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# Real-Time Mosaic-Aided Aerial Navigation: II. Sensor Fusion


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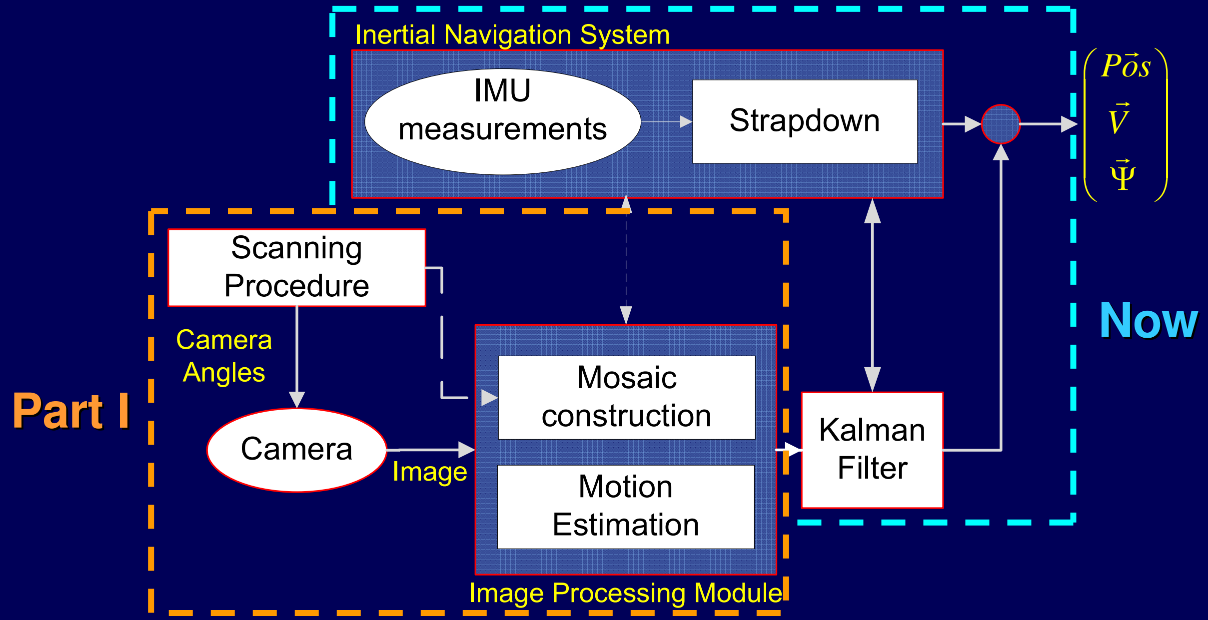
# Previously … In Part I

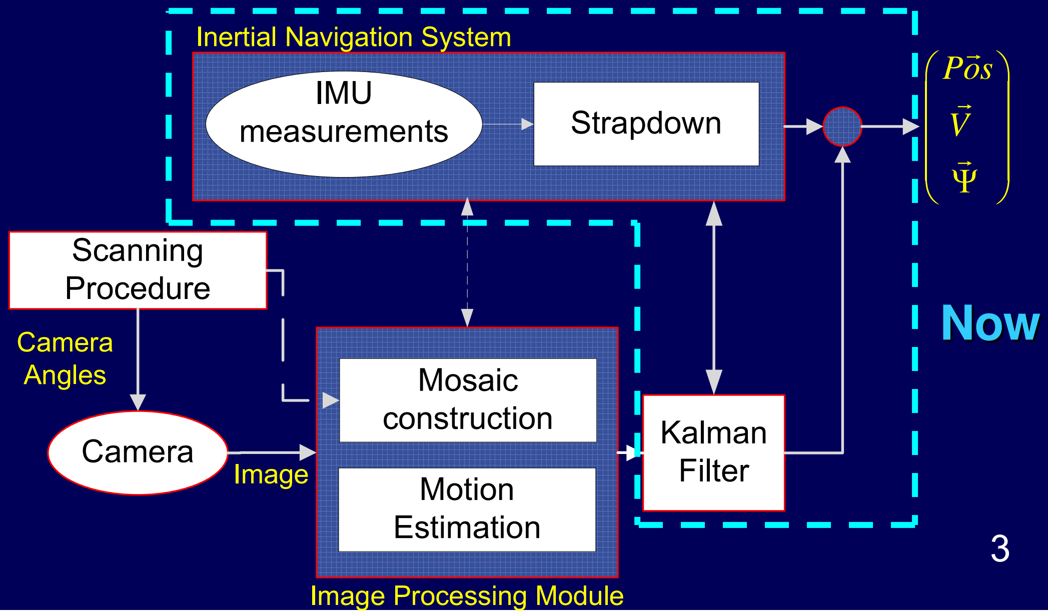
Introduction

- Camera scanning
- On-line mosaic construction
- Image-based motion estimation
  - Mosaicking improves estimation precision in challenging scenarios
    - Narrow camera FOV
    - Low-texture scene



# Contents

- Introduction
- Relative Motion Measurement Model
- Fusion with Navigation System
- Observability Analysis
- Performance Evaluation
- Summary



# Relative Motion Measurements Model



- Coordinate systems
  - L - Local Level Local North (LLLN)
  - B - Body
  - C - Camera

- Image-based motion estimation
  - $\vec{t}_{1\to 2}^{C_2}$ - translation (known up to some scale $\gamma$)
  - $R_{C_1}^{C_2}$ - rotation

- In **ideal** conditions, when there are no navigation errors and assuming perfect translation and rotation motion estimations:

$$\left[\overrightarrow{Pos}(t_2) - \overrightarrow{Pos}(t_1)\right]^{L_2} = \gamma T_{L_2}^{C_2} \vec{t}_{1\to 2}^{C_2}$$

$$T_{C_1}^{C_2} = R_{C_1}^{C_2}$$

  - $\overrightarrow{Pos}$ - Platform position
  - $T_M^N$ - DCM from system N to system M

# Relative Motion Measurements Model (Cont.)



$$\left[\overrightarrow{Pos}(t_2) - \overrightarrow{Pos}(t_1)\right]^{L_2} \times T_{L_2}^{C_2} \vec{t}_{1\to 2}^{\,C_2} = \vec{0}$$

$$T_{C_1}^{C_2}\left(R_{C_1}^{C_2}\right)^T = I$$

- In real conditions these constraints do not hold, due to
  - Navigation errors
  - Imperfect image-based motion estimations
- **Residual measurements** definition:

$$\left[\overrightarrow{Pos}_{Nav}(t_2) - \overrightarrow{Pos}_{Nav}(t_1)\right]^{L_2} \times T_{L_2,Nav}^{C_2} \hat{t}_{1\to 2}^{\,C_2} = \vec{z}_{translation}$$

$$T_{C_1,Nav}^{C_2}\left[\hat{R}_{C_1}^{C_2}\right]^T = I - \left[\vec{z}_{rotation}\right]_{\times}$$

Introduction

**Measurements Model**

Fusion with Navigation sys.

