Belief Space Planning for Autonomous Navigation while Modeling Landmark Identification

Research Thesis

In Partial Fulfillment of The Requirements for the Degree of Master of Science in Aerospace Engineering

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Contents

List of Figures

1 Abstract

We investigate the problem of autonomous navigation in unknown or uncertain environments, which is of interest in numerous robotics applications, such as navigation in GPS-deprived environments, mapping and 3D reconstruction, and target tracking. In lack of sources of absolute information (e.g. GPS), the robot has to infer its own state and create a model of the environment based on sensor observations, a problem known as simultaneous localization and mapping (SLAM). Moreover, it has to plan actions, in order to accomplish given goals while relying on information provided by the inference (estimation) process. The inferred state, e.g. robot poses and 3D landmarks, cannot be assumed perfectly known because the observations and dynamics are stochastic; hence, planning future actions has to take into account different sources of uncertainty. The corresponding problem is known as belief space planning (BSP).

An essential ingredient in SLAM and BSP problems is correct association of landmarks observed by robot sensors (e.g. camera), as incorrect association might lead to wrong estimation and to catastrophic results. In particular, the ability to re-identify a previously observed object can be challenging, especially considering images taken airborne or on the ground with shallow viewpoints – an object may look completely different when observed from different angles. Yet, state of the art BSP approaches typically consider perfect ability to re-identify an object. In this work we develop a viewpoint aware BSP approach by modeling re-identification aspects within the planning phase. We study our approach in simulation, considering the problem of autonomously reaching a goal with highest estimation accuracy in a GPS-deprived unknown environment.

2 Notations

x_k	Robot state (e.g. pose) at time t_k
l_j	Coordinates of landmark number j
L_k	World state, comprising landmarks $\{l_j\}$ mapped until time t_k
X_k	The joint state at time t_k: $X_k \doteq \{x_0, ..., x_k, L_k\}$ (assumes a static environment)
Z_i	Available observations at time t_i: $Z_i = \{z_{i,1}, ..., z_{i,n_i}\}$
n_i	Number of observed landmarks at time t_i
$z_{i,j}$	A measurement of the jth landmark at time t_i
X_i^o	Involved variables ($X_i^o \subseteq X_i$) in individual measurement models that correspond to observations Z_i
$Z_{0:k}$	All the available observations until time t$_k$: $Z_{0:k} \doteq \{Z_0, ..., Z_k\}$
u_i	Control action applied at time t_i
$u_{0:k}$	All controls until time t$_k$: $u_{0:k} \doteq \{u_o, ..., u_k\}$

We use $\epsilon \sim N(\mu, \Lambda^{-1})$ to denote a Gaussian random variable ϵ with mean μ and information matrix Λ (inverse of the covariance matrix)

3 Introduction

We investigate the problem of autonomous navigation in unknown or uncertain environments, which is of interest in numerous robotics applications, for example, it may be used for exploration under water, exploration in space or for navigation in any other GPS-deprived environments. Furthermore we want to plan a suitable control strategy in order to accomplish a given task, for example reaching a certain goal with highest estimation accuracy and possibly other objectives.

We use simultaneous localization and mapping (SLAM) in order to infer the state of the robot and map the environment. In order to navigate in unknown environment, and in lack of sources of absolute information (e.g. GPS) the robot has to infer its own state and create a model of the environment based on sensor observations.

In order to plan a suitable control strategy to accomplish a given task, we use belief space planning approach (BSP), while relying on information provided by the inference (estimation) process. The inferred state, e.g. robot poses and 3D landmarks, cannot be assumed perfectly known because the observations and dynamics are stochastic; hence, planning future actions has to take into account different sources of uncertainty.

Existing BSP approaches, consider perfect ability to re-identify an object. This assumption is far from the real world - the ability to re-identify a previously observed object can be challenging, an object may look completely different when observed from different angles. An identification of a specific object is possible under several conditions: it depends on the camera viewpoint, sensor capabilities and image processing capabilities. Incorrect association might lead to wrong estimation, wrong planning and to catastrophic results.

In this work we relax the aforementioned assumption in existing belief space planning approaches regarding object identification from arbitrary viewpoints. To that end, we model object identification both in SLAM part and planning part, and study our approach in simulation, considering the problem of autonomously reaching a goal with highest estimation accuracy in a GPS-deprived unknown environment.

4 Literature Review

In order to navigate in an unknown environment and in lack of sources of absolute information (such as GPS), the robot has to infer its own state and to create a model of the environment based on sensor observations. The corresponding problem is known as simultaneous localization and mapping (SLAM) and has been extensively investigated by the robotics community (Kaess et al., 2012 [5]; Konolige et al., 2010 [8]; Kaess et al., 2008 [6]).

On the other hand, active aspects of this problem, i.e. how to determine best robot actions, present numerous unresolved challenges that are of prime importance for next generation of autonomous robotic systems. The complexity stems from the fact that robot position cannot be assumed perfectly known because the observations and dynamics are stochastic: robot position after applying a motion command is different from the predicted position due to actuation noise, sensor observations are corrupted with noise, and thus robot state is always estimated with some error. This is especially the case when operating in lack of sources of absolute information, such as GPS. Planning future actions therefore has to take into account the different sources of uncertainty. Moreover, the robot does not know, a-priori, which measurements will be acquired and what the future sensor readings will be. A further source of complexity is that the motion strategy directly influences the amount and the quality of the acquired measurements, making planning and estimation even more coupled.

The corresponding problem is known as planning in the belief space, where the belief represents a distribution of possible realizations of robot state and may also include representation of the environment model (position of 3D landmarks in the environment) at appropriate time instances.

There are a few approaches dealing with planning under uncertainty: (a) Discretization - those approaches approximate the state space or the possible control space as finite sets. In this approach, the control action is selected from among a finite set of candidate control actions (Chaves et al., 2014 [1] ; Stachniss et al., 2005 [13]) (b) Prior knowledge of the environment - those approaches assume that prior knowledge of the environment is available and the belief represents only robot position. (Van Den Berg et al., 2012 [14] ; Prentice and Roy, 2009 [12]).

In particular, Van Den Berg et al., 2012 [14] consider there are sources of absolute information (such as GPS or known landmarks) and that the belief only represents uncertainty in robot state. Planning in the belief space is done in the continuous domain. They treat future measurements as random variables but do not model the uncertainty in the fact that a future measurement may be acquired or not.

The closest work to our research is the work by Indelman et al., 2015 [4], which assume a completely unknown or uncertain environment, and model this source of uncertainty as part of the belief space. They avoid discretization by working in continuous control and state space domain. However, an implicit assumption made in Indelman et al., 2015 [4] and other related approaches is the ability to identify a previously observed object or area from any viewpoint.

This assumption is far from the real world. Identification of a specific object is possible under several conditions: it depends on the camera viewpoint, sensor capabilities and image processing capabilities. A given object looks completely different when one observes it from a different angle. Even for a human it will be challenging to recognize the same object from a different direction without having a previous knowledge, all the more so it will be difficult for an autonomous robot.

Like in Indelman et al., 2015 [4], Chaves et al., 2015 [2] take into account the fact that a measurement may or may not be obtained, however, they do not model object re-identification or data association aspects.

Other recent works, model object identification from other aspects which are not related to object re-identification from different view directions: Kim and Eustice, 2014 [7] address an active SLAM problem, assume an unknown environment and model saliency of the measurement through the planning i.e. how unique and distinguishable are previously observed scenes/landmarks. Chaves et al., 2014 [1] consider discrete action space; they also model saliency, and prioritize trajectories that go through high-saliency regions. Patil et al., 2014 [11] work in a continuous domain and consider discontinuity aspects, for example due to camera field of view: a landmark within a camera field of view is modeled to be observed, while a landmark outside the field of view is not. However, that work focuses on modeling the probability of acquiring a measurement, without considering re-identification aspects.

In this work we relax the aforementioned assumption in existing belief space planning approaches regarding object identification from arbitrary viewpoints. We take into account that a future measurement may or may not be acquired, while enabling operation in unknown environments. We model object identification both in SLAM part and planning part, and focus on object re-identification from different viewpoint when the object is previously mapped. In this work we do not address data association aspects, and refer to recent research [10], which incorporates data association aspects within BSP. We use a discretized action space and consider a given set of candidate actions, provided by some black-box method, and choose the best control action sequence from this set while modeling landmark re-identification aspects.

5 Background

5.1 Computer vision algorithms

An essential ingredient in SLAM and BSP problems is correct association of landmarks observed by robot sensors (e.g. camera), as incorrect association might lead to wrong estimation and to catastrophic results. Identification of a specific object is possible under several conditions: it depends on the camera viewpoint, sensor capabilities and image processing capabilities.

The ability to re-identify a previously observed object can be challenging, an object may look completely different when observed from different angles. For example, in figure 1 there are different facets of the same house - even for a human it will be challenging to identify that those two views indeed represent the same object, all the more so, it will be challenging for a robot.

Figure 1: Same object from different view points.

In order to recognize an object and “understand” the image that was received in the robot’s camera, we need to use computer vision algorithms. The decision whether a landmark is recognized as a landmark that has been already seen before depends on the applied computer vision algorithm. To demonstrate key aspects, we consider the well known SIFT [9] and RANSAC [3] algorithms.

SIFT is an algorithm to detect and describe local features in images. This algorithm finds interest points (features) in an image and generates for each one a signature (feature description). In order to identify the object in another image, the features need to be detectable after possible changes like scaling, rotation and illumination. Each detected feature is described, in addition to the descriptor, by image coordinates, scale and orientation.

In the next step, the algorithm determines correspondences between descriptors in two views. This is done by comparing between the feature descriptors (based on Euclidean distance of their feature vectors). However, after this step, the matching results may contain false matches (outliers), for example because of noise, occlusions, blur or significant change in camera view. These false matches must be found and removed in order to avoid false recognition.

To identify and reject the false matches it is necessary to use a robust estimation algorithm, such as the RANSAC algorithm [3]. RANSAC is an algorithm for estimating parameters of a model from noisy data that contains outliers by random sampling. The basic version of the algorithm proceeds as follows:

1. Randomly sample a number of points which is sufficient for model description (for example - 2 points for a line)

2. Count the number of data points in agreement with model

3. Repeat steps 1 and 2 N times

4. Choose model with maximum support

However, in practice, computer vision algorithms are limited in their identification ability. This limitation defines the conditions in which two views of the same scene will be considered as the same object; for example, in SIFT algorithm, an object will be re-identified when the change in viewpoint direction is up to 30^o-40^o.

In sections 8.1 and 9 we will use this fact to model cases in which the landmark will be indeed re-identified.

5.2 SLAM

Simultaneous localization and mapping (SLAM), is an approach to enable a robot to navigate in and map an unknown environment. In this problem the robot constructs a map of the environment and simultaneously keeps track of its pose (position and orientation), using incoming sensor observations and the mapped environment thus far.

When using only odometry, meaning use of data from motion sensors to estimate change in position over time, the new pose is calculated from previous pose and the estimated motion. This method is sensitive to errors; thus, using only this method the solution will drift over time.

Therefore, in SLAM we use the environment to correct the pose of the robot. This is accomplished by extracting features from the environment (landmarks) and re-observing them when the robot moves around, observations that are also called loop closure measurements. A typical SLAM pipeline involves front-end (image matching, data association) and back-end (inference/optimization) processes. In this work we consider the front-end to ideally determine data association, i.e. no outliers, as already mentioned earlier.

There are various SLAM approaches which differ in the definition of what is estimated (state vector) and in the inference techniques. In this work we use a smoothing approach (for computational efficiency): the state vector in a smoothing approach to SLAM involves not just the most current robot location, but the entire robot trajectory up to the current time. In other words, the state vector includes: current state, past poses, and landmarks. We use Gauss-Newton optimization for inference. Another alternative is to use the well known Kalman smoother, which however, performs calculations in covariance and not in information form. Calculations in information form have computational advantages since the system in our case is sparse.

5.3 Planning

The robot needs to plan a suitable control strategy in order to accomplish a given task. The fact that robot observations and dynamics are stochastic makes planning complex, because after applying a motion command the actual robot position differs from the predicted one, e.g. due to actuation noise. Furthermore the robot does not know in advance which measurements will be acquired and what the future sensor readings will be.

The planning process relies on estimation and perception data, that can be represented by the belief - a probability distribution over the state conditioned on the currently available data. Decision making under uncertainty and belief space planning (BSP) then refer to approaches that consider belief evolution as a result of different candidate actions. Given an objective function, which typically comprises immediate costs such as uncertainty, path length and reaching a specific goal, the best/optimal control strategy can be calculated. This process involved evaluating, for each candidate action, the objective function and choosing the action that minimizes (or maximizes) the objective function. Each such evaluation requires calculating the posterior future beliefs for appropriate look ahead steps.

