Planning Under Uncertainty in the Continuous Domain: a Generalized Belief Space Approach

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International Conference on Robotics and Automation (ICRA), June 2014
Introduction

- Key components for autonomous operation include
  - **Perception**: Where am I? What is the surrounding environment?
  - **Planning**: What to do next?

Localization and mapping given robot motion

Planning (e.g. reach a goal)

Coupled problems

Estimation uncertainty and accuracy **varies** for different motion plans
Introduction – Motivating Example

- **Autonomous navigation** to different goals in unknown environment

Not accounting for uncertainty in planning

Accounting for uncertainty in planning
Related Work

- **Existing approaches often**
  - Assume known environment (e.g. map)
    [Prentice and Roy 2009], [Van den Berg et al. 2012], [Hollinger et al. 2013]
  - Discretize state and control space - performance depends on grid resolution
    [Stachniss et al. 2004], [Bryson and Sukkarieh 2008], [Valencia et al. 2012], [Kim and Eustice 2013]
  - Assume maximum likelihood observations
    [Miller et al. 2009], [Platt et al. 2010], [Patil et al. 2014]

- **Planning in the continuous domain - Generalized Belief Space (GBS)**
  - Probabilistic description of the robot and the environment states
  - Direct trajectory optimization approach (provides locally-optimal trajectories)
    - Environment is unknown/uncertain
    - Maximum likelihood observations assumption is avoided
    - Model the probability of acquiring a future observation (extends [Indelman et al. 2013])
Notations and Probabilistic Formulation

- **Joint state vector**
  \[ X_k \doteq \{ x_0, \ldots, x_k, L_k \} \]
  - Past & current robot states
  - Mapped environment

- **Joint probability distribution function**
  \[ p ( X_k | Z_k, U_{k-1} ) \]
  \[
p ( X_k | Z_k, U_{k-1} ) = \text{priors} \cdot \prod_{i=1}^{k} p ( x_i | x_{i-1}, u_{i-1} ) p ( z_i | X_i^o )
  \]
  - General observation model \( X_i^o \subseteq X_i \)

- **Computationally-efficient maximum a posteriori inference** e.g. [Kaess et al. 2012]
  \[ p ( X_k | Z_k, U_{k-1} ) \sim N ( X_k^*, \Sigma_k ) \]
Planning in the Generalized Belief Space

- Plan (locally) optimal control sequence over $L$ look-ahead steps: $u_{k:k+L-1}^\star$
  - By minimizing an objective function
  - Operating over the **generalized belief**
  - Model predictive control framework

**What is the generalized belief?**
- Probabilistic description of the **robot** and the **environment** states
- Generalized belief at planning time $t_k$: $gb(X_k) = p(X_k|Z_k, U_{k-1}) \sim N(X_k^\star, \Sigma_k)$

Known (from perception)
Planning in the Generalized Belief Space

- Generalized belief at the $l$-th look-ahead step

  - Describes the joint pdf (robot and environment states) at that time

  \[ gb(X_{k+l}) = p(X_{k+l}|Z_k, U_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1}) \]

  - Joint state at the $l$-th look ahead step
  - Past controls & measurements
  - Controls & measurements at the first $l$ look-ahead steps

  Planning time

  $l$-th look ahead step
Planning in the Generalized Belief Space

- Generalized belief at the $l$-th look-ahead step
  - Describes the joint pdf (robot and environment states) at that time
  
  \[
  gb(X_{k+l}) = p(X_{k+l} \mid Z_k, U_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1})
  \]

- Objective function can now involve uncertainty (e.g. covariance) in robot and environment states
  
  \[
  J_k(u_{k:k+L-1}) = \mathbb{E}_{Z_{k+1:k+L}} \left\{ \sum_{l=0}^{L-1} c_l (gb(X_{k+l}, u_{k+l}) + c_L (gb(X_{k+L})) \right\}
  \]

- For example, plan motion to minimize uncertainty in robot state
Generalized Belief Space (Cont.)