Observability Analysis

Performance Evaluation

Summary

# Relative Motion Measurements Model (Cont.)

- State vector definition

$$\vec{X} = \begin{bmatrix} \Delta\vec{P}^T & \Delta\vec{V}^T & \Delta\vec{\Psi}^T & \vec{d}^T & \vec{b}^T \end{bmatrix}^T \in \Re^{15\times 1}$$

- Continuous system matrix

$$\Phi_c = \begin{bmatrix} 0_{3\times3} & I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & A_s & 0_{3\times3} & T_L^B \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & -T_L^B & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix} \in \Re^{15\times 15}$$

  - $A_s$ - a skew-matrix constructed based on accelerometer sensors readings
  - $T_L^B$ - DCM from Body to Local Level Local North systems

# Relative Motion Measurements Model (Cont.)



- Measurements Equations

$$\begin{pmatrix} \vec{z}_{Translation} \\ \vec{z}_{Rotation} \end{pmatrix} = \begin{bmatrix} 0_{3\times3} & H^{Tr}_{\Delta V} & H^{Tr}_{\Delta\Psi} & H^{Tr}_{d} & H^{Tr}_{b} \\ 0_{3\times3} & 0_{3\times3} & H^{Rot}_{\Delta\Psi} & H^{Rot}_{d} & 0_{3\times3} \end{bmatrix} \begin{bmatrix} \Delta\vec{P} \\ \Delta\vec{V} \\ \Delta\vec{\Psi} \\ \vec{d} \\ \vec{b} \end{bmatrix} + \vec{v}$$

$$\Delta t = t_2 - t_1$$

## Translation terms

$$H^{Tr}_{\Delta V} = -\left[T^{C_2}_{L_2}\hat{t}^{C_2}_{1\to2}\right]_{\times} T^{L_1}_{L_2}\Delta t$$

$$H^{Tr}_{\Delta\Psi} = -\frac{1}{2}\left[T^{C_2}_{L_2}\hat{t}^{C_2}_{1\to2}\right]_{\times} T^{L_1}_{L_2} A_s\left(t_1\right)\left(\Delta t\right)^2$$

$$H^{Tr}_{d} = \frac{1}{6}\left[T^{C_2}_{L_2}\hat{t}^{C_2}_{1\to2}\right]_{\times} T^{L_1}_{L_2} A_s\left(t_1\right) T^{B_1}_{L_1}\left(\Delta t\right)^3$$

$$H^{Tr}_{b} = -\frac{1}{2}\left[T^{C_2}_{L_2}\hat{t}^{C_2}_{1\to2}\right]_{\times} T^{L_1}_{L_2} T^{B_1}_{L_1}\left(\Delta t\right)^2$$

## Rotation terms

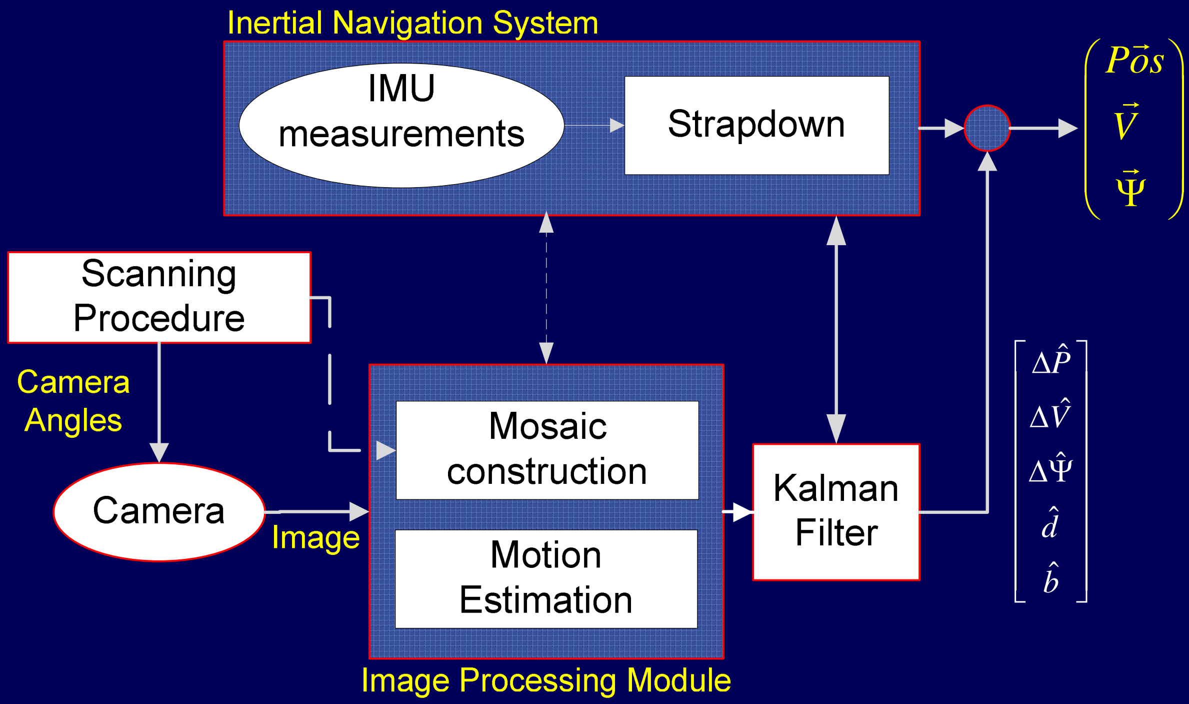
$$H^{Rot}_{\Delta\Psi} = \hat{R}^{C_2}_{C_1} T^{B_2}_{C_2} T^{L_2}_{B_2}\left(T^{E}_{L_2} T^{L_1}_{E} - I\right)$$

$$H^{Rot}_{d} = \hat{R}^{C_2}_{C_1} T^{B_2}_{C_2} T^{L_2}_{B_2} T^{B_1}_{L_1}\Delta t$$

# Relative Motion Measurements Model (Cont.)



- Remarks
  - Motion parameters may be estimated based on the homography or the fundamental matrices



# Implementation Details



- Adaptive translation measurement covariance

$$\vec{v}^{Tr} = \left[ Pos_{Nav}^{L_2}(t_2) - Pos_{Nav}^{L_2}(t_1) \right]_{\times} \Delta\hat{t}_{1\to 2}^{L_2} \quad , \quad \Delta\hat{t}_{1\to 2}^{L_2} = \hat{t}_{1\to 2}^{L_2} - t_{1\to 2}^{L_2}$$

$$R^{Tr} = -\left[ Pos_{Nav}^{L_2}(t_2) - Pos_{Nav}^{L_2}(t_1) \right]_{\times} R_{Est} \left[ Pos_{Nav}^{L_2}(t_2) - Pos_{Nav}^{L_2}(t_1) \right]_{\times}$$

- Measurement covariance matrix

$$R_k = \begin{bmatrix} R_k^{Tr} & 0 \\ 0 & R^{Rot} \end{bmatrix}$$

- Measurements-rejection mechanism is used to avoid fusion of low-quality measurements

# Implementation Details (Cont.)



- Fictitious Velocity (FV) measurement
  - Unobservable states in $\vec{X}$ are deteriorated due to imperfectness in image-based motion estimation $\left(\vec{t}_{1\to2}^{C_2}, R_{C_1}^{C_2}\right)$
  - Fictitious Velocity measurement is introduced
    - Goal – to let the filter "believe" the error along the flight heading is small
    - Implementation:

$$\left(\vec{V}^L\right)^T \Delta\vec{V} = 0$$

$$H^{FV} = \begin{bmatrix} 0_{1\times3} & \left(\vec{V}^L\right)^T & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} \end{bmatrix}$$

$$H_{Aug} = \begin{bmatrix} H^{Trans} \\ H^{Rot} \\ H^{FV} \end{bmatrix} \qquad R_{Aug} = \begin{bmatrix} R_{6\times6} & 0 \\ 0_{1\times6} & R^{FV} \end{bmatrix}$$

    - After the KF gain matrix is computed, the FV data is removed

# Observability Analysis



- Piece-Wise Constant System (PWCS) [Goshen-Meskin & Bar-Itzhack 1992]

$$\begin{cases} \vec{x}(k+1) = F_j \vec{x}(k) + B_j u(k) \\ \vec{z}_j(k) = H_j \vec{x}(k) \end{cases}$$

- For each time segment j=1,…,r the system matrices are constant
- At least n measurements in each segment
- Observability matrix in each segment

$$Q_j^T = \begin{bmatrix} H_j^T & \left(H_j^T F_j\right)^T & \dots & \left(H_j^T F_j^{n-1}\right)^T \end{bmatrix}$$

- Total Observability Matrix (TOM)

$$Q(r) = \begin{bmatrix} Q_1 \\ Q_2 F_1^{n-1} \\ \vdots \\ Q_r F_{r-1}^{n-1} F_{r-2}^{n-1} \dots F_1^{n-1} \end{bmatrix}$$

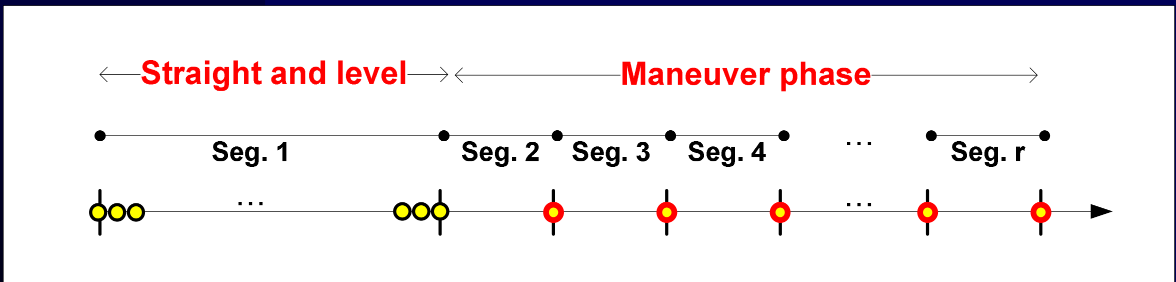
# Observability Analysis (Cont.)