6 Contribution

As mentioned in section 4, many existing approaches assume environment/map is known, while some recent approaches relax this assumption, thereby enabling operation in unknown environments. However, all existing BSP approaches typically consider perfect ability to re-identify an object from different view directions. In other words, these approaches assume that the view direction in which the object was observed does not matter and that the object will be identified from any view direction.

Yet, as discussed in section 5.1, the ability to re-identify a previously observed object can be challenging and depends on the view direction in which the object was observed, the computer vision algorithm and sensor capabilities. Therefore, the above assumption of ideal object identification, may lead to inconsistent uncertainty prediction with reality (inference), and hence, to incorrect planning, e.g. an incorrect (suboptimal) path might be chosen. The conclusion is that correct identification of landmarks is critical, as incorrect identification may lead to incorrect path choosing and catastrophic results.

In this work we enable operation in unknown environments without assuming perfect ability to re-identify an object. To do so, we model object re-identification and take into account that the object might not be identified when observed from completely different view directions, and develop a viewpoint aware BSP approach while considering both SLAM and planning aspects.

7 Probabilistic Formulation

7.1 SLAM

We define X_k as the joint state:

$$X_k \doteq \{x_0, ..., x_k, L_k\}, \tag{1}$$

where x_i is the robot state, and L_k is world state (for example, landmark coordinates).

The probability distribution function (pdf) over the joint state is:

$$p(X_k | Z_{0:k}, u_{0:k-1}), \tag{2}$$

where u_i and Z_i are, respectively, the control action and captured observations at time t_i. We consider Z_i to represent landmark observations and denote

$$Z_i \doteq \{z_{i,1}, ..., z_{i,n_i}\}, \tag{3}$$

where $z_{i,j}$ corresponds to observing landmark l_j at time t_i and n_i is the number of observed landmarks at that time.

The probabilistic motion model given control u_i and robot state x_i is:

$$p(x_{i+1} | x_i, u_i). \tag{4}$$

The probabilistic observation model for each measurement $z_{i,j}$ is:

$$p(z_{i,j} | x_i, l_j). \tag{5}$$

We consider the case of motion (4) and observation (5) models with additive Gaussian noise:

$$x_{i+1} = f(x_i, u_i) + w_i \quad , \quad w_i \sim N(0, \Sigma_w) \tag{6}$$

$$z_{i,j} = h(x_i, l_j) + v_{i,j} \quad , \quad v_{i,j} \sim N(0, \Sigma_v), \tag{7}$$

where Σ denotes a covariance matrix.

The pdf in equation (2) is defined by the belief at time t_k

$$b(X_k) \doteq p(X_k | Z_{0:k}, u_{0:k-1}), \tag{8}$$

and represented by a Gaussian:

$$b(X_k) = N(X_k^*, \Lambda_k^{-1}), \tag{9}$$

where the mean X_k^* is set to the maximum a posteriori (MAP) estimate, and the information matrix Λ_k is defined accordingly (as described in Indelman et al., 2015 [4]).

The joint pdf from equation (2) can be explicitly written as

$$p(X_k|Z_{0:k}, u_{0:k-1}) = \eta \cdot p(x_o) \cdot \prod_{i=1}^{k} \left[p(x_i|x_{i-1}, u_{i-1}) \prod_{j=1}^{n_i} p(z_{i,j}|x_i, l_j) \right], \quad (10)$$

where $p(x_0)$ is the prior information on x_0, $p(x_0) = N(\hat{x}_0, \Sigma_0)$, and η is a normalization constant. As seen, the expression in equation (10) includes priors, motion model (4) and the observation model (5) terms. The full derivation of equation 10 is provided in the Appendix.

In order to estimate X_k we use maximum a posteriori (MAP) inference:

$$X_k^* = \arg\max_{X_k} p(X_k|Z_{0:k}, u_{0:k-1}). \quad (11)$$

For Gaussian distributions, this involves solving a nonlinear least squares problem (see details in the Appendix):

$$X_k^* = \arg\max_{X_k} p(X_k|Z_{0:k}, u_{0:k-1}) = \arg\min_{X_k} - \ln\left(p(X_k|Z_{0:k}, u_{0:k-1}\right). \quad (12)$$

7.2 Belief Space Planning

In each step the robot needs to decide its next actions. In order to find the planning actions at planning time t_k, we optimize an objective function $J(u_{k:k+L-1})$ for L look-ahead steps. We consider a general objective function:

$$J(u_{k:k+L-1}) \doteq \mathop{\mathbb{E}}_{Z_{k+1:k+L}} \left\{ \sum_{l=0}^{L-1} c_l \left(b(X_{k+l}), u_{k+l}\right) \right\}, \quad (13)$$

where c_l is a general cost function that depends on the belief $b(X_{k+l})$ and on the control action u_{k+l}. The user-defined costs c_l can include different terms, with weights that specify the importance of each term. In this work, at each look ahead time l, the cost c_l includes three terms: distance to goal, pose uncertainty and control usage (see also [4]).

The belief at the lth look ahead step is defined as:

$$b(X_{k+l}) \doteq p(X_{k+l}|Z_{0:k}, u_{0:k-1}, Z_{k+1:k+l}, u_{k:k+l-1}), \quad (14)$$

where the joint state X_{k+l} includes the robot states and the observed world states until time $k+l$. The control actions and observations were separated:

- Until planning time t_k: $Z_{0:k}, u_{0:k-1}$ (which include the entire history of observations and actions, respectively, until the current time t_k)
- From planning time t_{k+1} for the first l look-ahead steps (until t_{k+l}): $Z_{k+1:k+l}, u_{k:k+l-1}$

In equation 13 we sum the costs for all L look ahead steps. However, since the future measurements $Z_{k+1:k+L}$ are unknown, we write the expectation operator to consider all possible realizations of these measurements.

For notational convenience we define the *history* of observations and actions as:

$$\mathcal{H}_k \doteq \{Z_{0:k}, u_{0:k-1}\}\,. \tag{15}$$

We will write again equation (14), using $\mathcal{H}_k$:

$$b(X_{k+l}) \doteq p(X_{k+l}|\mathcal{H}_k, Z_{k+1:k+l}, u_{k:k+l-1}). \tag{16}$$

This belief is represented by a Gaussian:

$$b(X_{k+l}) \sim N\left(X^*_{k+l}, \Lambda^{-1}_{k+l}\right), \tag{17}$$

where the mean X^*_{k+l} is set to the MAP estimate:

$$X^*_{k+l} = \underset{X_{k+l}}{\arg\max}\, b(X_{k+l}) = \underset{X_{k+l}}{\arg\min} - \ln b(X_{k+l}), \tag{18}$$

and the information matrix Λ_{k+l} is defined accordingly, (as described in Indelman et al., 2015 [4]).

Mathematical development of the belief in equation (16), while considering *ideal* ability of object re-identification, leads to:

$$p(X_{k+l}|\mathcal{H}_{k+l}) \propto p(X_k|H_k) \prod_{i=1}^{l} p\left(x_{k+i}|x_{k+i-1}, u_{k+i-1}\right) \prod_{j=1}^{n_i} p\left(z_{k+i,j}|x_{k+i}, l_j\right). \tag{19}$$

It can be seen that the belief, in equation (19), is composed of the belief at planning time t_k (obtained from SLAM), and motion and observation models for l look ahead steps. Also note that the above formulation assumes ideal data association.

Calculating the optimal control involves optimization of an objective function $J_k(u_{k:k+L-1})$:

$$u^*_{k:k+L-1} \doteq \left\{u^*_k, ..., u^*_{k+L-1}\right\} = \underset{u_{k:k+L-1}}{\arg\min}\, J_k(u_{k:k+L-1}). \tag{20}$$

There are two approaches, discrete and continuous, for calculating the optimal control $u^\star$. In the continuous approach, the optimal control is calculated iteratively in each step, using a minimization of an objective function - the optimization starts from a nominal control $u^{(0)}_{k:k+L-1}$, and, at each iteration (i), computes the delta vector $\Delta u_{k:k+L-1}$ that is used to update the current values of the controls. In the discrete mode, in which we are using in this work, a set of controls are planned offline. We approximate the control space as finite sets, the control action is selected from among a finite set of candidate control actions.

8 Approach

As explained in Section 6, in reality object re-identification is not perfect. Yet, state of the art BSP approaches typically consider perfect ability to re-identify an object (as can be seen in equation (19)).

Correct identification of objects is critical - the assumption of ideal object identification may lead to inconsistent uncertainty prediction with reality (inference), incorrect planning and therefore incorrect path choosing.

In this work we are not assuming perfect ability to re-identify an object. We take into account that the object might not be identified when observed from completely different view directions. We are modeling object re-identification and develop a viewpoint aware BSP approach while considering both SLAM and planning aspects.

8.1 Concept

We define the event of acquiring a measurement as an event in which the robot recognizes the object. The ability to identify an object depends on the view direction in which the object is being observed and on the image processing ability (as explained in Section 5.1).

In each observation we define the LOS (Line Of Sight) to the object, which is the straight line between the camera of the robot and the observed image. We also define a *cone of identification* in which the object can be identified using image processing algorithms, it means that all the line of sights inside this cone will lead to object recognition. Recall from section 5.1, in SIFT algorithm, an object will be re-identified when the change in viewpoint direction is up to 30^o-40^o.

A description of LOS and cone of identification is given in figure 2 - in this example, the robot is a UAV and the object is the house.

In figure 3 there is a description of possible states of the robot:

1. The first state A, is the first time the robot observes the object. The LOS of this observation and cone of identification in which the object can be identified, are showed in yellow.

2. The second state B, describes a situation in which the robot observes the object from a different view direction, while the new LOS is inside cone of identification that corresponds to state A. In this case the object will be identified as the object that was observed in state A.

3. The third state C, describes a situation in which the robot observes the object from a different view direction, but now, the new LOS is outside the cone of identification which was defined in state A. Therefore, the object will not be identified in this case, although it is observed in the same facet as in state A (because, the LOS angle in state C, is not close enough to LOS angle in state A, in order to identify the object by computer vision algorithms).

4. The last state D, describes a situation in which the robot observes the object from a completely different view direction, in different facet than in state A, and a completely different scene. In this case it is clear that the robot will not identify the object as the same object from the first state A.

Note that it is not sufficient to model only whether an object is within the field of view of the camera, as done in [11]. For example, in states C and D, the object is in the field of view of the camera, but still is not recognized, as it being observed from different view directions which are not allowing the identification.

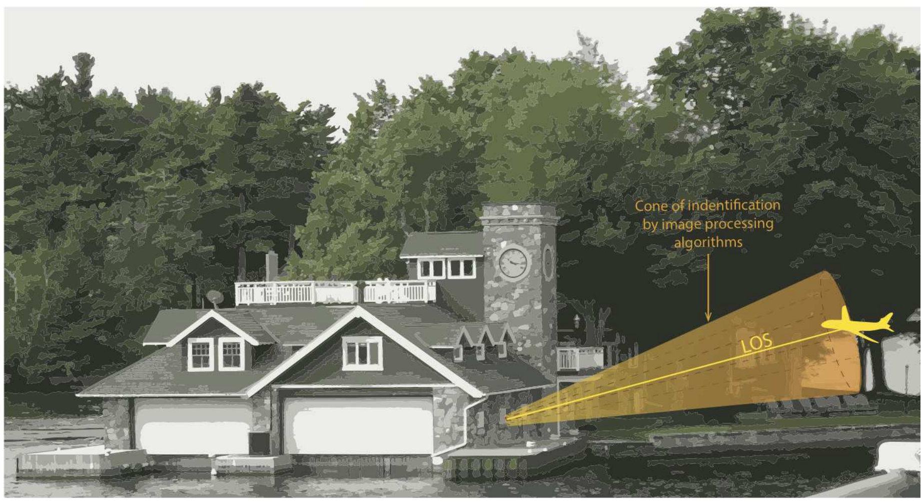

Figure 2: LOS Description.

Figure 3: Recognition of an object.

The concept of determination about object re-identification, is described in figure 4.

At planning time t_k the robot needs to examine some candidate sequence of actions in order to determine what is the optimal trajectory. In each future step, the robot observes landmarks in the environment, in order to infer it's state from an appropriate posterior distribution. Therefore, in order to calculate the posterior, it needs to determine in each future step whether the observed object is recognized.

In order to determine whether an object is re-identified, we need to calculate the examined LOS and the preserved LOS's from SLAM and planning parts. We use the current state of the robot and landmark coordinates, in order to calculate the examined LOS at time t_{k+i}.