\[ gb \left( X_{k+l} \right) = p \left( X_{k+l} \mid Z_k, U_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1} \right) \]

- Modeling future observations \( Z_{k+1:k+l} \):
  - Treated as random variables [Van den Berg et al. 2012]
  - Will a future observation be actually acquired?
  - Model probabilistically acquisition of \( Z_{k+1:k+l} \) by random binary variables \( \Gamma_{k+1:k+l} \)

- Marginalize out \( \Gamma_{k+1:k+l} \) to get \( gb \left( X_{k+l} \right) \)

\[ gb \left( X_{k+l} \right) = \sum_{\Gamma_{k+1:k+l}} p \left( X_{k+l}, \Gamma_{k+1:k+l} \mid Z_k, U_0, \ldots, U_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1} \right) \]

- Expensive! Instead – Expectation Maximization (details soon)
Planning in the Generalized Belief Space

\[ J_k (u_{k:k+L-1}) \equiv \mathbb{E}_{Z_{k+1:k+L}} \left\{ \sum_{l=0}^{L-1} c_l (gb (X_{k+l}), u_{k+l}) + c_L (gb (X_{k+L})) \right\} \]

- How to calculate locally-optimal control?

\[ u^*_{k:k+L-1} = \arg \min_{u_{k:k+L-1}} J_k (u_{k:k+L-1}) \]

**Dual-layer iterative optimization**

**Outer layer** - Inference over \( u_{k:k+L-1} \)

Starting from initial guess \( u_{k:k+L-1}^{(0)} \)

At each iteration:
- Compute \( \Delta u_{k:k+L-1} \)
- Update control

\[ u_{k:k+L-1}^{(i+1)} = u_{k:k+L-1}^{(i)} + \Delta u_{k:k+L-1} \]

**Inner layer** - Inference over the belief

For each look-ahead step, for a given control

\[ gb(X_{k+l}) \sim N (X_{k+l}^*, \Sigma_{k+l}) \]

- As a function of random variables \( Z_{k+1:k+l} \)
- EM formulation to avoid marginalizing over the latent variables \( \Gamma_{k+1:k+l} \)
**Outer Layer:** Inference over the Control

Iterative optimization over the nonlinear objective function $J_k (u_{k:k+L-1})$

\[ J_k (u_{k:k+L-1}) = \mathbb{E} \left\{ \sum_{l=0}^{L-1} c_l \left( gb \left( X_{k+l} \right) , u_{k+l} \right) + c_L \left( gb \left( X_{k+L} \right) \right) \right\} \]

- Involves:
  - Calculating gradient $\nabla J_k$
  - Evaluating objective function $J_k$ for different control values

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**Outer layer** - Inference over $u_{k:k+L-1}$

Starting from initial guess $u_{k:k+L-1}^{(0)}$

At each iteration:

- Compute $\Delta u_{k:k+L-1}$
- Update control

\[ u_{k:k+L-1}^{(i+1)} = u_{k:k+L-1}^{(i)} + \Delta u_{k:k+L-1} \]

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**Inner layer** - Inference over the belief

For each look-ahead step, for a given control $gb(X_{k+l}) \sim N \left( X_{k+l}^*, \Sigma_{k+l} \right)$

- As a function of random variables $Z_{k+1:k+l}$
- EM formulation to avoid marginalizing over the latent variables $\Gamma_{k+1:k+l}$
**Inner Layer: Inference Over the Belief**

Given **current** controls $u_{k:k+L-1}$, for each look ahead step $l$:

- Compute the Gaussian approximation $X_{k+l}^*, \Sigma_{k+l}$ such that

  $$
  gb(X_{k+l}) \sim N \left( X_{k+l}^*, \Sigma_{k+l} \right)
  $$

- **EM formulation:**

  $$
  X_{k+l}^* = \arg \min_{X_{k+l}} \mathbb{E}_{\Gamma_{k+1:k+l} | X_{k+l}} \left[ -\log p \left( X_{k+l}, \Gamma_{k+1:k+l} | Z_k, U_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1} \right) \right]
  $$