- In our case

$$\vec{X}(k+1) = \Phi_{d_j}\vec{X}(k)$$

$$\begin{pmatrix} \vec{Z}_j^{Trans} \\ \vec{Z}_j^{Rot} \end{pmatrix} = \begin{pmatrix} H_j^{Tr} \\ H_j^{Rot} \end{pmatrix}\vec{X}(k)$$

  - Each segment may have less than n measurements
    - Measurements frequency is not as high as desired
  - Examined scenario
    - Straight and Level (SL) flight + maneuver phase
    - Maneuver phase is divided into segments
      - Worst case – one measurement per segment




$$\Rightarrow Q(r) = \begin{bmatrix} Q_1 \\ H_2\Phi_{d_2}\Phi_{d_1}^{n-1} \\ \vdots \\ H_r\Phi_{d_r}\ldots\Phi_{d_2}\Phi_{d_1}^{n-1} \end{bmatrix}$$

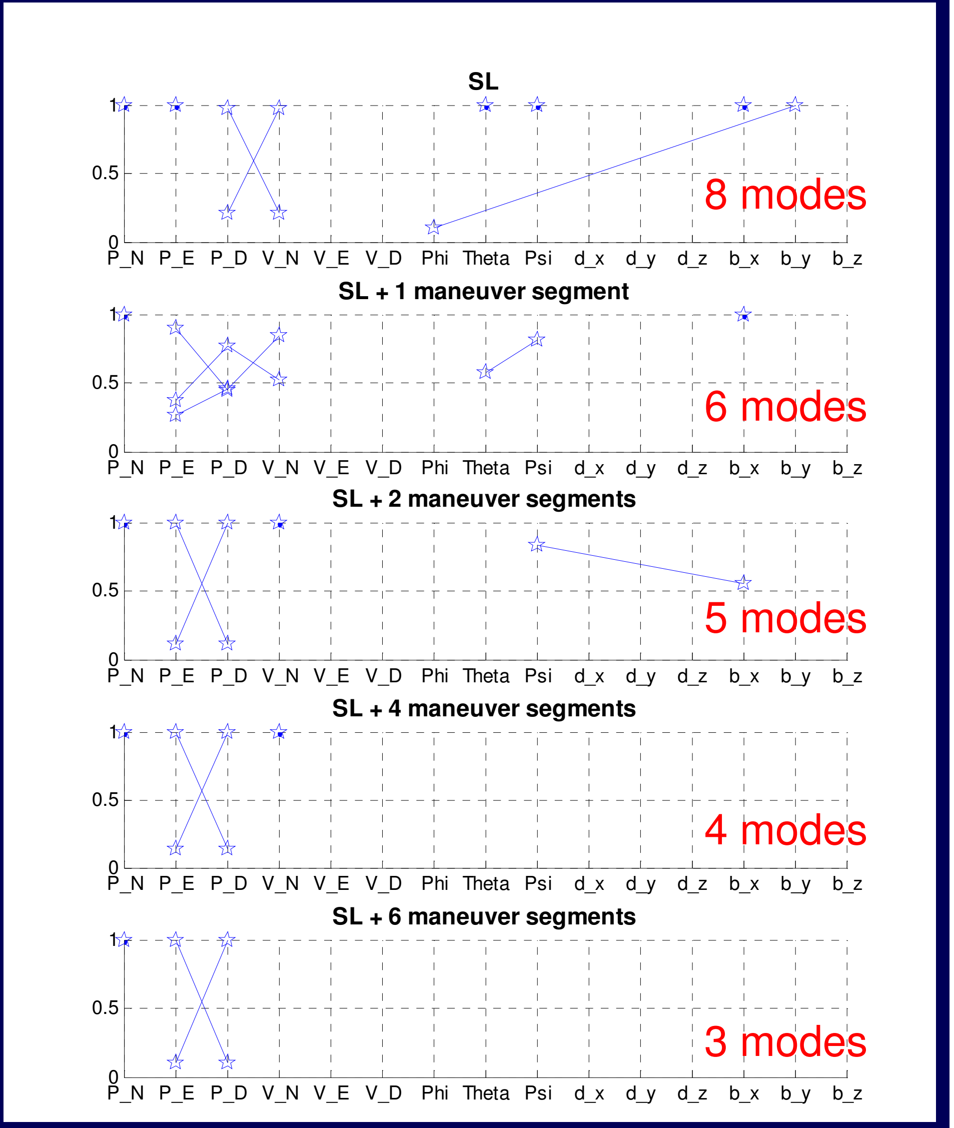
# Observability Analysis (Cont.)

- Number of observable modes – rank of $Q(r)$
- Unobservable modes components – Nullspace of the Observability Grammian

$$G = Q(r)^T Q(r)$$

- Analysis Results
  - Position terms are always unobservable
  - After several maneuver segments other states become observable
- Problematic estimation of some states in realistic scenarios

Unobservable modes



# Performance Study

Introduction

Measurements Model

Fusion with Navigation sys.

Observability Analysis

**Performance Evaluation**

Summary

## Ideal Measurements

## Two-view Aided Navigation

## Mosaic Aided Navigation

- Assumed initial navigation errors and IMU errors

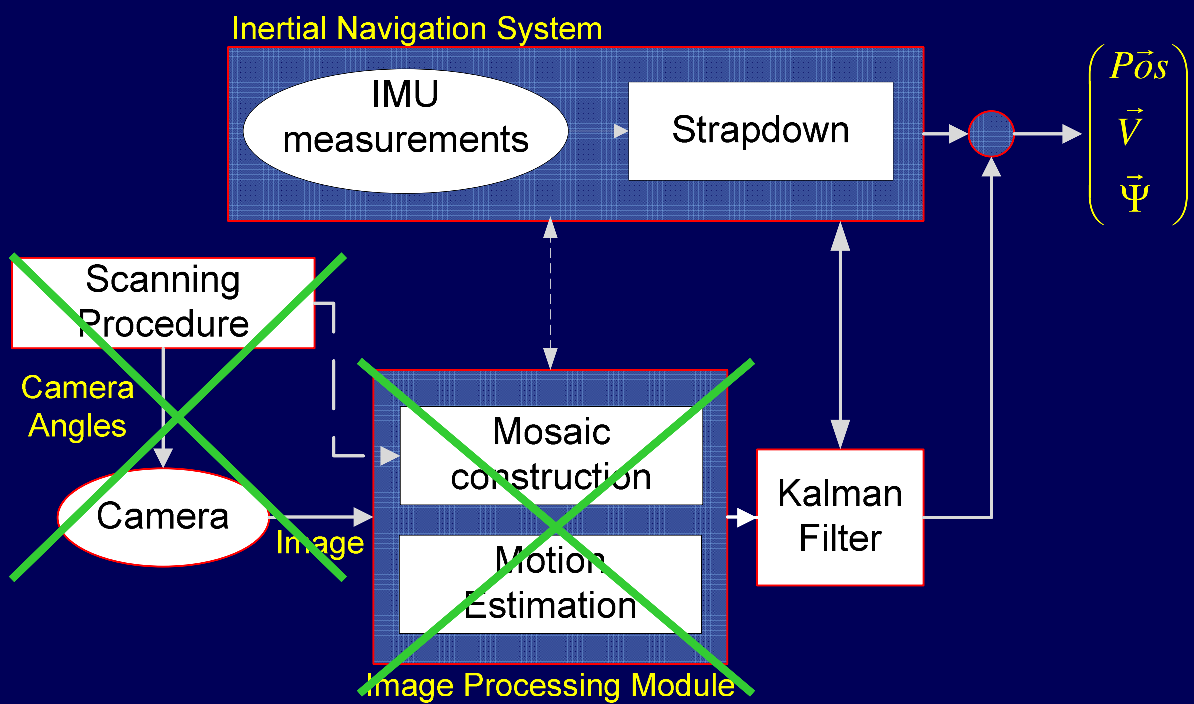
| Description | Value | Units |
|---|---|---|
| Initial **position** error $(1\sigma)$ | $(100 \quad 100 \quad 100)^T$ | m |
| Initial **velocity** error $(1\sigma)$ | $(0.3 \quad 0.3 \quad 0.3)^T$ | m/s |
| Initial **attitude** error $(1\sigma)$ | $(0.1 \quad 0.1 \quad 0.1)^T$ | deg |
| IMU **drift** $(1\sigma)$ | $(1 \quad 1 \quad 1)^T$ | deg/hr |
| IMU **bias** $(1\sigma)$ | $(1 \quad 1 \quad 1)^T$ | mg |