The preserved LOS's are calculated from the estimated in the SLAM stage (which continues until planning time t_k; described by the blue part in figure 4), and from the expected data in the planning part (begins at planning time t_k and continues until time t_{k+i-1}; described by the orange part in figure 4).

We check if the examined LOS is inside one of previous cones of identification that were defined by the preserved LOS's. If so, the object is re-identified; otherwise, the object is not re-identified. Note that if the object is not re-identified, it will be classified as a new object. Therefore it will not add new information about the robot's state, and will not cause a reduction in the covariance.

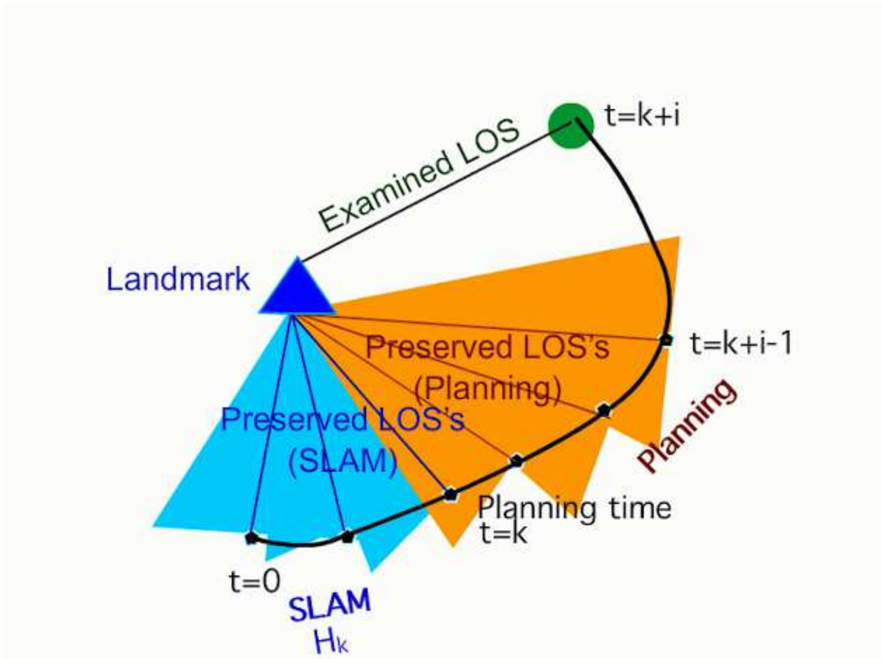

Figure 4: Concept description.

8.2 Viewpoint Aware BSP

In this section we will provide mathematical derivation of the concept mentioned above in section 8.1.

8.2.1 Define joint probability

In order to model object re-identification, we define a new binary random variable $\gamma_{i,j}$ for each observation $z_{i,j}$, in order to represent the event of this measurement being acquired. The new variable $\gamma_{i,j}$ indicates whether a measurement of landmark j is acquired at time i. If the landmark is recognized (the measurement is acquired), $\gamma_{i,j} = 1$, otherwise $\gamma_{i,j} = 0$ (as described in figure 5). We define also $\Gamma_i \doteq \{\gamma_{i,j}\}_{j=1}^{n_i}$, where n_i is the number of possible observations at time t_i.

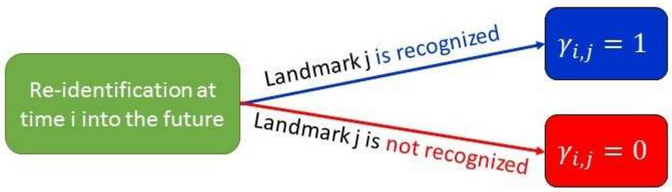

Figure 5: $\gamma_{i,j}$ definition.

In order to solve the Gaussian approximation of the belief (17) we need to take into account that we do not know ahead of time whether or not a future observation $z_{k+l,j}$ will be obtained, meaning whether the landmark j will be re-identified at time $k+l$, and what the measurement will be. Therefore, we treat $z_{i,j}$ and Γ_i as random variables.

We define a joint probability density over the random variables in our problem:

$$p(X_{k+l}, \Gamma_{k+1:k+l}, Z_{k+1:k+l}|\mathcal{H}_k, u_{k:k+l-1}). \tag{21}$$

By using the chain rule on equation (21), we get:

$$p\left(X_{k+l}, \Gamma_{k+1:k+l}|\mathcal{H}_k, Z_{k+1:k+l}, u_{k:k+l-1}\right) p\left(Z_{k+1:k+l}|\mathcal{H}_k, u_{k:k+l-1}\right). \tag{22}$$

We take the assumption that $p\left(Z_{k+1:k+l}|\mathcal{H}_k, u_{k:k+l-1}\right)$ is uninformative (which is a fairly standard assumption, because we can not say much about future measurements given only the controls and the past measurements) and get:

$$p(X_{k+l}, \Gamma_{k+1:k+l}, Z_{k+1:k+l}|\mathcal{H}_k, u_{k:k+l-1}) \propto \\ p\left(X_{k+l}, \Gamma_{k+1:k+l}|\mathcal{H}_k, Z_{k+1:k+l}, u_{k:k+l-1}\right). \tag{23}$$

Recall from section 7.2, in order to use the objective function in equation (13), we need to know the belief $b(X_{k+l})$, but as Z_i and Γ_i are treated as random variables, we only know the joint probability function in equation (23).

Therefore, in order to get the belief, $b(X_{k+l})$ we marginalize the latent variables $\Gamma_{k+1:k+l}$ and get:

$$b(X_{k+l}) = p(X_{k+l}|\mathcal{H}_k, Z_{k+1:k+l}, u_{k:k+l-1}) = \\ \sum_{\Gamma_{k+1:k+l}} p\left(X_{k+l}, \Gamma_{k+1:k+l}|\mathcal{H}_k, Z_{k+1:k+l}, u_{k:k+l-1}\right). \tag{24}$$

For example, in case of only one measurement, in only one time step, $\Gamma_{k+1} = \{\gamma_{k+1,1}\}$, the last term will be expanded as:

$$\sum_{\mathit{\Gamma}_{k+1}} p\left(X_{k+1}, \mathit{\Gamma}_{k+1} | \mathcal{H}_k, Z_{k+1}, u_k\right) = \sum_{\gamma_{k+1,1}} p\left(X_{k+1}, \gamma_{k+1,1} | \mathcal{H}_k, Z_{k+1}, u_k\right). \quad (25)$$

Since $\gamma_{k+1,1}$ is a binary random variable, i.e. can be either 1 or 0, equation (25) can be explicitly written as:

$$\sum_{\gamma_{k+1,1}} p\left(X_{k+1}, \gamma_{k+1,1} | \mathcal{H}_k, Z_{k+1}, u_k\right) =$$
$$p\left(X_{k+1}, \gamma_{k+1,1} = 1 | \mathcal{H}_k, Z_{k+1}, u_k\right) + p\left(X_{k+1}, \gamma_{k+1,1} = 0 | \mathcal{H}_k, Z_{k+1}, u_k\right). \quad (26)$$

8.2.2 Examination of joint probability derivation

In this section we examine the final result of the joint probability derivation while modeling object re-identification. The derivation of the joint probability in equation (23) will give the final result (full derivation is detailed in section 8.2.3):

$$p\left(X_{k+l}, \mathit{\Gamma}_{k+1:k+l} | \mathcal{H}_{k+l}\right) \equiv p\left(X_{k+l}, \mathit{\Gamma}_{k+1:k+l} | \mathcal{H}_k, Z_{k+1:k+l}, u_{k:k+l-1}\right) \propto$$
$$p\left(X_k | \mathcal{H}_k\right) \prod_{i=1}^{l} p\left(x_{k+i} | x_{k+i-1}, u_{k+i-1}\right) p\left(Z_{k+i}, \Gamma_{k+i} | \mathcal{H}_{k+i-1}, X^o_{k+i}\right), \quad (27)$$

where $\mathcal{H}_{k+l} \doteq \{u_{0:k+l-1}, Z_{0:k+l}\}$, and with

$$p\left(Z_{k+i}, \Gamma_{k+i} | \mathcal{H}_{k+i-1}, X^o_{k+i}\right) = p\left(Z_{k+i} | X^o_{k+i}, \Gamma_{k+i}\right) p\left(\Gamma_{k+i} | \mathcal{H}_{k+i-1}, X^o_{k+i}\right), \quad (28)$$

which can be further expanded in terms of individual observations $z_{k+i,j} \in Z_{k+i}$, i.e.:

$$p\left(Z_{k+i}, \Gamma_{k+i} | \mathcal{H}_{k+i-1}, X^o_{k+i}\right) = \prod_{j=1}^{n_i} p\left(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j}\right) p\left(\gamma_{k+i,j} | \mathcal{H}_{k+i-1}, x_{k+i}, l_j\right). \quad (29)$$

Plugging in equation (29) into equation (27) we get:

$$\boxed{\begin{aligned} p\left(X_{k+l}, \mathit{\Gamma}_{k+1:k+l} | \mathcal{H}_{k+l}\right) \propto p\left(X_k | \mathcal{H}_k\right) \prod_{i=1}^{l} p\left(x_{k+i} | x_{k+i-1}, u_{k+i-1}\right) \cdot \\ \cdot \prod_{j=1}^{n_i} p\left(z_{k+i,j} | x_{k+i}, l_j, \gamma_{k+i,j}\right) p\left(\gamma_{k+i,j} | \mathcal{H}_{k+i-1}, x_{k+i}, l_j\right). \quad (30) \end{aligned}}$$

We assume that the variables $\gamma_{k+i,j}$ are statistically independent. Also note the difference with respect to the derivation in section 7.2 , equation (19) and in Indelman et al., 2015 [4], where the last term in the above equation does not account for the history $\mathcal{H}_{k+i-1}$.

It can be seen that the final result of the derivation in equation (30), is composed of three main terms:

The first two terms are the same as seen in equation (19) when assuming ideal object identification. $p\left(X_k|\mathcal{H}_k\right)$ is the inference in the past until planning time t_k (SLAM) and $p\left(x_{k+i}|x_{k+i-1},u_{k+i-1}\right)$ is motion model for l future states from planning time t_k.

The last term is now composed of two different terms (differently than in equation (19) when assuming ideal object identification). The first term $p\left(z_{k+i,j}|x_{k+i},l_j,\gamma_{k+i,j}\right)$ represents the probability to get a certain measurement $z_{k+i,j}$, given robot state x_{k+i}, landmark coordinates l_j, and data about object identification $\gamma_{k+i,j}$ - whether the measurement is acquired or not. In the case of object identification (meaning measurement acquirement) - the term will turn into the basic measurement model as in equation (19), and we only need to calculate the probability of acquiring a measurement. If the object is not identified - this term is uninformative, because, when the measurement is not acquired, and therefore loop closure does not achieved, it is useless to consider the probability of achieving a certain measurement. The second term $p\left(\gamma_{k+i,j}|\mathcal{H}_{k+i-1},x_{k+i},l_j\right)$ represents the probability of acquiring a measurement j at time $k+i$, given the history $\mathcal{H}_{k+i-1}$, robot state x_{k+i} and landmark coordinates l_j.

In this work, as opposed to existing BSP approaches which are described in section 7.2 (and for example like in previous work of [4]), we are modeling the event of acquiring a measurement as an event which depends on the past estimated robot states, world states and observations. Therefore the term $p\left(\gamma_{k+i,j}|\mathcal{H}_{k+i-1},x_{k+i},l_j\right)$ includes the history $\mathcal{H}_{k+i-1}$. The concept of determination whether a measurement is acquired, meaning whether a landmark is re-identified, is described using figure 4 in section 8.1.

Note that the term $\mathcal{H}_{k+i-1}=\{H_k,u_{k:k+i-2},z_{k+1:k+i-1}\}$ includes the information that is needed in order to determine whether the measurement has been acquired in two levels. First, it includes the information in the SLAM part until time t_k (H_k, described using blue color in figure 4). Second, it also includes the expected information about the measurements and controls, from planning time t_k until time t_{k+i} in the future ($u_{k:k+i-2},z_{k+1:k+i-1}$, described using orange color in figure 4).

In order to determine whether a measurement is acquired, we use the data about robot state x_{k+i} and landmark coordinates l_j, at the examined time step t_{k+i}, to compute the current LOS. The preserved LOS's (which is composed of estimated LOS's in the SLAM part, and the expected LOS's in the planning part) can be obtained from the history $\mathcal{H}_{k+i-1}$. Note that X^*_{k+i-1} can be obtained from $p\left(X_{k+i-1}|\mathcal{H}_{k+i-1}\right)=N\left(X^*_{k+i-1},\Lambda^{-1}_{k+i-1}\right)$.

