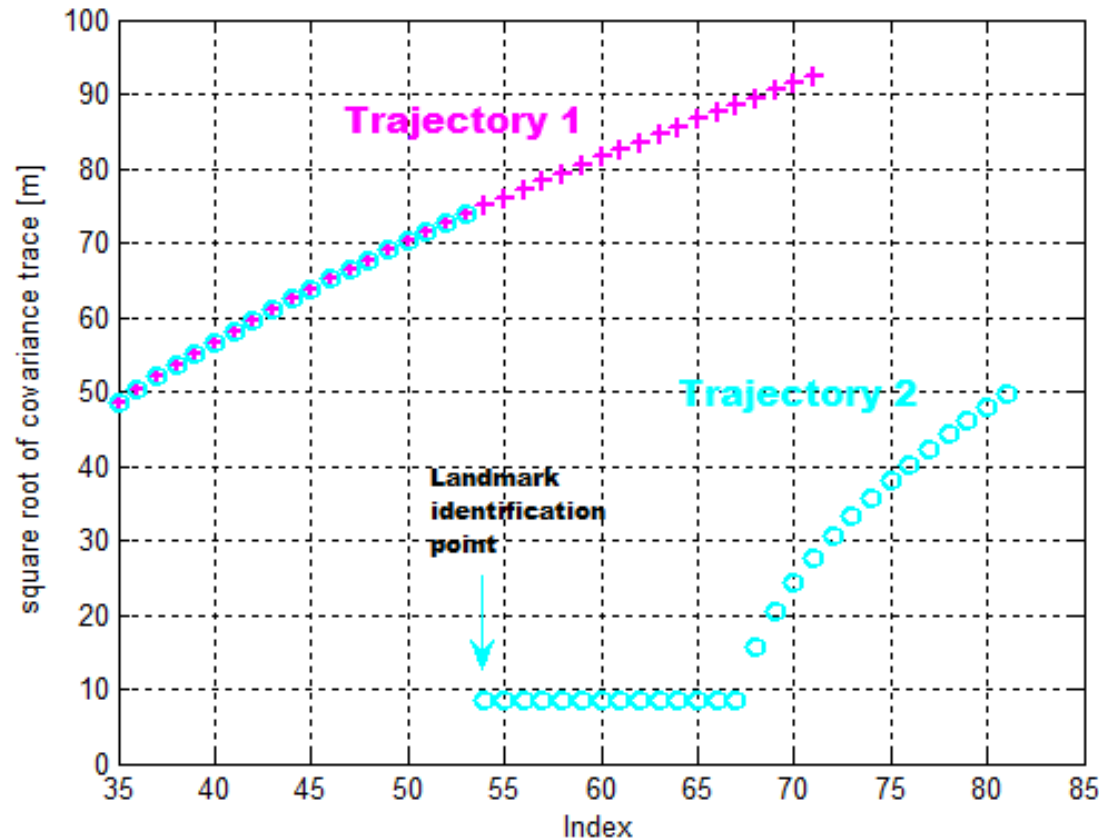
### Trajectory 1

Landmark isn't identified

Estimate covariance keep growing



**Consistent with SLAM**



### Trajectory 2

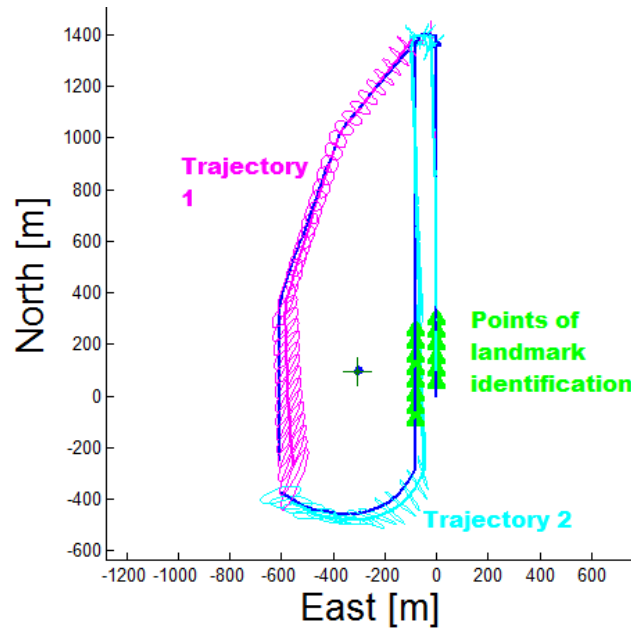
Landmark is identified

Estimate covariance drops at identification point