- Platform trajectory – Straight and level north heading flight

# Performance Study: Ideal Measurements

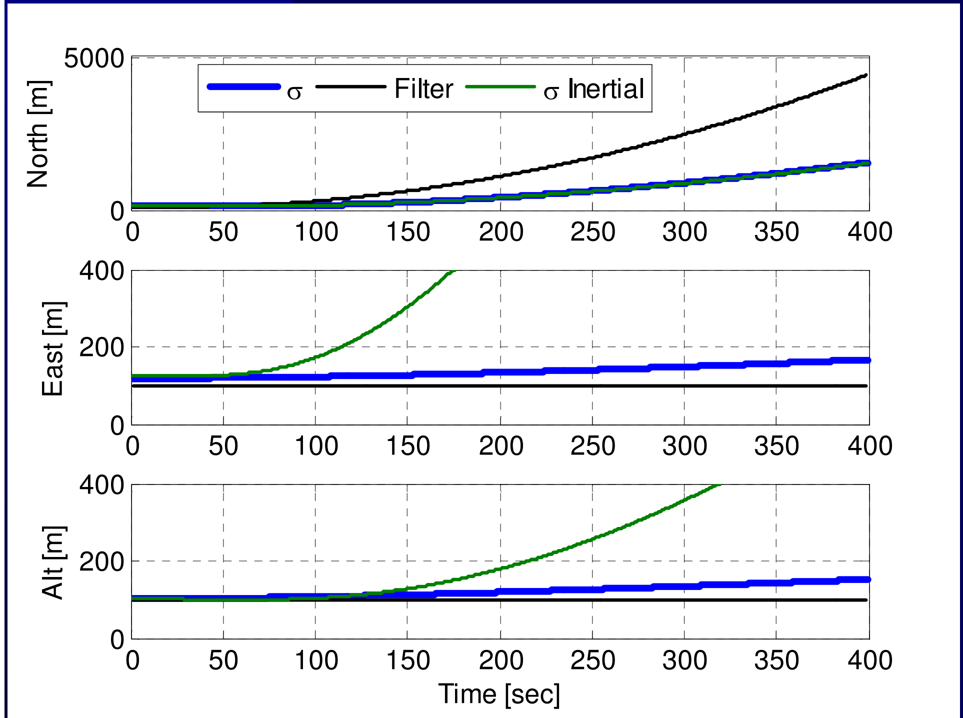


- Ideal relative motion estimations, computed based on platform true trajectory (not image-based measurements)
  - Best possible performance

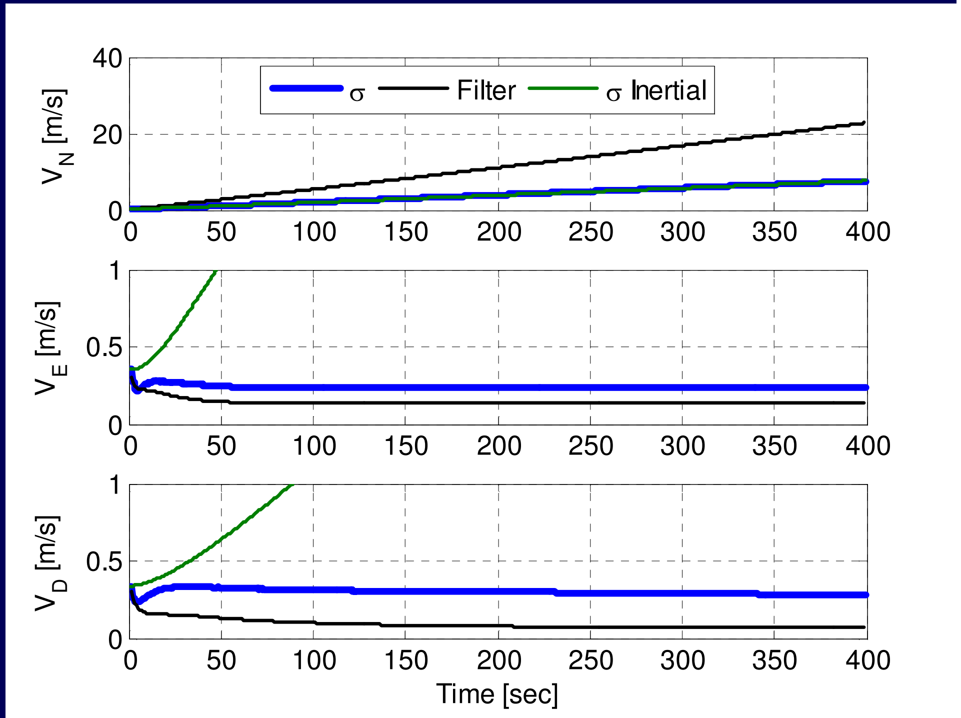


# Ideal Measurements (Cont.)

- Monte-Carlo results
  - Straight and level <u>north</u> heading flight
  - Comparison to inertial scenario