Now, we have all the information in order to determine whether a measurement is acquired, as explained in section 8.1: if the examined LOS is inside one of previous cones of identification then the object is identified and the measurement is acquired, else, the measurement is not acquired.

The determination process about measurement acquirement is described in figure 6.

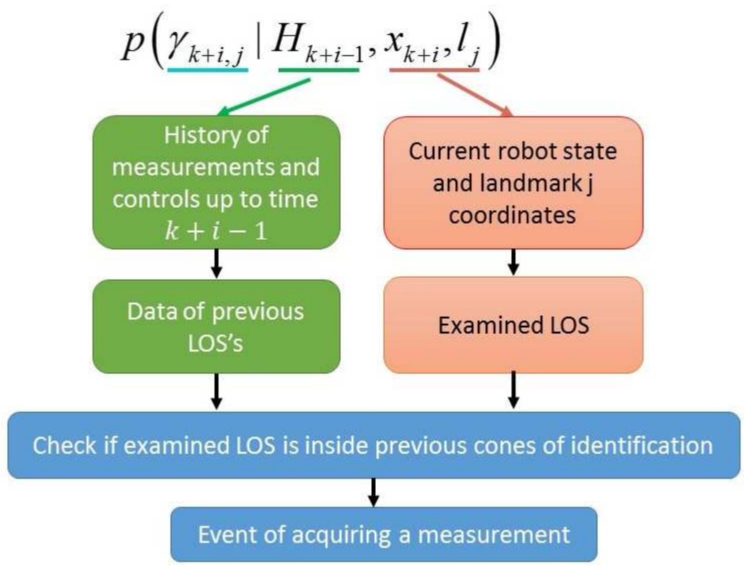

Figure 6: Determination process about measurement acquirement.

8.2.3 Full derivation of joint probability

In this section we describe the full derivation of the joint probability in equation (23), which its final result was presented in section 8.2.2.

The derivation of equation (23) proceeds in a recursive manner as follows

$$p\left(X_{k+l}, \mathit{\Gamma}_{k+1:k+l} | \mathcal{H}_k, Z_{k+1:k+l}, u_{k:k+l-1}\right) = \frac{p\left(Z_{k+l}, \Gamma_{k+l} | \mathcal{H}_{k+l-1}, u_{k+l-1}, X_{k+l}, \Gamma_{k+1:k+l-1}\right) p\left(X_{k+l}, \Gamma_{k+1:k+l-1} | \mathcal{H}_{k+l-1}, u_{k+l-1}\right)}{p\left(Z_{k+l} | \mathcal{H}_{k+l-1}, u_{k+l-1}\right)}, \tag{31}$$

where $\mathcal{H}_{k+l-1} \doteq \{u_{0:k+l-2}, Z_{0:k+l-1}\}$.

We assume that the term $p\left(Z_{k+l} | \mathcal{H}_{k+l-1}, u_{k+l-1}\right)$ is uninformative, therefore we can write equation (31) as:

$$p\left(X_{k+l},\mathit{\Gamma}_{k+1:k+l}|\mathcal{H}_k,Z_{k+1:k+l},u_{k:k+l-1}\right)\propto$$
$$p\left(Z_{k+l},\Gamma_{k+l}|\mathcal{H}_{k+l-1},u_{k+l-1},X_{k+l},\Gamma_{k+1:k+l-1}\right)p\left(X_{k+l},\Gamma_{k+1:k+l-1}|\mathcal{H}_{k+l-1},u_{k+l-1}\right). \tag{32}$$

We now proceed with derivation of the two terms in the above equation separately.

Term $p\left(X_{k+l},\Gamma_{k+1:k+l-1}|\mathcal{H}_{k+l-1},u_{k+l-1}\right)$ Applying chain rule we get

$$\begin{aligned}
&p\left(X_{k+l},\Gamma_{k+1:k+l-1}|\mathcal{H}_{k+l-1},u_{k+l-1}\right)=\\
&p\left(x_{k+l}|\mathcal{H}_{k+l-1},u_{k+l-1},X_{k+l-1},\Gamma_{k+1:k+l-1}\right)p\left(X_{k+l-1},\Gamma_{k+1:k+l-1}|\mathcal{H}_{k+l-1},u_{k+l-1}\right)=\\
&\qquad p\left(x_{k+l}|x_{k+l-1},u_{k+l-1}\right)p\left(X_{k+l-1},\Gamma_{k+1:k+l-1}|\mathcal{H}_{k+l-1}\right),
\end{aligned} \tag{33}$$

where in the last equation (33) we used the Markovian property of the motion model and the fact that future control does not impact past/current state.

We also used the assumption $L_k \equiv L_{k+l}$, which means that the world state in planing time k is the same state as in future time $k+l$. Therefore $X_{k+l} = X_{k+l-1} \cup x_{k+l}$.

Term $p\left(Z_{k+l},\Gamma_{k+l}|\mathcal{H}_{k+l-1},u_{k+l-1},X_{k+l},\Gamma_{k+1:k+l-1}\right)$ Also here, we apply chain rule and get

$$\begin{aligned}
&p\left(Z_{k+l},\Gamma_{k+l}|\mathcal{H}_{k+l-1},u_{k+l-1},X_{k+l},\Gamma_{k+1:k+l-1}\right)=\\
&p\left(Z_{k+l}|\mathcal{H}_{k+l-1},u_{k+l-1},X_{k+l},\Gamma_{k+1:k+l}\right)p\left(\Gamma_{k+l}|\mathcal{H}_{k+l-1},u_{k+l-1},X_{k+l},\Gamma_{k+1:k+l-1}\right)=\\
&\qquad p\left(Z_{k+l}|X^o_{k+l},\Gamma_{k+l}\right)p\left(\Gamma_{k+l}|\mathcal{H}_{k+l-1},X^o_{k+l}\right).
\end{aligned} \tag{34}$$

The first term in the last equation (34) was obtained by recalling the measurement model. To get to the second term we employed simplification by assuming Γ_{k+l} and $\Gamma_{k+1:k+l-1}$ are statistically independent. One can simplify further, and consider only the history at planning time k, i.e. $p\left(\Gamma_{k+l}|\mathcal{H}_k,X^o_{k+l}\right)$.

Plugging in into Eq. (31) ($p\left(X_{k+l},\mathit{\Gamma}_{k+1:k+l}|\mathcal{H}_{k+l}\right)$) Combining the two terms we get

$$\begin{aligned}
&p\left(X_{k+l},\mathit{\Gamma}_{k+1:k+l}|\mathcal{H}_{k+l}\right)\propto p\left(X_{k+l-1},\Gamma_{k+1:k+l-1}|\mathcal{H}_{k+l-1}\right)\cdot\\
&\quad\cdot p\left(x_{k+l}|x_{k+l-1},u_{k+l-1}\right)p\left(Z_{k+l}|X^o_{k+l},\Gamma_{k+l}\right)p\left(\Gamma_{k+l}|\mathcal{H}_{k+l-1},X^o_{k+l}\right)
\end{aligned} \tag{35}$$

As seen, a recursive formulation is obtained. Proceeding in a similar manner all the way until $l = 1$ we get

$$p\left(X_{k+l}, \mathit{\Gamma}_{k+1:k+l} | \mathcal{H}_{k+l}\right) \equiv p\left(X_{k+l}, \mathit{\Gamma}_{k+1:k+l} | \mathcal{H}_k, Z_{k+1:k+l}, u_{k:k+l-1}\right) \propto$$
$$p\left(X_k | \mathcal{H}_k\right) \prod_{i=1}^{l} p\left(x_{k+i} | x_{k+i-1}, u_{k+i-1}\right) p\left(Z_{k+i}, \Gamma_{k+i} | \mathcal{H}_{k+i-1}, X^o_{k+i}\right). \quad (36)$$

9 Implementation details

We define the event of acquiring a measurement as an event in which the robot recognizes the landmark, this will happen in the following conditions:

1. The landmark is in camera's field of view
2. The landmark is in a suitable range for detection
3. The landmark is recognized as a known landmark

A recognition of a landmark can occur if the landmark has been seen before from a similar view direction, we use LOS (line of sight) in order to define the view direction of the landmark from the robot's camera point of view (as explained in section 8). In order to define whether a landmark can be recognized, we need to preserve all the line of sights from the past.

We choose to represent the LOS, separated into vertical and horizontal plane (for implementation convenience) as described in figure (7). The calculations in this work were made in horizontal and vertical planes, but another option is to perform similar calculations in 3D.

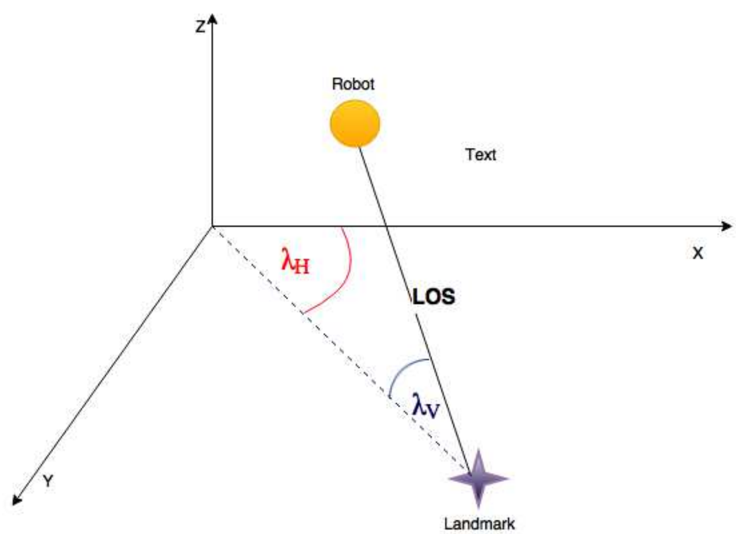

Figure 7: LOS Description.

In order to calculate λ_H and λ_V we will write in detail the robot's position vector and landmark's position vector:

$$\hat{x}_i = \begin{pmatrix} \hat{x}_R \\ \hat{y}_R \\ \hat{z}_R \end{pmatrix} \; \hat{l}_j = \begin{pmatrix} \hat{x}_L \\ \hat{y}_L \\ \hat{z}_L \end{pmatrix}$$

The relative position of the landmark with respect to robot's position is:

$$RelativePosition = \hat{l}j - \hat{x}_i = \begin{pmatrix} \hat{x}_{Rel} \\ \hat{y}_{Rel} \\ \hat{z}_{Rel} \end{pmatrix} = \begin{pmatrix} \hat{x}_L - \hat{x}_R \\ \hat{y}_L - \hat{y}_R \\ \hat{z}_L - \hat{z}_R \end{pmatrix}$$

λ_H is the horizontal angle of the LOS:

$$\lambda_H = \arctan \frac{\hat{y}_{rel}}{\hat{x}_{rel}}$$

λ_V is the vertical angle of the LOS:

$$\lambda_V = \arctan \frac{\hat{z}_{rel}}{\sqrt{\hat{x}_{rel}^2 + \hat{y}_{rel}^2}}$$

Note that λ_H,λ_V are a function of estimated values of robot's position and of landmark's position.

Recall the notion of cone of identification (see section 8.1). In order to represent the cone of identification we define an equal radius for each plane (vertical and horizontal) in which the computer vision algorithms will still be able to recognize the observed object, and mark it as r_{ImP} .