**Trajectory 2 has lower estimate covariance though it is longer → preferred**  
**Consistent with SLAM**

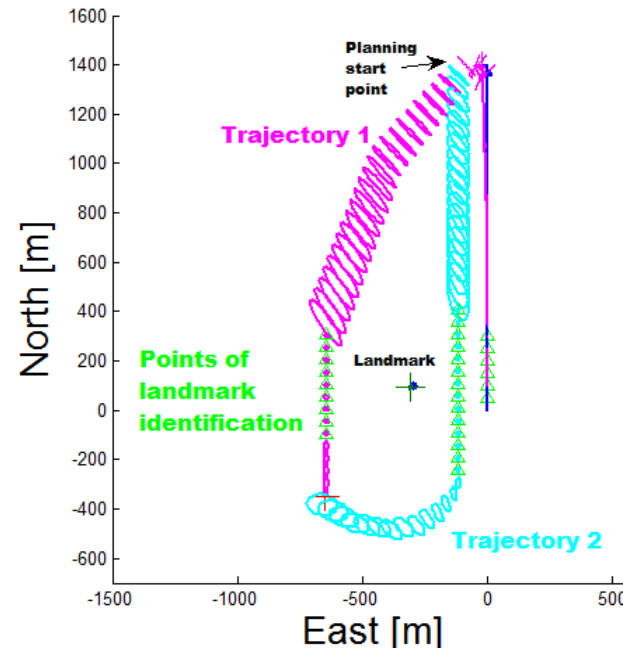
# Results - summary

**SLAM**



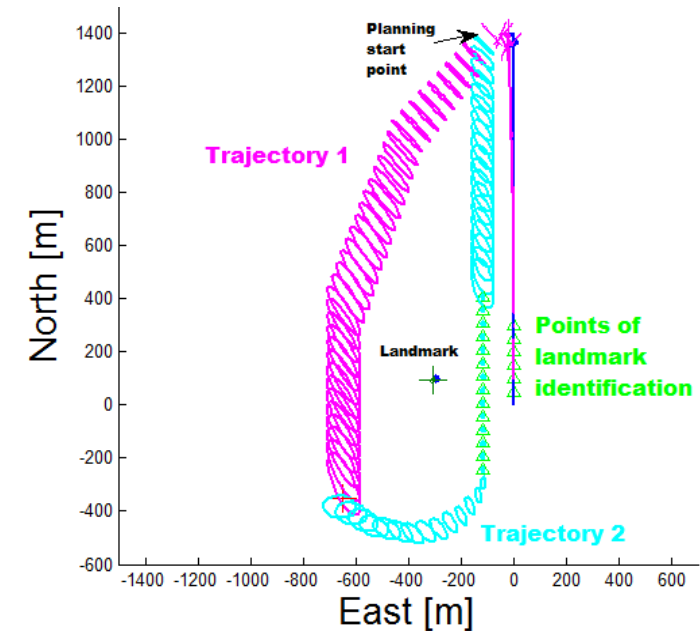
**In reality – the landmark is re-identified only in trajectory 2**