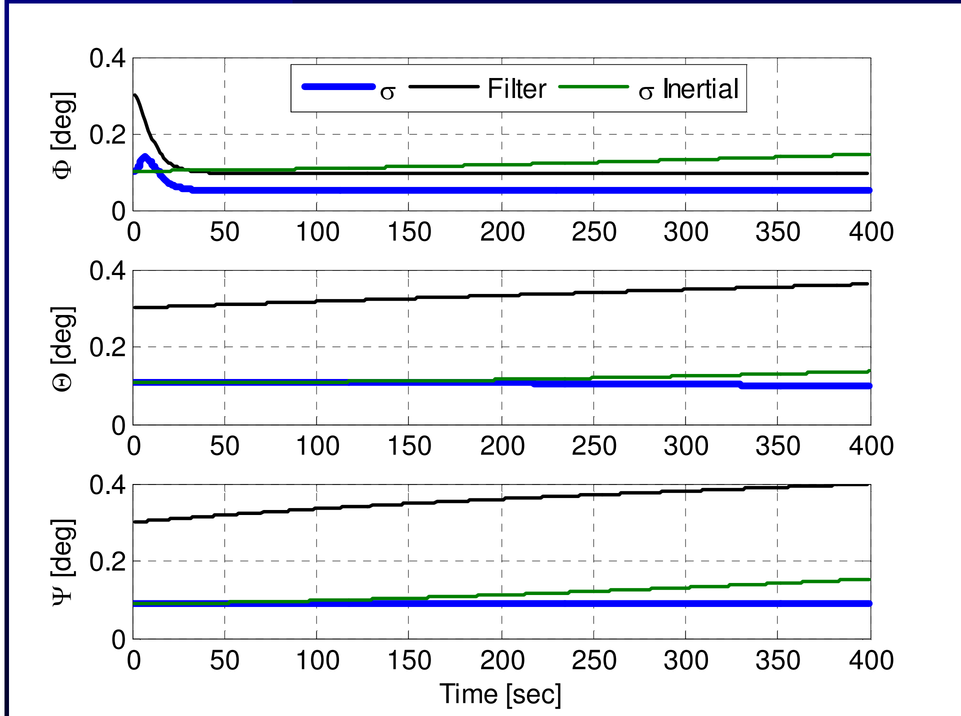

Position Errors



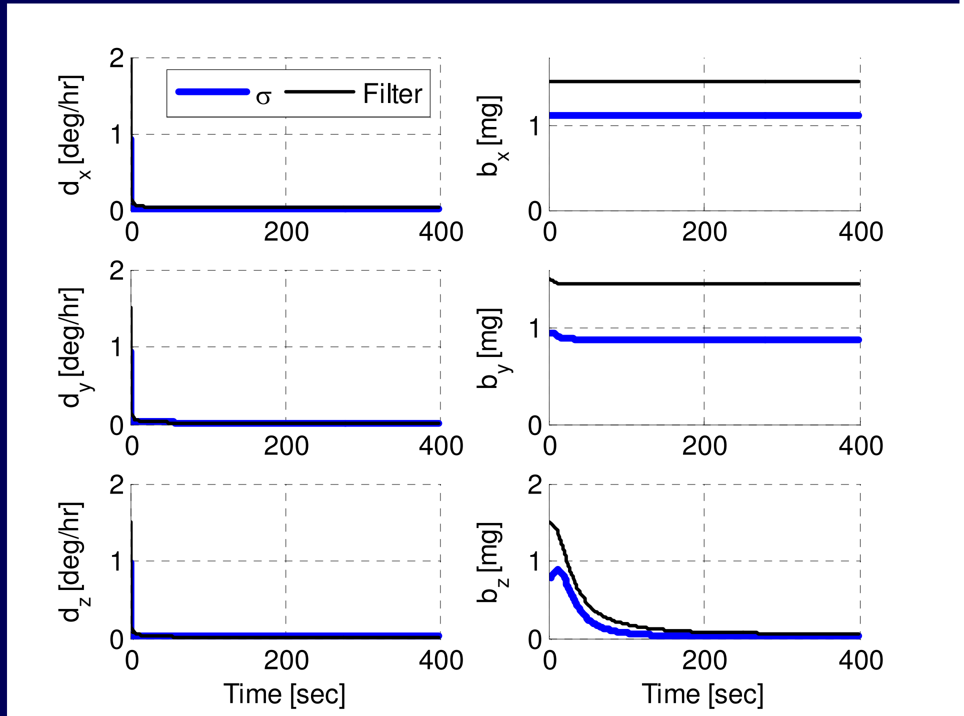

Velocity Errors

# Ideal Measurements (Cont.)

- Monte-Carlo results
  - Straight and level north heading flight
  - Comparison to inertial scenario




Euler Angles Errors




Drift and Bias Estimation Errors

# Ideal Measurements (Cont.)

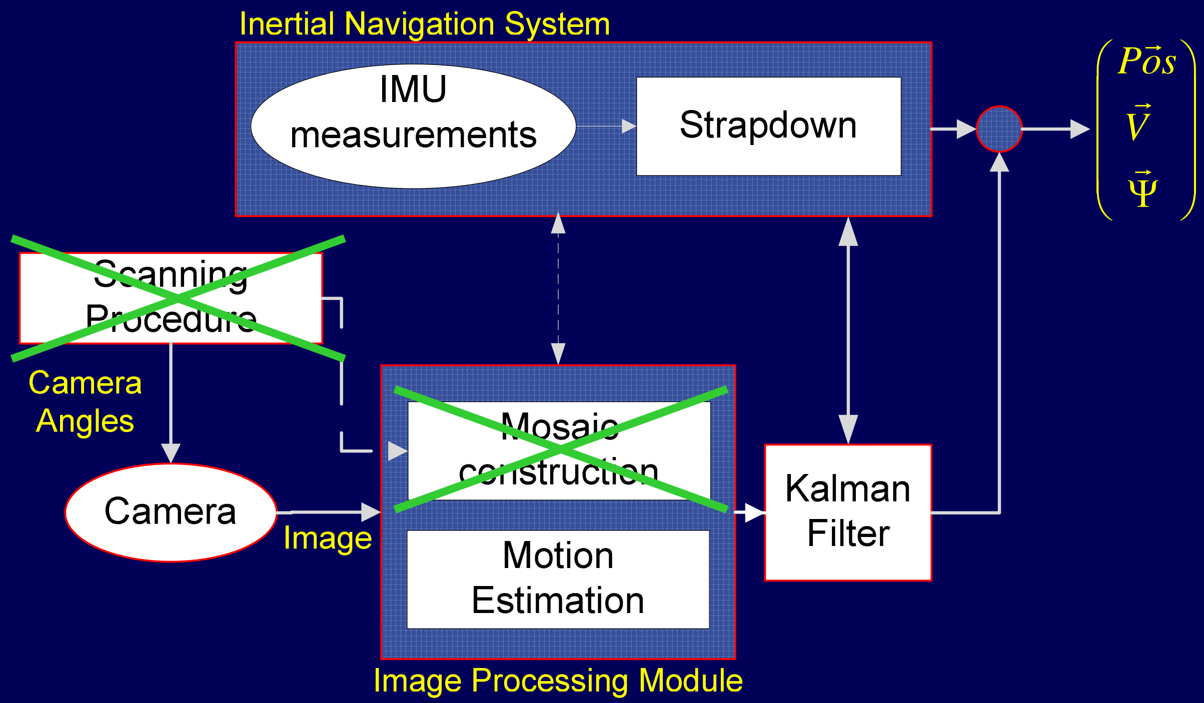


- Conclusions
  - Position and velocity errors perpendicular to the flight heading are considerably reduced and nearly nullified, respectively
  - Roll angle error estimation
  - Drift estimation in all axes
  - Bias estimation in z axis
- Increased observability while performing maneuvers
  - Pitch angle error estimation
  - Bias estimation in y axis

# Performance Study: Two-view Aided Navigation



- Motion estimation based on consecutive camera-captured images
  - The images were acquired from Google Earth
  - Without mosaic image construction