We define:

$\lambda_{V_{i,j}}$ is λ_V of landmark j in time t_i

We preserve all former line of sights in order to define if the landmark is recognized. Therefore:

$\lambda_{V_{s,j}}$ are former angles that were preserved at time t_s of landmark j

$d_{sV,j}$ is the difference between λ_V from the current examined time t_i ($\lambda_{V_{i,j}}$), and all former λ_V ($\lambda_{V_{s,j}}$), of landmark j:

$$d_{sV,j} \triangleq \left| \lambda_{V_{i,j}} \text{-} \lambda_{V_{s,j}} \right|, \; \forall s \in [1, ..., i-1] \tag{37}$$

$d_{V.j}$ is the minimum difference between λ_V at current examined time t_i ($\lambda_{V_{i,j}}$), to one of the former λ_V ($\lambda_{V_{s,j}}$), of landmark j:

$$d_{V,j} = min(d_{sV.j}) \tag{38}$$

Similarly, we define for horizontal plane:

$\lambda_{H_{i,j}}$ is λ_H of landmark j at time t_i

$\lambda_{H_{s,j}}$ are former angles that were preserved at time t_s of landmark j

$d_{sH,j}$ is the difference between λ_H at current examined time t_i ($\lambda_{H_{i,j}}$), and all former λ_H ($\lambda_{H_{s,j}}$), of landmark j:

$$d_{sH,j} \triangleq \left|\lambda_{H_{i,j}}\text{-}\lambda_{H_{s,j}}\right|, \forall s \in [1, ..., i-1] \tag{39}$$

$d_{H,j}$ is the minimum difference between λ_H at the current examined time t_i ($\lambda_{H_{i,j}}$), to one of the former λ_H ($\lambda_{H_{s,j}}$), of landmark j:

$$d_{H,j} = min(d_{sH,j}) \tag{40}$$

The probability to acquire a measurement j at time t_i ($\gamma_{i,j}$), given history $\mathcal{H}_{i-1}$, robot state x_i and landmark's coordinates l_j is modeled as

$$p(\gamma_{i,j} = 1|\mathcal{H}_{i-1}, x_i, l_j) = \begin{cases} 1 & \text{if} \quad (d_{V,j} < r_{ImP} \text{ and } d_{H,j} < r_{ImP}) \\ 0 & \text{else} \end{cases}$$

10 Results

In order to show the effect of modeling landmark identification from different view directions, and using viewpoint aware BSP approach, we are using a simulation and compare the influence of applying the new addition (as detailed in sections 8 and 9), to a simulation which uses the existing BSP approach.

The simulation, which models both SLAM and planning parts, considers a robot, for example an unmanned aerial vehicle (UAV), that has a camera and a range sensor. The robot operates in an unknown 3D area and has to reach a specific goal. The robot uses its sensors to observe landmarks in the environment and to estimate its own position. We currently use imagery and range measurements (the measurements are corrupted with Gaussian noise, according to the measurement noise covariance Σ_v from equation (7)). Note that we do not solve the problem of data association in the simulation and assume it is given.

In this work, we use discretization of the action space, a set of controls are planned offline in order to examine a given trajectory and investigate the evolution of the objective function along the trajectory.

We examine two predefined trajectories in order to validate the assumption in this work: taking into account landmark view directions, will have a great influence on the objective function $J_k(u_{k:k+L-1})$, and therefore on trajectory selection. The objective function that were defined for trajectory examination is $J_k = \sqrt{trace(\Lambda_{k+L}^{-1})}$, i.e. square root of covariance trace in the end of the trajectory.

10.1 Scenario

We check two predefined trajectories that differ in the landmarks view directions and their lengths, but reaching the same target. The trajectories are shown in figure 8.

Planning occurs at time k. By that time, the robot performs SLAM for estimating robot pose and mapping the environment, by observing landmarks in its field of view and mapping them. In each landmark observation, it determines whether the landmark is recognized, using current and preserved LOS's (as described in sections 8 and 9).

Now (at planning time k) it has two candidate trajectories to choose from. In each planing step, the robot determines about the expected landmarks observations, whether the landmarks are recognized. The decision about landmark re-identification (when applying viewpoint aware BSP), is done using examined LOS, preserved LOS's from SLAM part and expected LOS's of planning part until examination point (as explained in figure 4, section 8.1).

The trajectories differ in their path length, and view directions of the landmarks:

Trajectory 1 has shorter path, but the landmarks view directions in planning part, are completely different than in SLAM part.

Trajectory 2 has longer path, but the landmarks view directions in planning part, are similar to landmarks view directions in SLAM part.

We expect the version of simulation which contains the existing BSP approach with ideal landmark identification, to prefer Trajectory 1 which is shorter. When applying the new viewpoint aware BSP approach, which takes into account landmark's view direction in order to recognize the landmark, we expect to prefer Trajectory 2, though it is longer, similarly to what we expect in reality.

In order to compare between the trajectories, we calculate the position covariance along the trajectories.

We will first examine an environment which includes only one landmark, and then proceed to examine the results in an environment which includes multiple landmarks.

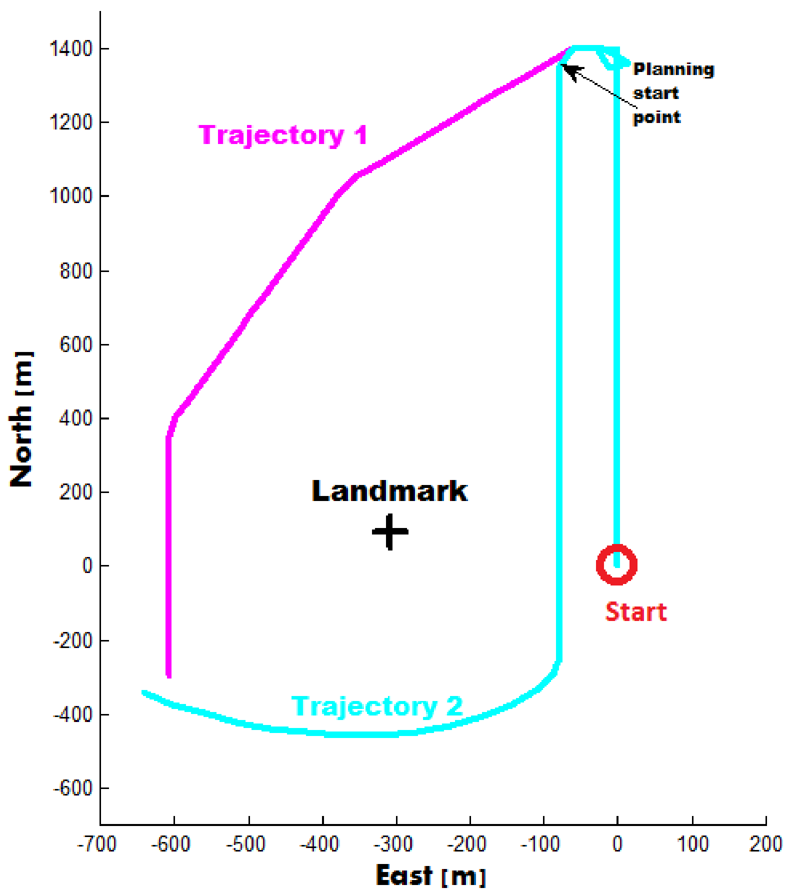

Figure 8: Trajectories definition.

In the figures below we will describe the trajectories that were examined.

We shall use the following different marks in the figures and their representation:

Blue line - true trajectory of the robot

Magenta line - estimated trajectory of the robot, for Trajectory 1

Magenta ellipse - position covariance in Trajectory 1

Cyan line - estimated trajectory of the robot, for Trajectory 2

Cyan ellipse - position covariance in Trajectory 2

Green triangles - points in the trajectory in which the landmark is identified

10.2 Results examination - one landmark

In this section, we will examine the results in an environment which includes one landmark.

In order to make a comparison, we examine three cases: SLAM , SLAM + Planning when assuming ideal landmark identification, SLAM + Planning when modeling landmark identification.

10.2.1 SLAM

This case represents the results we expect to have in practice from a SLAM system. In this case, the trajectory of the robot is calculated in SLAM mode along the trajectory (the robot does not perform planning). Meaning, at each step k, the robot performs SLAM in order to estimate it's state X_k^* and covariance Λ_k^{-1}, as shown in section 7.1, equation (9).

Figure 9 shows the estimated trajectories X_k^* in 2D plane (which is calculated according to equation (11)), with the ellipses indicating the covariance Λ_k^{-1} (from equation (9)). Figure 10 depicts the square root of the covariance Λ_k^{-1} trace, for the two trajectories.

As seen, in Trajectory 1, in which the path is going through new view directions of the landmark, the landmark is not identified, as expected to happen in real world (sections 5.1, 8.1). Therefore, the covariance keeps growing until the end point and is not reduced.

In contrast, in Trajectory 2, the path is going through similar landmark's view directions, to the view directions that the landmark was observed in the beginning of the trajectory. Therefore the landmark is identified (as can be seen by the green triangles in figure 9) and the covariance drops at the identification point.

This is compatible to what was explained in figure 3, section 8.1.

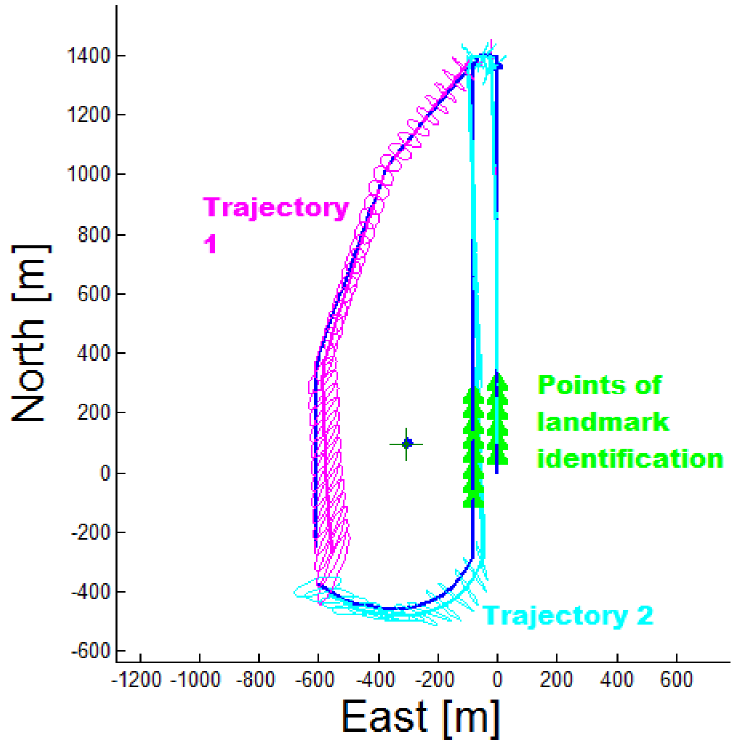

Figure 9: One landmark, SLAM.

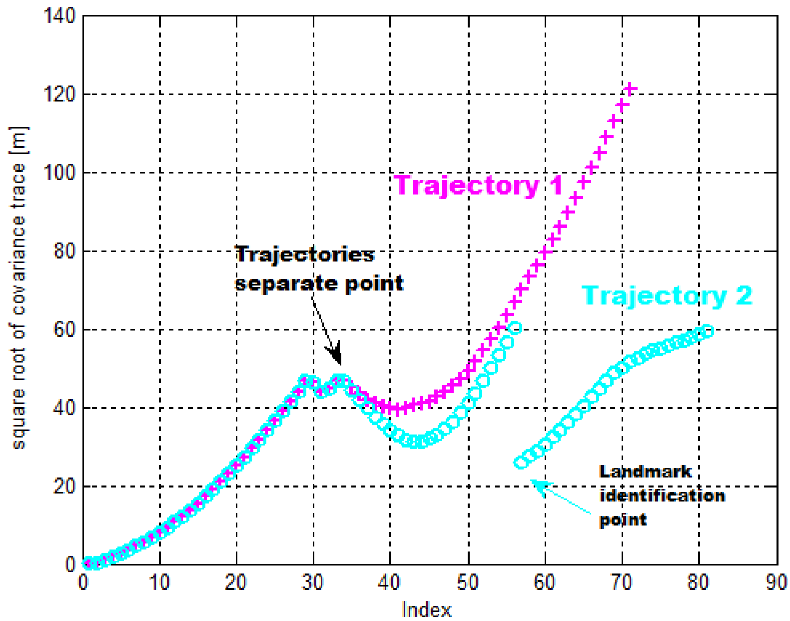

Figure 10: One landmark, SLAM, square root of covariance trace.

10.2.2 SLAM + Planning, Assuming ideal landmark identification

In this case, the robot performs SLAM until planning start point. From planning start point, the robot performs planning, i.e. it needs to choose a trajectory from amongst predefined trajectories, which we assume that are given. The trajectories can be also calculated online.

In this part, we do not apply our contribution of taking into account the landmark's view direction in the planning part. Landmark identification is defined only by distance to the landmark, when it is in the field of view of the camera. In other words, this part represents the existing BSP approach which assumes ideal landmark identification.

Figure 11 describes the estimated trajectories in 2D plane, the ellipses indicates the position covariance. In the SLAM part of the trajectory, the estimated trajectory is a plot of X_k^* which is calculated according to equation (11) and the covariance is Λ_k^{-1} from equation (9). In the planning part the estimated trajectory is a plot of X_{k+l}^* and the covariance is Λ_{k+l}^{-1} which are described in equation (17). Figure 12 describes the square root of covariance trace for the two trajectories, where the covariance is Λ_k^{-1} in SLAM and Λ_{k+l}^{-1} in planning.

It is shown that both in Trajectory 1 and Trajectory 2, the landmark is identified in a certain point of the path and thus the covariance decreases. This is in contrary to what happens in SLAM and real world (section 10.2.1 figures 9 and 10), in which the landmark is not identified when the path of the trajectory goes threw a completely different view directions, as in trajectory 1.