**Planning - Without applying object identification**



**When not applying object identification – the landmark is re-identified incorrectly also in trajectory 1**

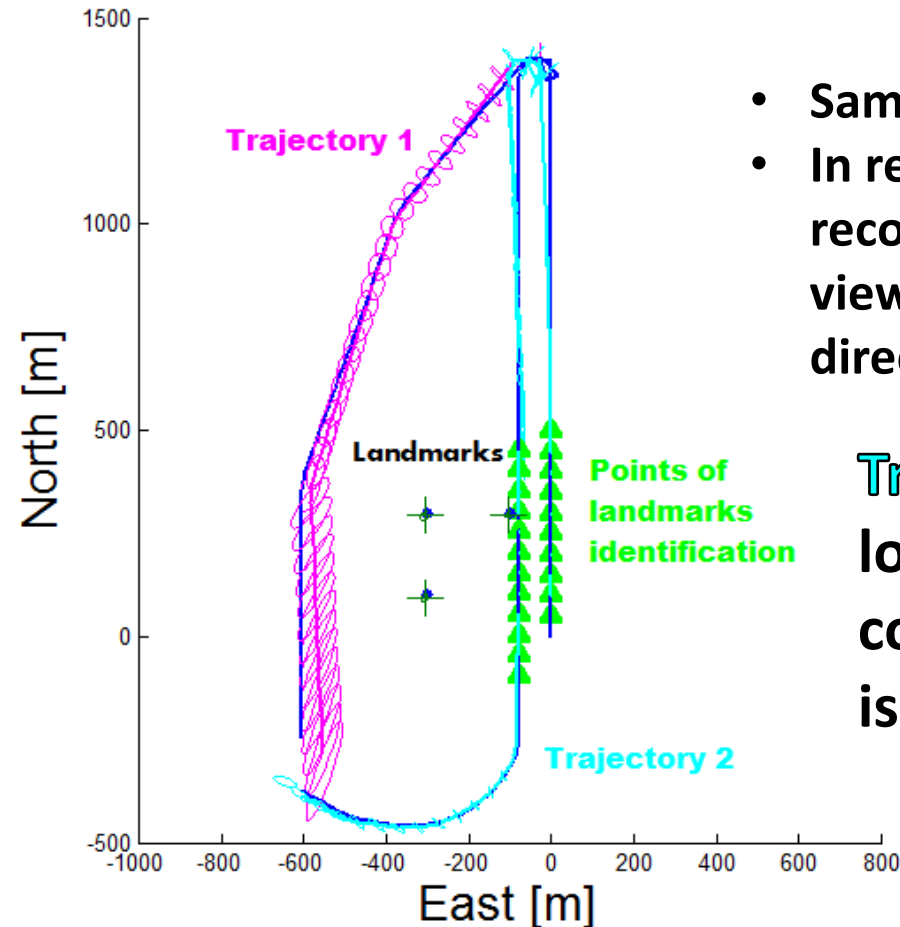
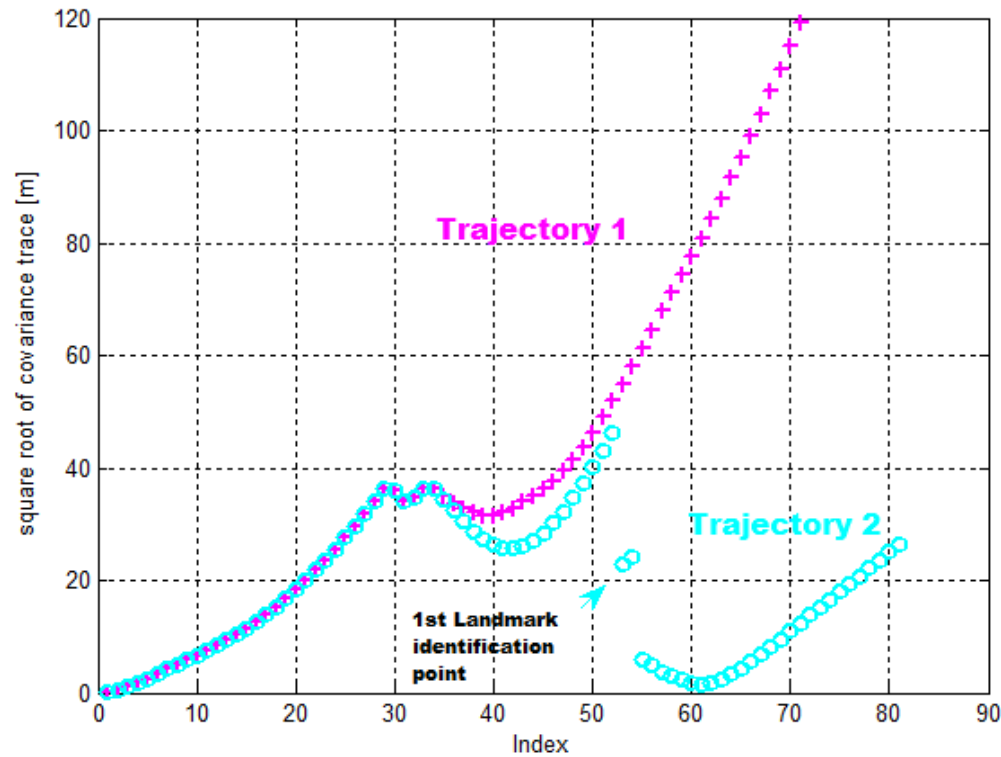
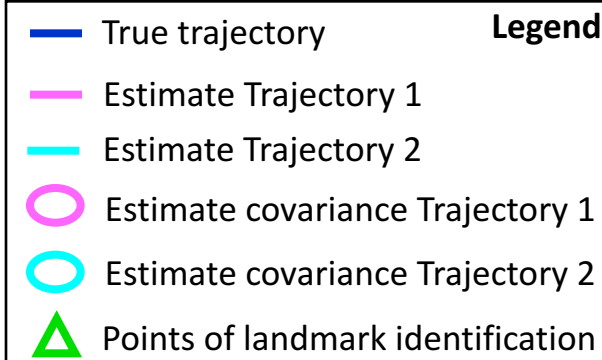
**Planning – With applying object identification**