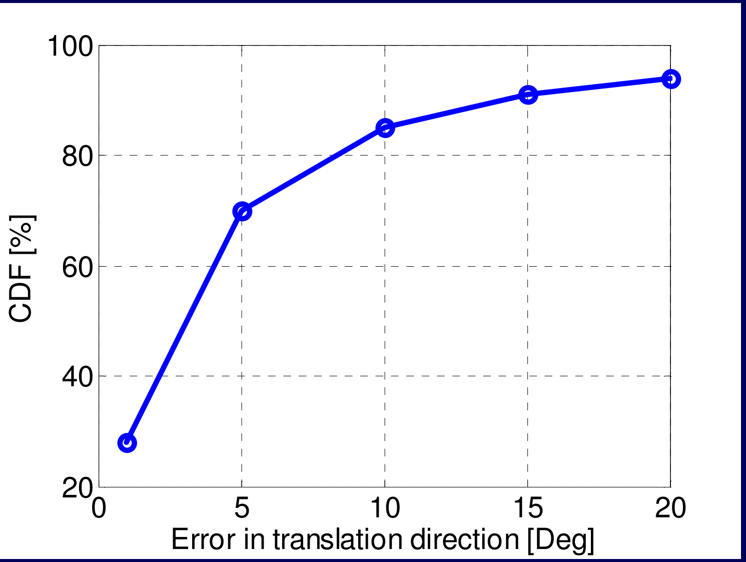


# Two-view Aided Navigation (Cont.)



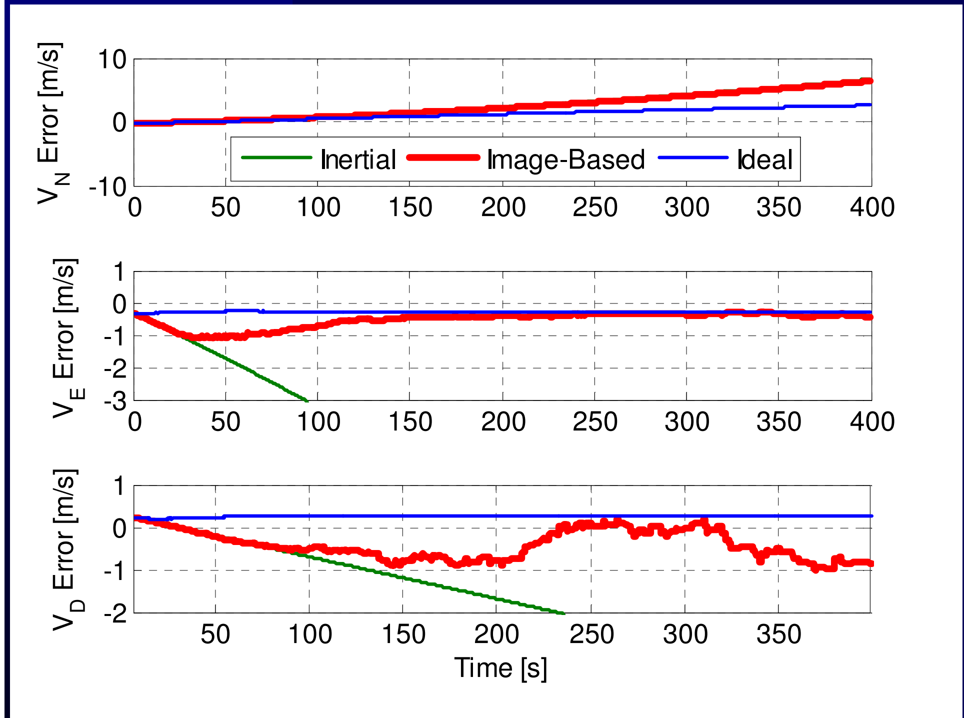
- **Wide** field-of-view camera

Cumulative Distribution Function (CDF) of translation motion estimation error

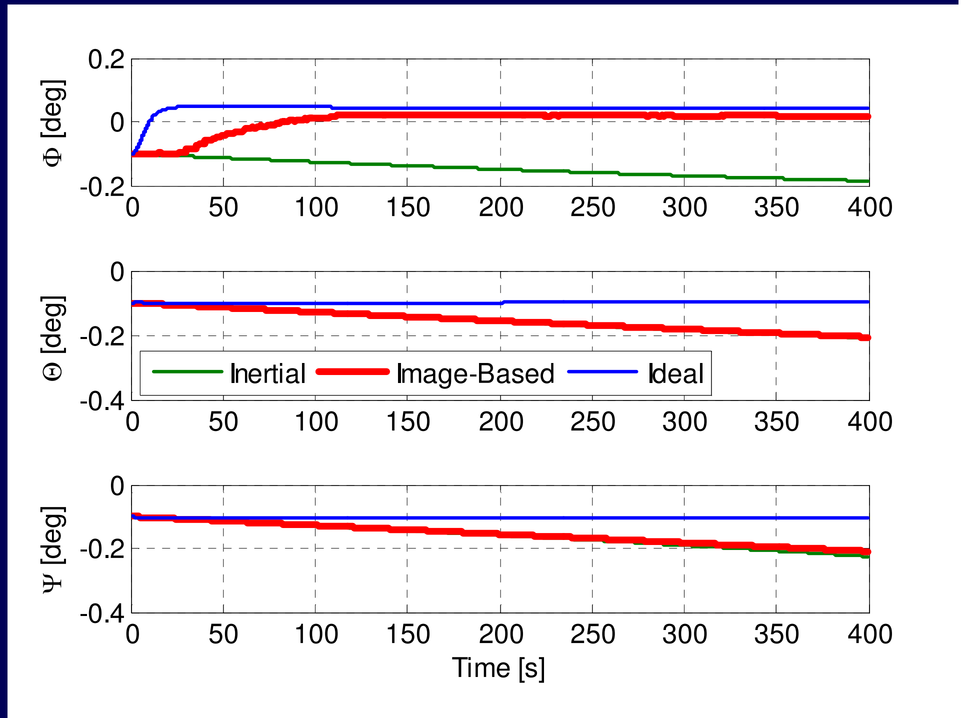


# Two-view Aided Navigation (Cont.)

- With Fictitious Velocity (FV) measurement
- Comparison to
  - Ideal relative motion measurements
  - Inertial scenario