The incorrect landmark identification leads to incorrect drop of the covariance, which is inconsistent with reality, and finally to an incorrect decision about the trajectory selection. Therefore it is incorrectly deduced that Trajectory 1, which has a shorter path and assumed to has lower covariance, is preferred.

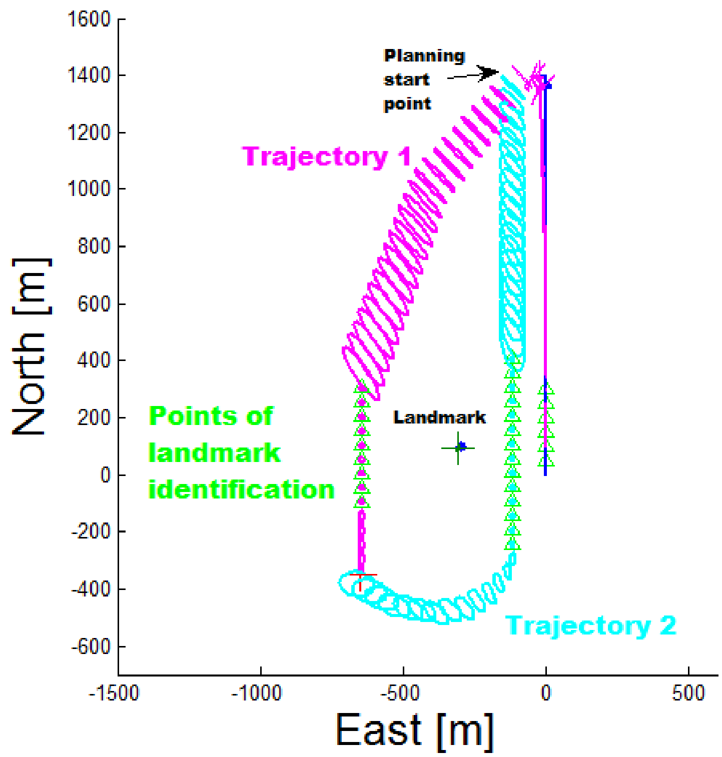

Figure 11: One landmark, Slam+Planning , **Without** modeling landmark re-identification. .

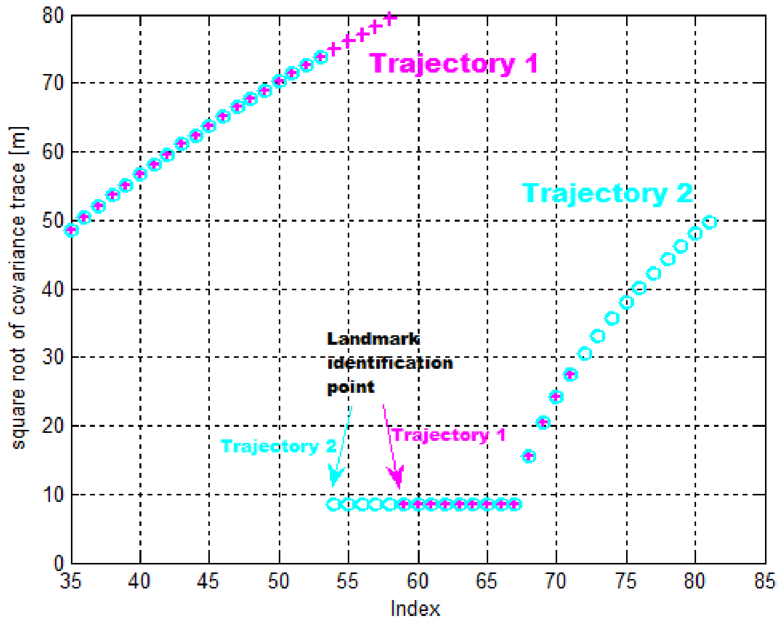

Figure 12: One landmark, Slam+Planning, square root of covariance trace, **Without** modeling landmark re-identification.

10.2.3 SLAM + Planning, Modeling landmark identification

In this case, the robot performs SLAM until planning start point in which the trajectories are separated, from planning start point, the robot performs planning, according to the predefined trajectory.

In this part, we apply our contribution of taking into account landmark's view direction in the planning part (viewpoint aware BSP approach). The landmark identification is defined by distance to the landmark, and in addition, the landmark will be identified in a condition that the landmark is being viewed from a similar view direction as explained in sections 8.1 and 9.

Figure 13 describes the trajectory in 2D plane; the ellipses indicates the position covariance. Figure 14 describes the square root of covariance trace for the two trajectories.

In the SLAM part the estimated trajectory is a plot of X_k^* which is calculated according to equation (11) and the covariance is Λ_k^{-1} from equation (9). In the planning part the estimated trajectory is a plot of X_{k+l}^* and the covariance is

Λ_{k+l}^{-1} which are described in equation (17).

It is shown that in Trajectory 1, in which the path goes through a completely different path than in the SLAM part, the landmark is not recognized and the covariance continues to evolve. In Trajectory 2 which goes through a path in which the landmark's view directions are similar to the view directions that the landmark was observed in the SLAM part, the landmark is being recognized and the covariance drops to a lower level, until the point where the landmark can not be recognized anymore - from that point, the covariance evolves again.

When using our viewpoint aware BSP approach, the results fit the results of the SLAM part (section 10.2.1, figures 9 and 10), and therefore model better the reality. This is also aligned with the explanation given using figure 3 from section 8.1, and figure 1 from section 5.1.

Here, the trajectory that is identified as a better trajectory is Trajectory 2 (as in SLAM part, section 10.2.1), which has lower covariance but a longer path compared to Trajectory 1. This is in contrast to the results without applying view directions (section 10.2.2).

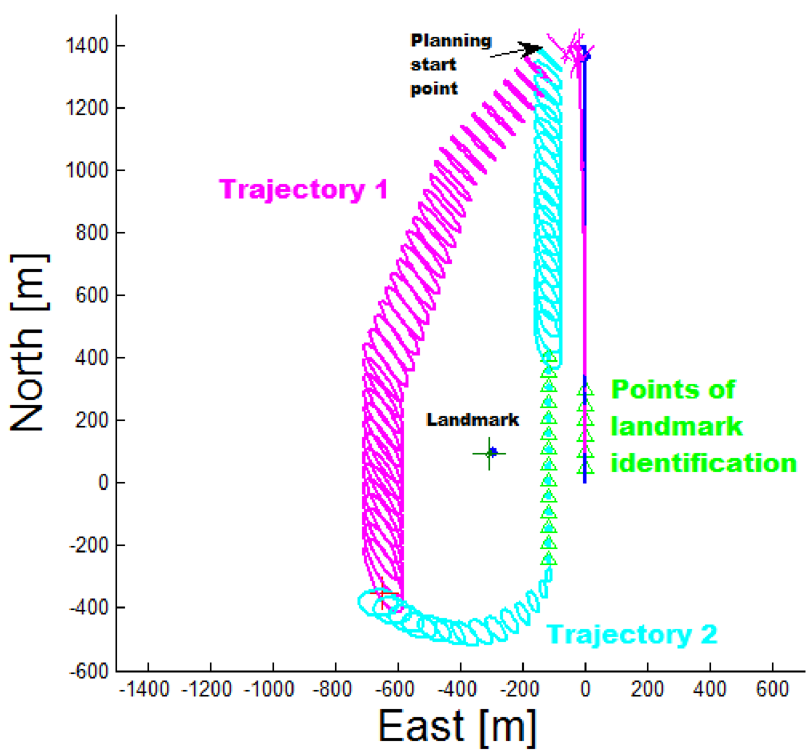

Figure 13: One landmark, Slam+Planning, **With** modeling landmark re-identifications.

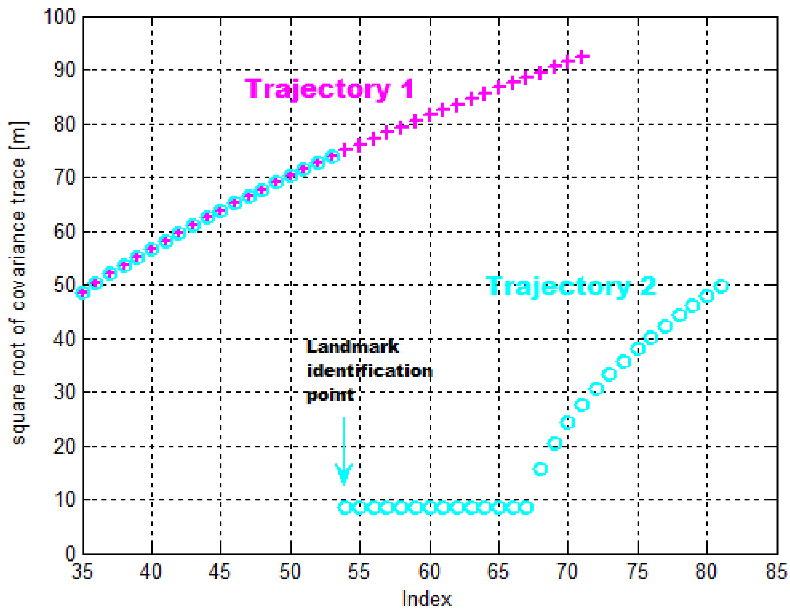

Figure 14: One landmark, Slam+Planning, square root of covariance trace, **With** modeling landmark re-identification.

10.2.4 Results conclusions - one landmark

We divided the results into three parts: SLAM , SLAM + Planning when assuming ideal landmark identification, SLAM + Planning when modeling landmark re-identification. Square root of covariance trace of the different parts are summarized in table 1.

In the SLAM part, in which the robot does not perform planning, and represents the reality, the landmark is re-identified only in trajectory 2 where the path goes through a similar view directions of the landmark. Trajectory 2 is preferred because it has lower covariance though it is longer.

In the SLAM+Planning part, when assuming ideal landmark identification, the landmark is re-identified incorrectly in trajectory 1, which goes through a path with completely different view directions of the landmark, and cause an incorrect drop of the covariance. Trajectory 1 is incorrectly preferred, because it has shorter path, and assumed to has lower covariance**. The results in this part are not consistent with SLAM part and reality.**

In the SLAM+Planning part, when modeling landmark re-identification, the landmark is re-identified only in trajectory 2, in which the path goes through

	Ideal identification	**Modeling re-identification**
Trajectory 1	27.54	92.34
Trajectory 2	49.72	49.72

Table 1: Square root of covariance trace - Summary

a similar view directions of the landmark, similarly to SLAM and reality. Trajectory 2 is preferred because it has lower covariance though it is longer. **The results in this part are consistent with SLAM part and reality.**

We note that the covariance was calculated in different ways in SLAM and Planning part according to implementation convenience. Therefore the form of the covariance trace is slightly different, but the tendency is similar and is satisfying for proving the desirable issues.

10.3 Results examination - multiple landmarks

In this section, we will examine the results in an environment which includes multiple landmarks.

As in previous section 10.2, in order to make a comparison, we examine three cases: SLAM , SLAM + Planning when assuming ideal landmark identification, SLAM + Planning when modeling landmark re-identification.

The results are compatible with the results that were presented in section 10.2 using an environment that includes one landmark. As can be seen:

In SLAM part (figures 15 and 16) in which the robot does not perform planning and represents the reality, the landmarks are re-identified only in trajectory 2 in which the path goes through a similar view directions of the landmarks. Trajectory 2 is preferred because it has lower covariance though it is longer.

In the SLAM+Planning part, when assuming ideal landmark identification (figures 17 and 18) the landmarks are re-identified incorrectly in trajectory 1, which goes through a path with completely different view directions of the landmarks, and cause an incorrect drop of the covariance. Trajectory 1 is incorrectly preferred, because it has shorter path, and assumed to has lower covariance. The results in this part are not consistent with SLAM part and reality.

In the SLAM+Planning part, when modeling landmark re-identification, the landmarks are re-identified only in trajectory 2 in which the path goes through a similar view directions of the landmarks, similarly to SLAM and reality. Trajectory 2 is preferred because it has lower covariance though it is longer. The results in this part are consistent with SLAM part and reality.