**When applying object identification – the landmark is re-identified only in trajectory 2, similarly to reality**

# Results – SLAM

## Multiple landmarks



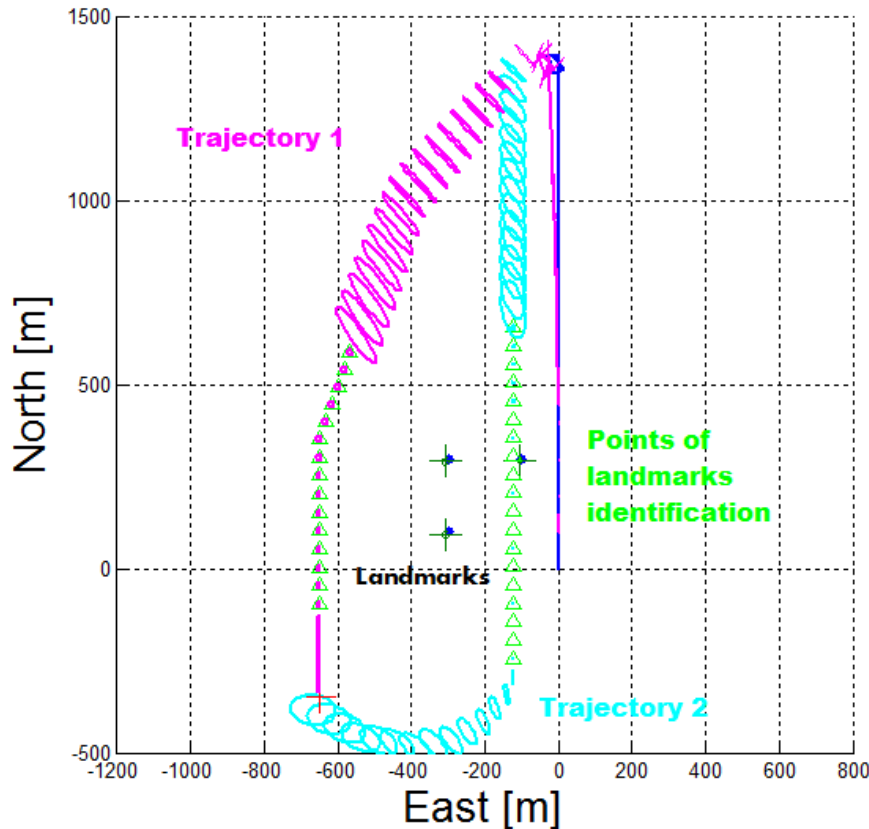
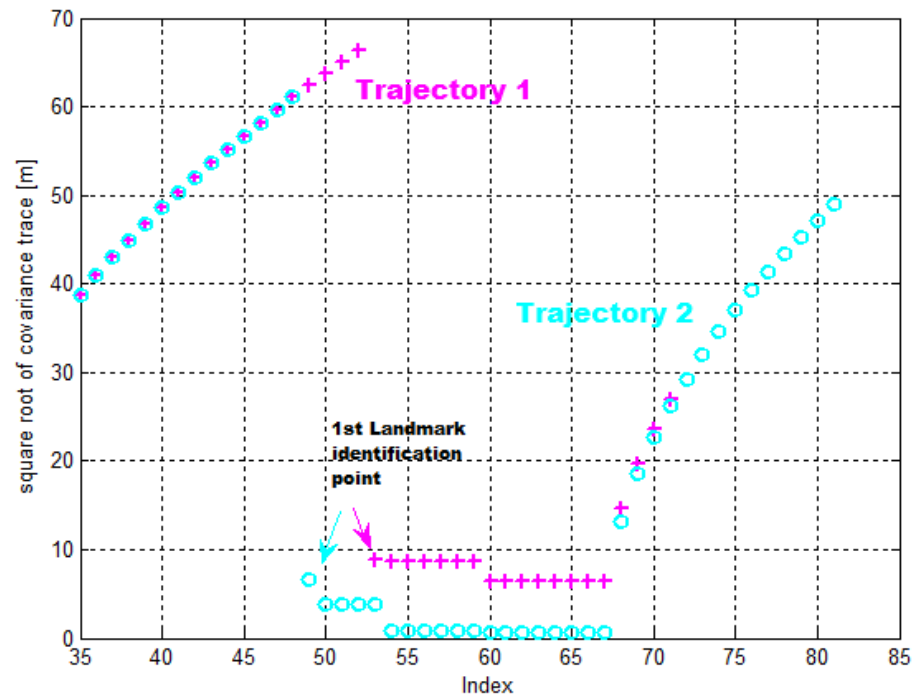
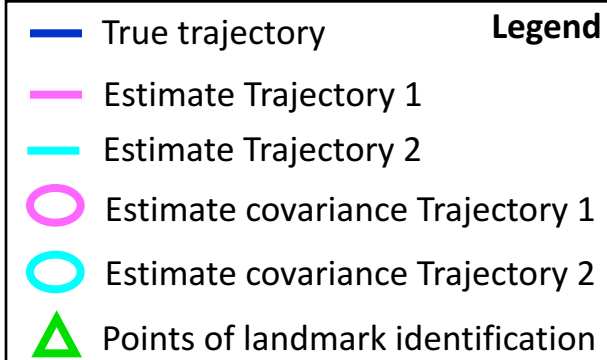
- Same as one landmark
- In reality: Landmark is recognized only where already viewed from similar view directions