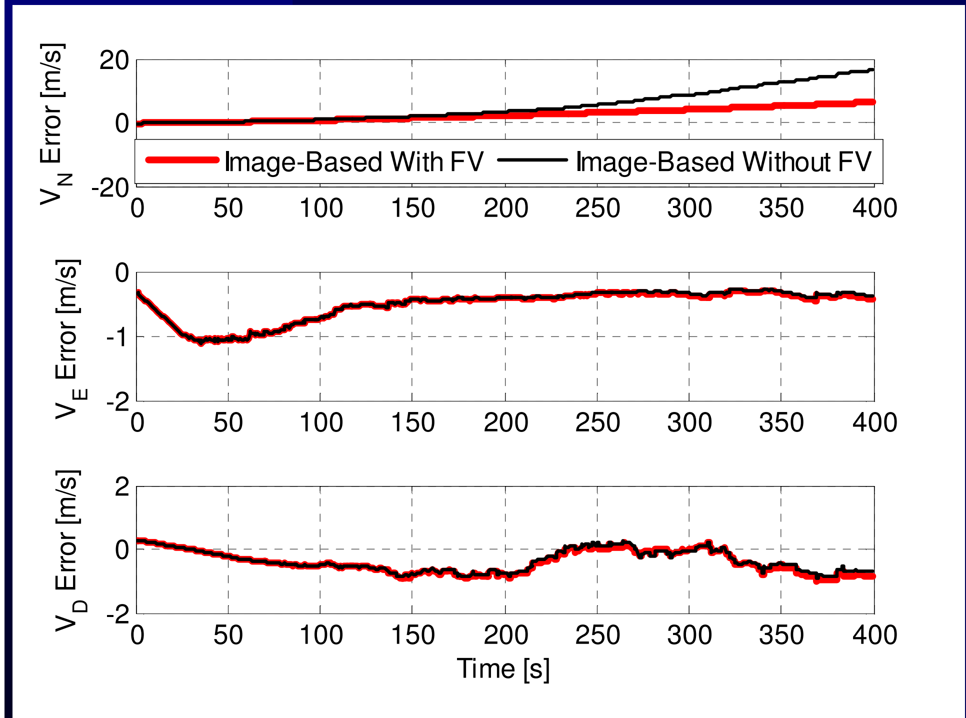

Velocity Errors



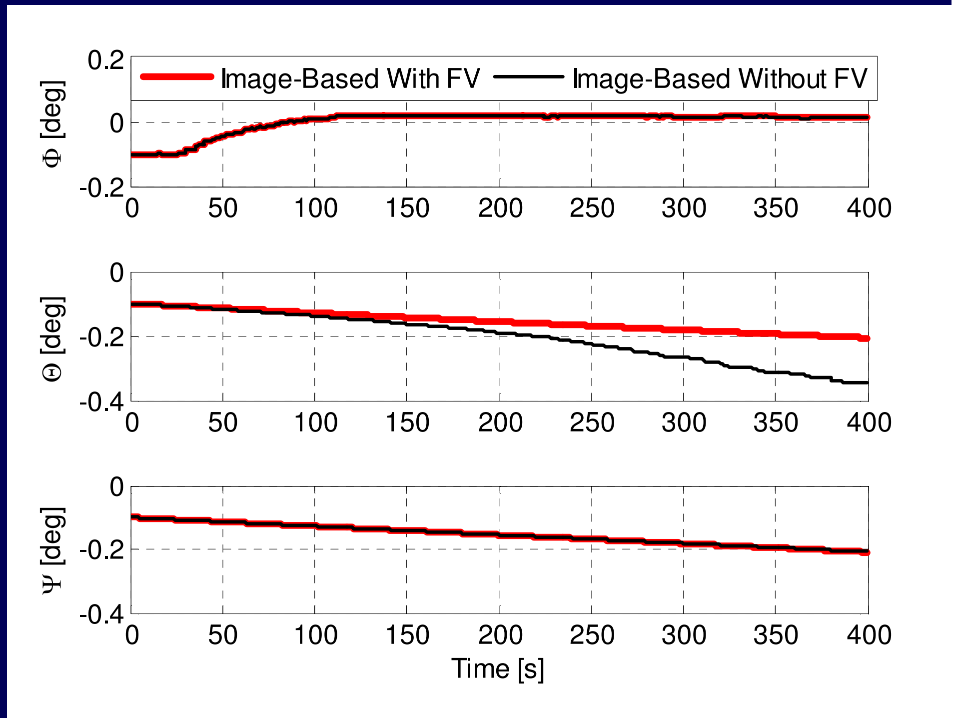

Euler Angles Errors

# Two-view Aided Navigation (Cont.)

- Fictitious Velocity (FV) measurement influence
  - Real images, with FV
  - Real images, without FV
- Drift is not estimated in all cases



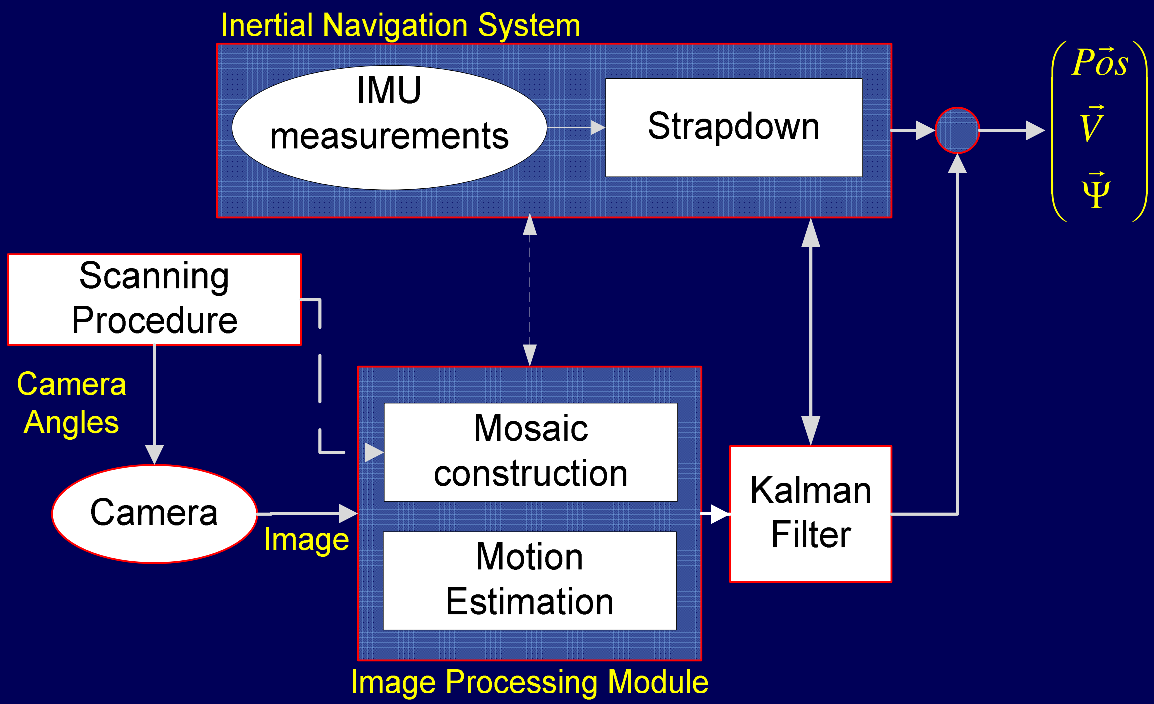

Velocity Errors




Euler Angles Errors

# Performance Study

## Mosaic Aided Navigation



# Mosaic Aided Navigation (Cont.)



- Mosaic construction based on images from camera scanning
- Motion estimation between a new captured image and a mosaic
  - Downward-Looking images only
  - Increased overlapping region

Original Overlapping Area

Additional Overlapping Area

Mosaic

New image

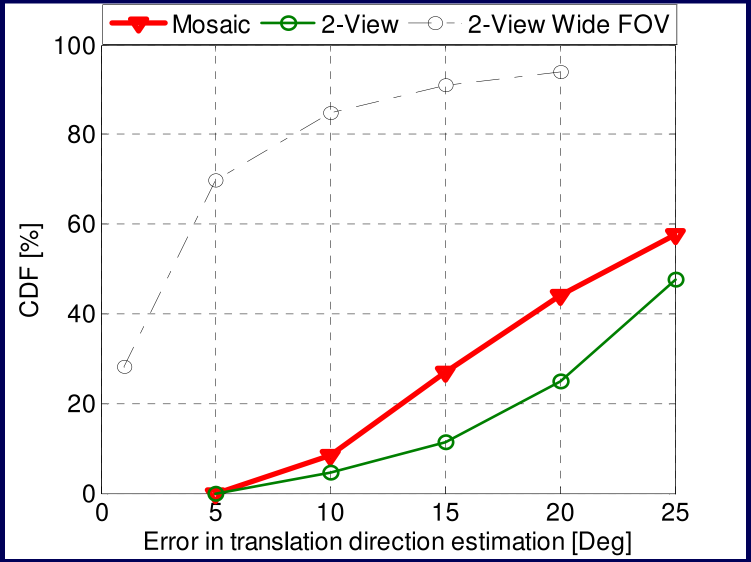
# Mosaic Aided Navigation (Cont.)



- Narrow field of view (FOV) camera: $5^0 \times 3^0$
- Low-texture type scenes
  - Example image acquired from Google Earth

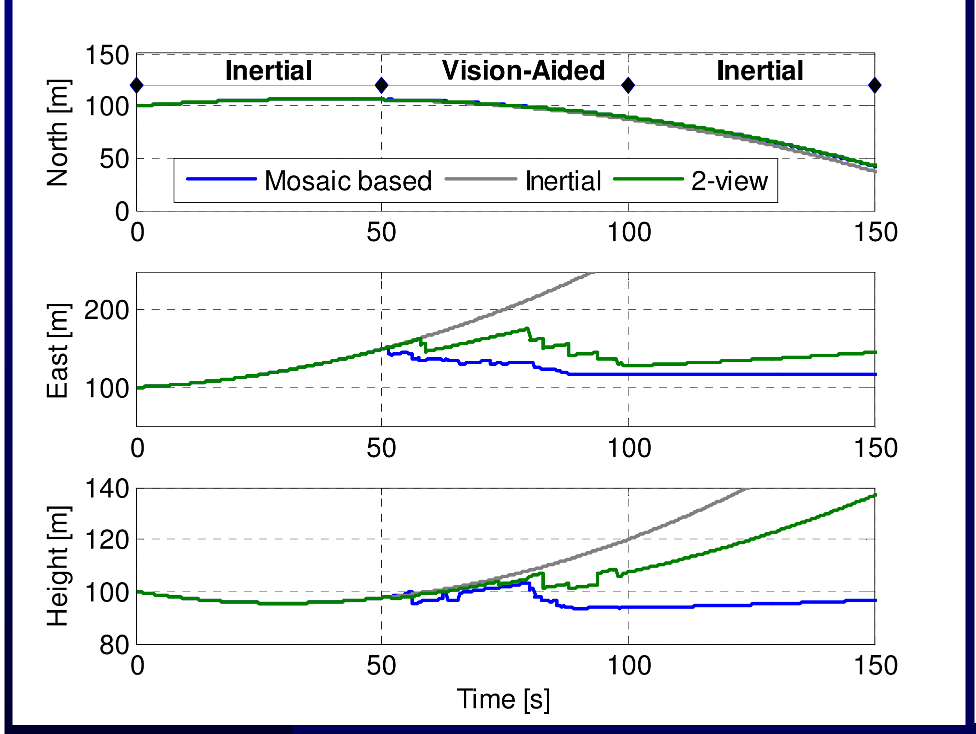
  - Superior mosaic-based motion estimation precision