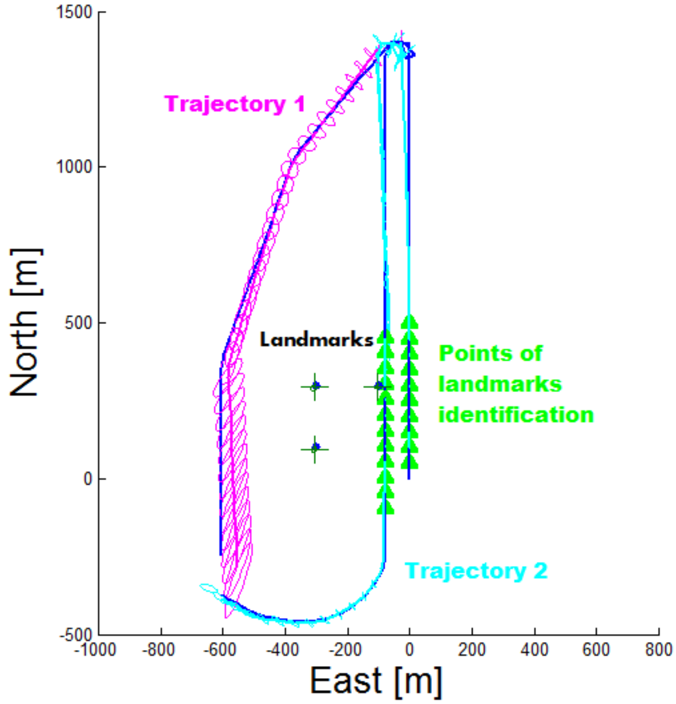

Figure 15: Multiple landmarks, SLAM.

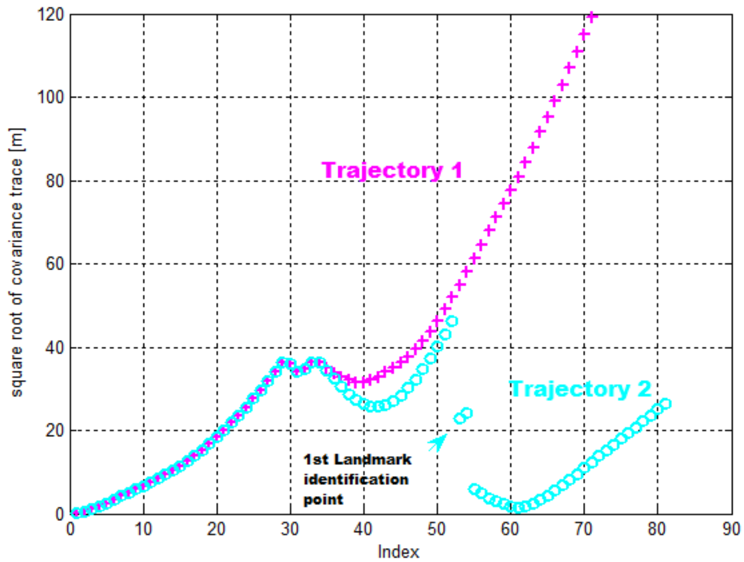

Figure 16: Multiple landmarks, SLAM, square root of covariance trace.

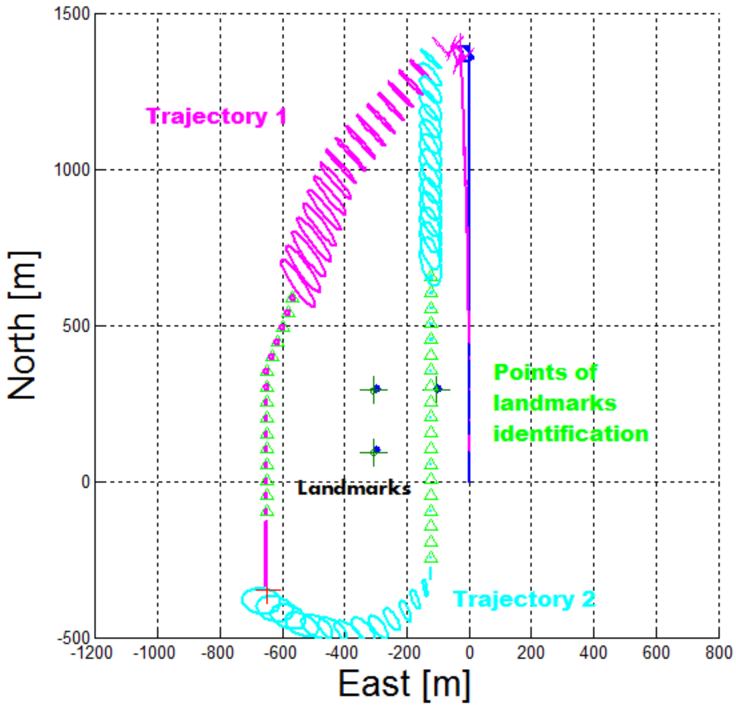

Figure 17: Multiple landmarks, Slam+Planning, **Without** modeling landmark re-identification.

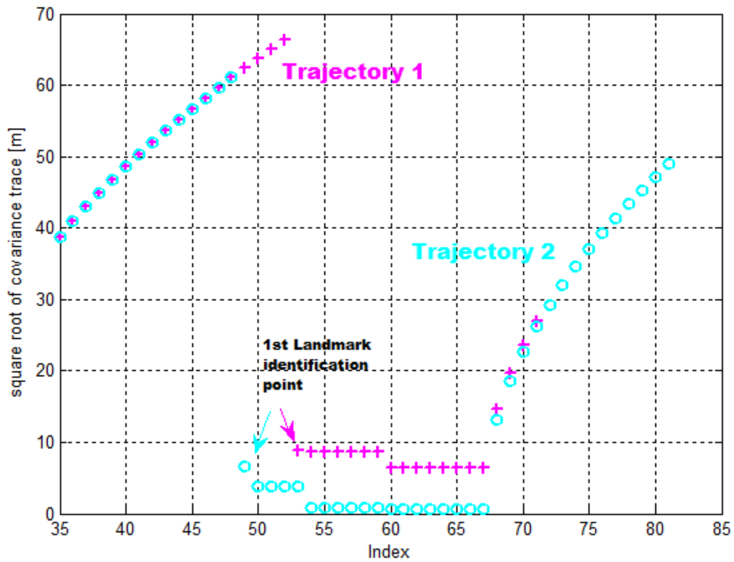

Figure 18: Multiple landmarks, Slam+Planning, square root of covariance trace, **Without** modeling landmark re-identification.

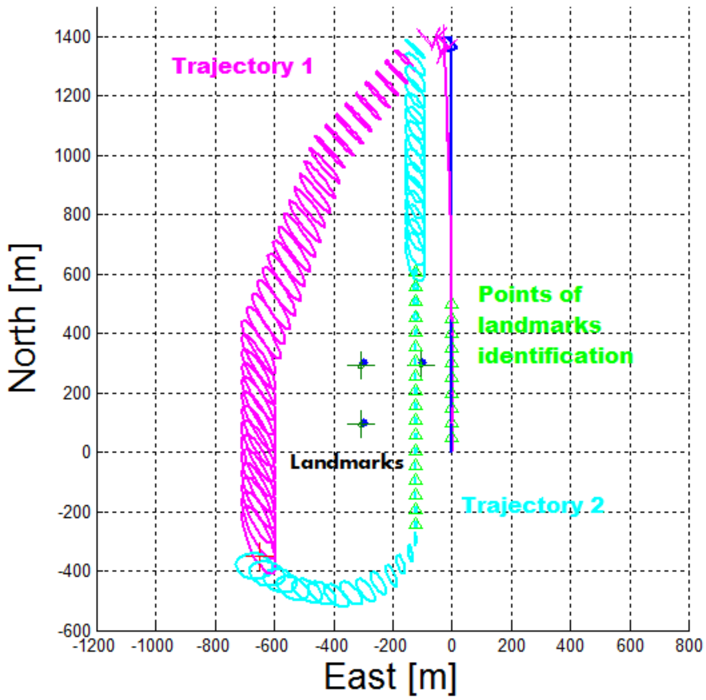

Figure 19: Multiple landmarks, Slam+Planning, **With** modeling landmark re-identification.

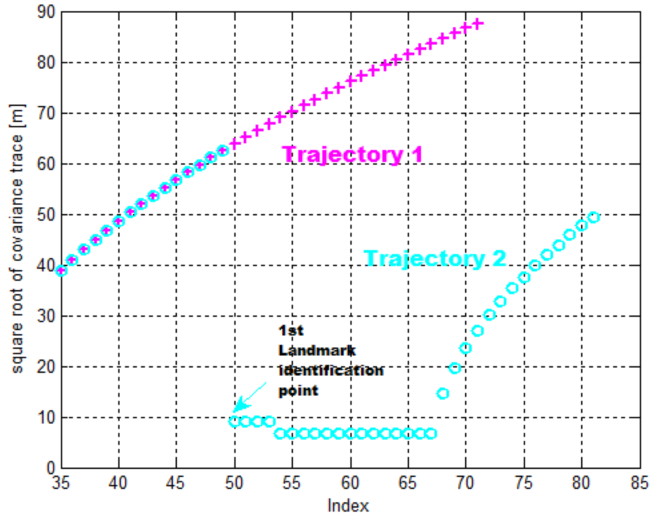

Figure 20: Multiple landmarks, Slam+Planning, square root of covariance trace, **With** modeling landmark re-identification.

11 Conclusions

In this work we investigated the problem of autonomous navigation in unknown or uncertain environments. We considered the problem of autonomously reaching a goal with highest estimation accuracy and possibly other objectives.

We used simultaneous localization and mapping (SLAM) in order to infer the state of the robot and map the environment, and used belief space planning approach (BSP) in order to plan a suitable control strategy to accomplish a given task, while relying on information provided by the inference (estimation) process. The observations and dynamics are stochastic, therefore the inferred state cannot be assumed perfectly known, and planning has to take into account different sources of uncertainty.

While existing BSP approaches consider perfect ability to re-identify an object, in this work we relax this assumption, model object re-identification and develop a viewpoint aware BSP approach, while considering both SLAM and planning aspects. That is because the assumption about ideal object identification is far from real world, as the ability to re-identify a previously observed object can be challenging, and depends a lot on the view direction in which the object is observed.

While studying the working concept of computer vision algorithms and in order to model object re-identification, we defined the cone of identification, in which, change in object's view direction, still makes the identification of an object possible. We defined a new binary random variable which defines whether a measurement is being acquired or not, and developed the BSP equations while taking into account object re-identification from different view directions, and the fact that the event of acquiring a measurement is unknown on planning time.

We examined the effects of the new addition using simulation, considering the problem of autonomously reaching a goal with highest estimation accuracy in a GPS-deprived unknown environment. We deduced as expected, that using the existing BSP approach which assuming ideal object identification, may lead to inconsistent uncertainty prediction with reality (inference), incorrect planning and therefore incorrect path choosing. When we modeled object re-identification and used a viewpoint aware BSP approach, the results were consistent with reality.

We conclude that correct identification of landmarks is critical, and incorrect identification may lead to catastrophic results.

The BSP approach that was developed in this work, enables the usage of belief space planning while considering realistic object identification, when it is being viewed from different view directions, and will lead to much more realistic planning and path choosing.

12 Appendix - Complete Derivation of SLAM Equations

Mathematical development of equation (2) will lead to:

$$p(X_k|Z_{0:k}, u_{0:k-1}) = p(X_k|u_{0:k-1}, Z_{0:k-1}, Z_k). \tag{41}$$

Using Bayes rule, we can write equation (41) as:

$$\frac{p(Z_k|X_k, u_{0:k-1}, Z_{0:k-1})\cdot p(X_k|u_{0:k-1}, Z_{0:k-1})}{p(Z_k|u_{0:k-1}, Z_{0:k-1})}. \tag{42}$$

Applying Markov assumption we get:

$$\frac{p(Z_k|X_k^o)\cdot p(X_k|u_{0:k-1}, Z_{0:k-1})}{p(Z_k|u_{0:k-1}, Z_{0:k-1})}. \tag{43}$$

Where X_k^o are involved variables ($X_k^o \subseteq X_k$) in individual measurement models that correspond to observations Z_k. We assume that the term $p(Z_k|u_{0:k-1}, Z_{0:k-1})$ is uninformative, therefore we can write equation (43) as:

$$\eta_k \cdot p(Z_k|X_k^o)\cdot p(X_k|u_{0:k-1}, Z_{0:k-1}), \tag{44}$$

while η_k is a normalization constant.