**Trajectory 2 has lower estimate covariance though it is longer → preferred**

# Results – Planning

## Multiple landmarks

### Without applying object identification



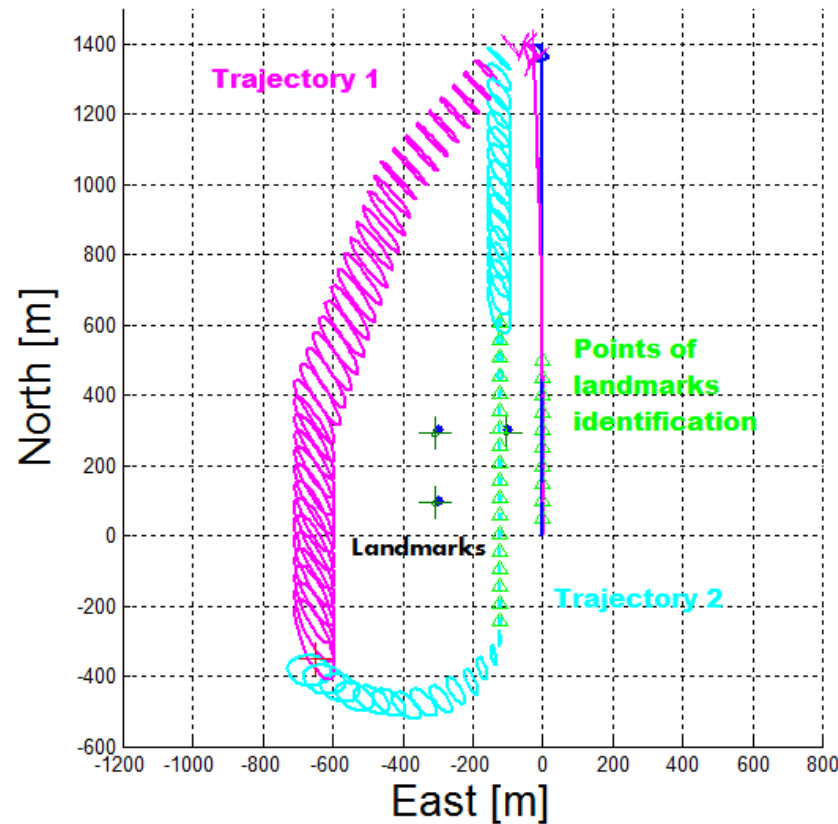
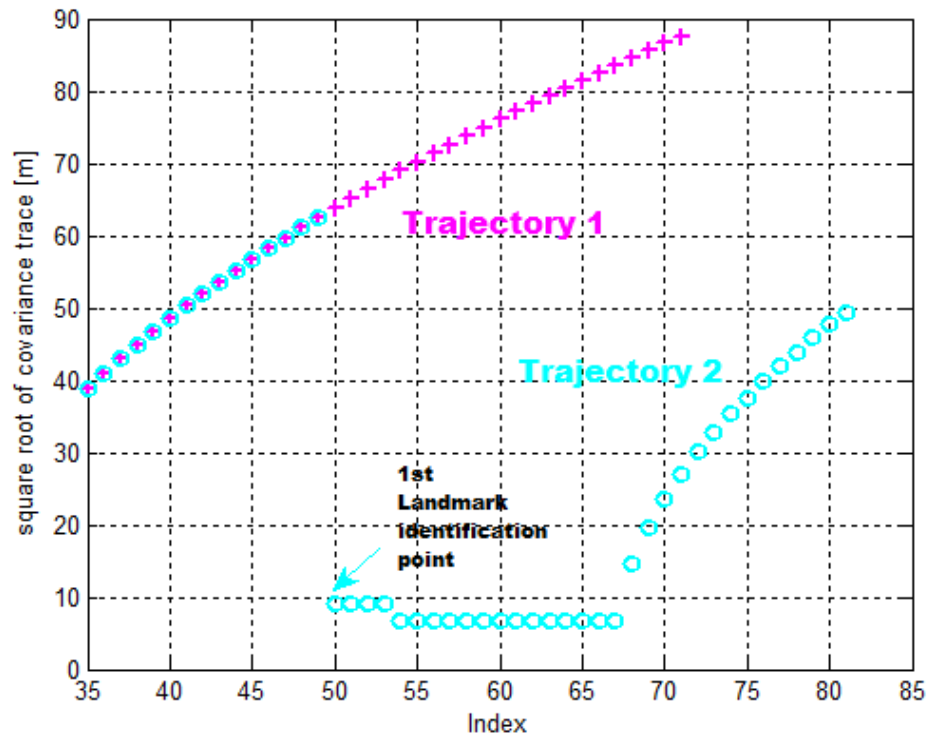
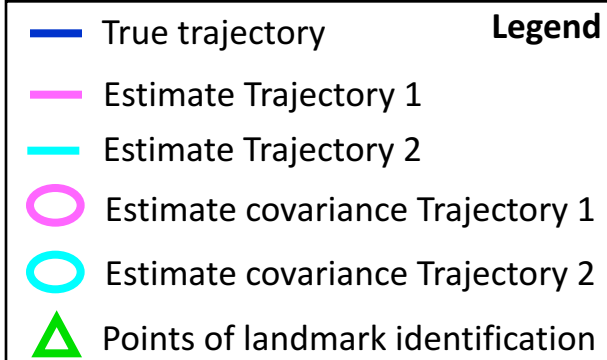
- Same as one landmark
- When not applying object identification: landmark is recognized incorrectly from new view directions

**Trajectory 1 has lower estimate covariance → incorrectly preferred**

# Results – Planning

## Multiple landmarks

### With applying object identification



- Same as one landmark
- Similar to reality:  
Landmark is recognized only where already viewed from similar view directions