Cumulative Distribution Function (CDF) of translation motion estimation error

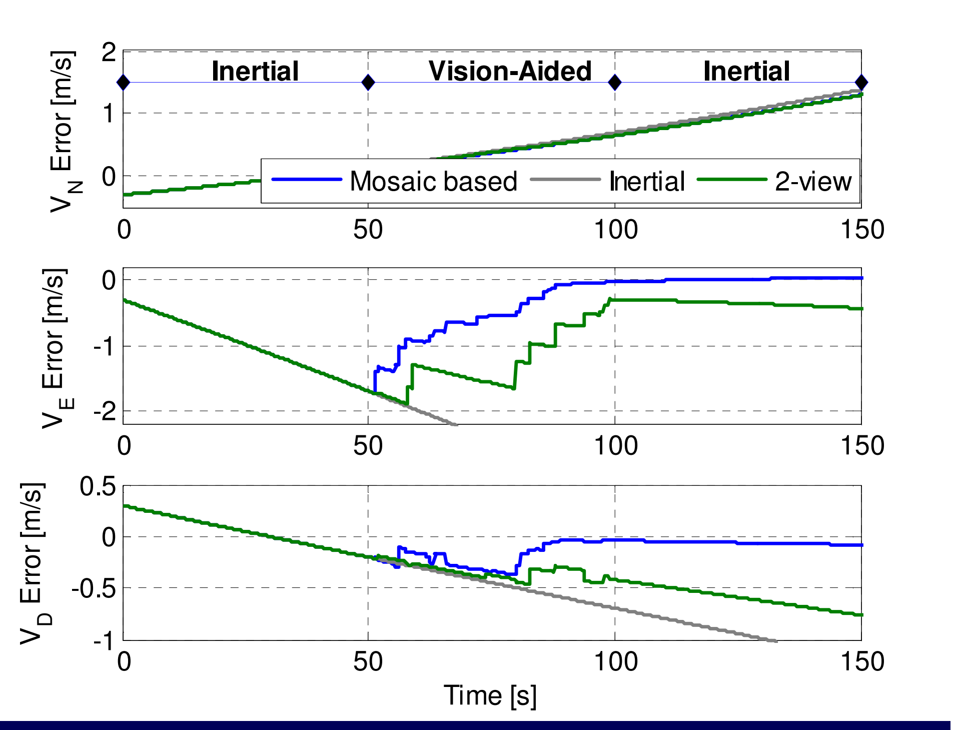


# Mosaic Aided Navigation (Cont.)

- Straight and level north heading trajectory
- Measurements fusion between $50 \leq t \leq 100$
- Inertial navigation elsewhere



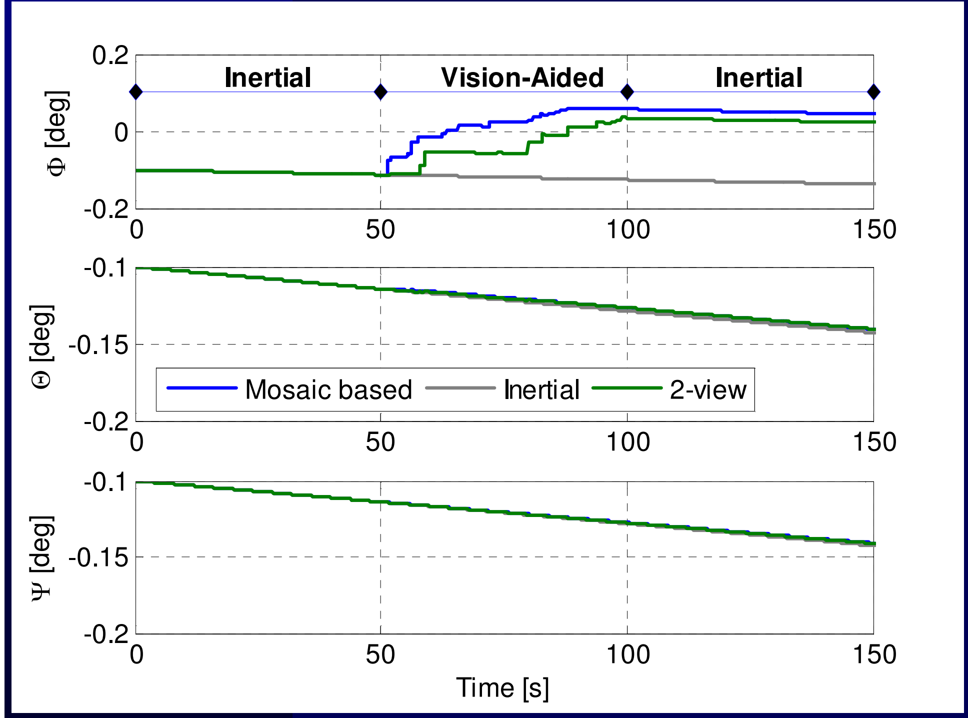

Position Errors

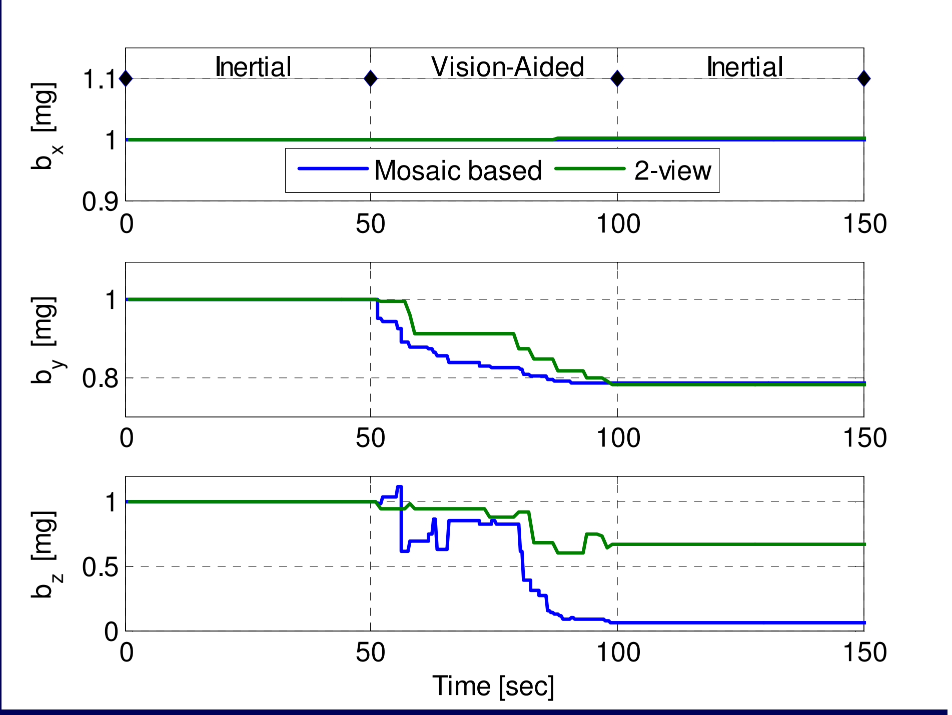



Velocity Errors

# Mosaic Aided Navigation (Cont.)

- Straight and level north heading trajectory
- Measurements fusion between $50 \leq t \leq 100$
- Inertial navigation elsewhere




Euler Angles Errors

Bias Estimation Errors

# Summary



- Mosaic-aided navigation method was presented:
  - Camera scanning
  - Mosaic construction
  - Mosaic-based motion estimation fusion with an INS
- The method does not require any a-priori information and does rely on external sensors, apart from the camera sensor
- The method may be applied also for two-view motion estimation
- Observability analysis
- Performance evaluation
  - Statistical study based on ideal motion estimations
  - Two-view aided navigation for wide FOV cameras
  - Improved performance of mosaic-aided navigation for narrow FOV cameras

Thank you …