Note that the measurements are given until time t_{k-1}, therefore the world is mapped until time t_{k-1} :$L_k \equiv L_{k-1}$, and, $X_k = \{X_{k-1}, x_k\}$. Using this fact and the chain rule on equation (44) we get:

$$\eta_k \cdot p(Z_k|X_k^o)\cdot p(x_k|u_{0:k-1}, Z_{0:k-1}, X_{k-1})\cdot p(X_{k-1}|u_{0:k-1}, Z_{0:k-1}). \tag{45}$$

Using Markov assumption on equation (45) we get:

$$\eta_k \cdot p(Z_k|X_k^o)\cdot p(x_k|u_{k-1}, x_{k-1})\cdot p(X_{k-1}|u_{0:k-2}, Z_{0:k-1}). \tag{46}$$

The derivation proceeds in a recursive manner as follows:

$$= ... = \eta \cdot p(x_o)\cdot \prod_{i=1}^{k} p(x_i|u_{i-1}, x_{i-1})\cdot p(Z_i|X_i^o), \tag{47}$$

while η is a normalization constant, $p(x_0)$ is the prior information on x_0, $p(x_0) \sim N(\hat{x}_0, \Sigma_0)$.

The final result of the mathematical development of the probability distribution function from equation (2) is:

$$p(X_k|Z_{0:k}, u_{0:k-1}) = \eta \cdot p(x_o) \cdot \prod_{i=1}^{k} \left[p(x_i|x_{i-1}, u_{i-1}) \prod_{j=1}^{n_i} p(z_{i,j}|x_i, l_j) \right]. \tag{48}$$

It is received when expanding the term $p(Z_i|X_i^o)$ in equation (47) for all measurements available in time t_i.

n_i is the number of observed landmarks at time t_i.

In order to estimate X_k we use Maximum a posteriori (MAP) estimate:

$$X_k^* = \arg\max_{X_k} p(X_k|Z_{0:k}, u_{0:k-1}). \tag{49}$$

For Gaussian distributions, involves solving a nonlinear least squares problem:

$$X_k^* = \arg\max_{X_k} p(X_k|Z_{0:k}, u_{0:k-1}) = \arg\min_{X_k} -\ln\left(p(X_k|Z_{0:k}, u_{0:k-1}\right). \tag{50}$$

We use the Mahalanobis norm, $\|x-\mu\|_{\Sigma}^2 = (x-\mu)^T \Sigma^{-1}(x-\mu)$ to write again equation (50):

$$\begin{gathered} X_k^* = J(X_k) = \arg\min_{X_k} \Big\{ \|x_0 - \widehat{x}_0\|_{\Sigma_0}^2 + \\ + \sum_i \left[\|x_i - f(x_{i-1}, u_{i-1})\|_{\Sigma_W}^2 + \sum_j \|z_{i,j} - h(x_i, l_j)\|_{\Sigma_v^{i,j}}^2 \right] \Big\}. \end{gathered} \tag{51}$$

This equation is solved using Gauss-Newton optimization, which includes the following steps:

1. Linearization by first order Taylor expansion
2. Reorganize equation to get a term in the form of $\mathcal{A} \cdot \Delta X_k - \breve{b}$
3. Solve for ΔX_k
4. Update linearization point $\bar{X}_k \leftarrow \bar{X}_k + \Delta X_k$

Linearization of equation (51) by first order Taylor expansion leads to:

$$\begin{gathered} J(\bar{X}_k + \Delta X_k) \approx \arg\min_{\bar{X}_k + \Delta X_k} \Big\{ \|\Delta x_0\|_{\Sigma_0}^2 + \\ + \sum_i \Big[\left\| \bar{x}_i - f(\bar{x}_{i-1}, u_{i-1}) + 1 \cdot \Delta x_i - \nabla_{x_{i-1}} f \cdot \Delta x_{i-1} \right\|_{\Sigma_W}^2 \\ + \sum_j \|z_{i,j} - h(\bar{x}_i, l_j) - \nabla_{x_i} h \cdot \Delta x_i\|_{\Sigma_v^{i,j}}^2 \Big] \Big\}. \end{gathered} \tag{52}$$

with Jacobians:

$$\nabla_{x_{i-1}} f = \frac{\partial f}{\partial x_{i-1}}|_{\bar{x}_{i-1}, u_{i-1}} \tag{53}$$

$$\nabla_{x_i} h = \frac{\partial h}{\partial x_i}|_{\bar{x}_i, l_j} \tag{54}$$

In the next step, reorganize equation (52) in order to get a term in the form of $J(\bar{X}_k + \Delta X_k) \approx \left\| \mathcal{A} \cdot \Delta X_k - \breve{b} \right\|^2$ and calculate optimal increment ΔX_k by solving the linear equation $\mathcal{A} \cdot \Delta X_k = \breve{b}$.

Update linearization point by $\bar{X}_k \leftarrow \bar{X}_k + \Delta X_k$ and repeat process until convergence.

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במקרה השני בו מתבצע BSP מרגע תחילת התכנון, ללא החידוש של מידול זיהוי האובייקט, האובייקטים מזוהים בשני המסלולים, גם במסלול הקצר יותר בו כיווני הראיה שונים לגמרי מהכיוונים בהם נצפו האובייקטים בעבר. זיהוי האובייקטים במקרה זה אינו תואם את המציאות וגורם לירידה בשגיאת השערוך במסלול זה, שאף היא אינה תואמת את הירידה הצפויה במציאות ולכן גורמת להעדפה שגויה של המסלול הקצר יותר.

במקרה השלישי בו מתבצע BSP מרגע תחילת התכנון, כאשר כולל את החידוש של מידול זיהוי האובייקט, האובייקטים מזוהים בצורה נכונה התואמת את המציאות, כלומר רק במסלול בו כיווני הראיה דומים לכיוונים בהם נצפו האובייקטים בעבר. לכן המסלול המועדף נבחר להיות המסלול הארוך בו נצפים האובייקטים מכיוונים דומים, כפי שצפוי במציאות.

לסיכום, זיהוי לא נכון של האובייקטים, שאינו תואם את מה שקורה במציאות עלול להביא לירידה בשגיאת השערוך, שאינה תואמת את הצפוי במציאות ולכן לבחירה לא נכונה של מסלול ולתוצאות הרסניות. הגישה שפותחה בעבודה זו מאפשרת שימוש בגישת ה BSP תוך התחשבות ביכולת הריאלית לזיהוי אובייקט כאשר נצפה מכיוונים שונים.

<u>תקציר</u>

עבודה זו עוסקת בחקר הבעיה של ניווט אוטונומי בסביבה שאינה ידועה. ניווט אוטונומי בסביבה שאינה ידועה נדרש באפליקציות רבות למשל בחלל, מתחת למים או בכל סביבה שבה לא ניתן להשתמש ב GPS. במקרה שבו אין מקורות מידע נוספים, הרובוט נדרש לשערך את מצבו ולמפות את הסביבה על ידי שימוש במדידות המתקבלות מהסנסורים שלו (למשל מצלמה). בעיה זו נקראת SLAM. כאשר נעשה שימוש רק בנתוני התנועה לצורך שערוך המצב של הרובוט, פתרון הניווט נסחף לאורך זמן עקב רעשים. לכן, בבעיה זו, משתמשים בנתוני הסביבה המתקבלים מן המדידות על מנת להקטין את שגיאת השערוך. הקטנת שגיאת השערוך מתאפשרת על ידי צפייה על אובייקטים בסביבה, ולאחר מכן צפייה חוזרת עליהם מכיוון אחר כאשר הרובוט ממשיך בתנועה.

בנוסף, הרובוט נדרש לתכנן פעולות בקרה מתאימות על מנת להשלים משימה נתונה, תוך כדי הסתמכות על הנתונים המשוערכים. נתוני התנועה של הרובוט סטוכסטיים והמדידות המתקבלות הינן רועשות ולכן המצב המשוערך של הרובוט והאובייקטים בסביבה אינו ודאי. לכן בתכנון פעולות הבקרה העתידיות נדרש להתחשב במקורות אי הודאות השונים. בעיה זו נקראת BSP – Belief Space Planning (תכנון במרחב המצב).

תכנון פעולות הבקרה נעשה על ידי אופטימיזציה של פונקציית מחיר אשר יכולה לכלול לדוגמא אילוצים לגבי מינימום שגיאת השערוך, אורך מסלול והגעה למטרה מסוימת. גישות קיימות העוסקות בתכנון במרחב המצב, מניחות שקיים מידע מוקדם על הסביבה, גישות אחרות מניחות שאין מידע מוקדם על הסביבה, אבל כל הגישות הקיימות מניחות זיהוי אובייקט אידיאלי, כאשר נצפה מכיוונים שונים. ההנחה שאובייקט יזוהה בהכרח כאשר נצפה מכיוונים שונים לגמרי, אינה נכונה- יכולת זיהוי אובייקט תלויה מאוד בכיוונים בהם נצפה האובייקט, ביכולות המדידים וביכולות אלגוריתמי עיבוד התמונה.

בעבודה זו, אנו לא מניחים שזיהוי האובייקט אידיאלי. אנו ממדלים את זיהוי האובייקט ומפתחים גישת ה-BSP תוך התחשבות בכך. אנו מתמקדים בזיהוי אובייקט מכיווני צפייה שונים, בהנחה שנתון מיהו האובייקט בשדה הראייה. על מנת למדל את זיהוי האובייקט אנו מגדירים "קונוס זיהוי" סביב קו הראייה בין המצלמה לסצנה הנצפית, אשר מגדיר את כיווני הצפייה מהם האובייקט יזוהה בצפייה חוזרת. לצורך פיתוח משוואות ה- BSP תוך התחשבות במידול זיהוי האובייקט, אנו מגדירים משתנה אקראי בינארי חדש אשר מציין האם זוהה או לא זוהה האובייקט.

אנו משתמשים בסימולציה על מנת לבחון את ההשפעות של מידול זיהוי האובייקט על התוצאות. הסימולציה כוללת רובוט עם מצלמה (ומד מרחק) בסביבה שאינה מוכרת. אנו בוחנים שני מסלולים נתונים מראש, כאשר לכל אחד מהם דרך שונה אך מגיעים לאותה מטרה. בחלק הראשון הדרך זהה ומתבצע בה SLAM, עד אשר מגיעים לנקודה נתונה שהיא נקודת תחילת התכנון בה הרובוט מתחיל לבצע תכנון BSP, החל מנקודה זו כל מסלול ממשיך בדרך אחרת. מסלול אחד, נע בדרך קצרה יותר, אך אשר עוברת דרך כיווני ראיה חדשים בהם טרם נצפו האובייקטים. המסלול השני הינו ארוך יותר, אך עובר דרך כיווני ראייה דומים לכיווני הראייה בהם נצפו האובייקטים בחלק הראשון של ה- SLAM.

על מנת לבחון את החידוש במידול זיהוי האובייקט, אנו בוחנים את המסלולים הנתונים, בהיבט של הקטנת שגיאת השערוך. אנו בוחנים שלושה מקרים:

1. לאורך כל המסלול מתבצע רק SLAM, לא מתבצע תכנון, מקרה זה מייצג את המציאות.
2. מתבצע תכנון BSP החל מנקודת תחילת התכנון, על פי גישות BSP קיימות המניחות זיהוי אובייקט אידיאלי.
3. מתבצע תכנון BSP החל מנקודת תחילת התכנון, על פי גישת BSP הכוללת את החידוש של מידול זיהוי האובייקט.

מבחינת התוצאות בשלושת המקרים השונים, עולה כי במקרה הראשון בו מתבצע SLAM בלבד ואשר מייצג את המציאות, האובייקטים מזוהים רק כאשר נצפים מכיוונים דומים לכיוונים בהם נצפו בעבר, כצפוי. כאשר האובייקטים מזוהים שוב, שגיאת השערוך צונחת ולכן המסלול המועדף הוא המסלול בו האובייקטים מזוהים שנית, על אף שמסלול זה הוא ארוך יותר.

המחקר נעשה בהנחיית פרופסור ואדים אינדלמן בפקולטה להנדסת אוירונוטיקה וחלל

תודות

ברצוני להביע את הערכתי ותודתי הרבה לפרופ׳ ואדים אינדלמן על ההנחיה וההכוונה בכל שלבי המחקר. תודה על העצות הסבלנות וההסברים המפורטים. העבודה יחדיו על המחקר הייתה עבורי מהנה ומעניינת במיוחד.

למשפחתי הנפלאה, אני מודה לכם על העידוד והסבלנות לאורך כל הדרך. תודה מיוחדת לבעלי עידן על התמיכה הבלתי מוגבלת. לגילי והדר, תודה לכן על שגרמתן לתקופה להיות כה מהנה ומיוחדת.

ניווט אוטונומי באמצעות שערוך ותכנון
תחת אי ודאות
תוך מידול זיהוי אובייקט

חיבור על מחקר

לשם מילוי חלקי של הדרישות לקבלת התואר מגיסטר למדעים בהנדסת אוירונוטיקה וחלל

שירה הר-נס

הוגש לסנט הטכניון - מכון טכנולוגי לישראל

טבת תשע״ז, חיפה, ינואר 2017