**Trajectory 2 has lower estimate covariance though it is longer → preferred**

# Conclusions

We developed a viewpoint aware BSP approach  
and modeled object re-identification

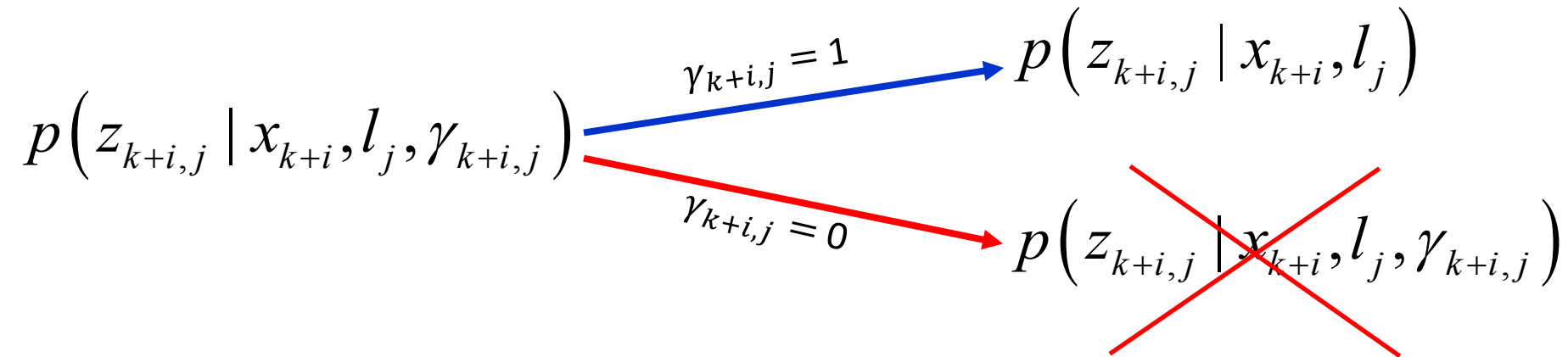


**Correct identification of landmarks is critical**  
**Uncertainty prediction consistent with reality (inference)**  
**Correct planning and path choosing**



# Thank you

# Why $p(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j})$ is uninformative?



Observation model:

$$z_{i,j} = h(x_i, l_j) + v_{i,j}(\gamma_{i,j}) \quad v_{i,j} : N(0, \Sigma_v)$$

$$\Sigma_v = \begin{cases} \Sigma_v & \gamma_{i,j} = 1 \\ \rightarrow \infty & \gamma_{i,j} = 0 \end{cases}$$

# How we use the belief $b(X_{k+l})$ when the future measurements are unknown?

$$b(X_{k+l}) : N(X_{k+l}^*, \Sigma_{k+l})$$

- We solve  $X_{k+l}^* = \underset{X_{k+l}}{\operatorname{argmax}}(b(X_{k+l}))$  using optimization method Non linear least squares
- In order to find the covariance  $\Sigma_{k+l}$  we do not need to know the measurements, only the fact that they were acquired or not
- We assume Maximum likelihood assumption:

$$z = h(\bar{x}) \rightarrow x^* = \bar{x}$$

- Where  $\bar{x}$  is the predicted value of  $x$ , according to motion model

# Marginalization

$X_k$  – All robot ( $x_{1:k}$ ) until time  $k$  and world states ( $l_{1:j}$ )

$H_k \triangleq \{Z_{0:k}, u_{0:k-1}\}$  Past measurements and controls

$\gamma_{i,j}$  – Event of acquiring measurement  $j$  at time  $i$

$\Gamma_i \triangleq \{\gamma_{i,j}\}_{j=1}^{n_i}$

Notations

$$p(X_{k+l}|H_{k+l}) = \sum_{\Gamma_{k+1:k+l}} p(X_{k+l}, \Gamma_{k+1:k+l} | H_{k+l})$$

For example, for only one observation ( $\Gamma_{k+1}$ ):

$$p(X_{k+1}|H_{k+1}) = p(X_{k+1}, \gamma_{k+1,1} = 1 | H_{k+1}) + p(X_{k+1}, \gamma_{k+1,1} = 0 | H_{k+1})$$