- **Gauss Newton method**

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**Outer layer** - Inference over $u_{k:k+L-1}$

Starting from initial guess $u_{k:k+L-1}^{(0)}$

At each iteration:

- Compute $\Delta u_{k:k+L-1}$
- Update control

\[ u_{k:k+L-1}^{(i+1)} = u_{k:k+L-1}^{(i)} + \Delta u_{k:k+L-1} \]

**Inner layer** - Inference over the **belief**

For each look-ahead step, for a **given** control

$$
gb(X_{k+l}) \sim N \left( X_{k+l}^*, \Sigma_{k+l} \right)
$$

- As a function of random variables $Z_{k+1:k+l}$
- EM formulation to avoid marginalizing over the latent variables $\Gamma_{k+1:k+l}$
Inner Layer: Inference Over the Belief

Given current controls $u_{k:k+L-1}$, for each look ahead step $l$:

- Compute the Gaussian approximation $X^*_{k+l}, \Sigma_{k+l}$ such that

$$gb(X_{k+l}) \sim N(X^*_{k+l}, \Sigma_{k+l})$$

- EM formulation:

$$X^*_{k+l} = \arg\min_{X_{k+l}} \mathbb{E}_{\Gamma_{k+1:k+l}|\bar{X}_{k+l}} \left[-\log p(X_{k+l}, \Gamma_{k+1:k+l}| Z_k, U_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1})\right]$$

- Gauss Newton method

- Next - we show the above formulation:
  - Guides the robot towards informative distant 3D points (outside sensing range)
    - Loop closures to reduce uncertainty
  - Alternative formulation using signed distance function [Patil et al. 2014]
Limited Sensing Range

- Illustrative (toy) example

- **Without** modeling probability of acquiring future observations:
  - 3D points outside sensing range contribute **zero** gradient to $\nabla J_k$
  - Robot will **not be guided** to re-observe these points (i.e. no loop closures)
Back to Inner Layer

\[ X_{k+l}^* = \arg \min_{X_{k+l}} \mathbb{E}_{\Gamma_{k+1:k+l} \mid \bar{X}_{k+l}} \left[-\log p(X_{k+l}, \Gamma_{k+1:k+l} \mid Z_k, U_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1})\right] \]

- Joint pdf:
  \[ p(X_k \mid Z_k, U_{k-1}) \prod_{i=1}^l p(x_{k+i} \mid x_{k+i-1}, u_{k+i-1}) p(Z_{k+i}, \Gamma_{k+i} \mid X_{k+i}^o) \]

- Inference over the belief:

\[ X_{k+l}^* = \arg \min_{X_{k+l}} \|X_k - X_k^*\|_I_k^2 + \sum_{i=1}^l \|x_{k+i} - f(x_{k+i-1}, u_{k+i-1})\|_{\Omega_w}^2 \]

\[ + \sum_{i=1}^l \sum_{j=1}^{n_i} p(\gamma_{k+i,j} = 1 \mid \bar{X}_{k+l}) \|z_{k+i,j} - h(X_{k+i,j}^o)\|_{\Omega_{w,j}}^2 \]

Probability of observing the j-th 3D point
\[ X_{k+l}^* = \arg \min_{X_{k+l}} \mathbb{E}_{\Gamma_{k+1:k+l} | \bar{X}_{k+l}} \left[ -\log p (X_{k+l}, \Gamma_{k+1:k+l} | Z_k, U_{k-1}, Z_{k+1:k+l}, u_{k:k+l-1}) \right] \]

- Joint pdf:
\[
p(X_k | Z_k, U_{k-1}) \prod_{i=1}^{l} p(x_{k+i} | x_{k+i-1}, u_{k+i-1}) p (Z_{k+i}, \Gamma_{k+i} | X_{k+i}^o)\]

- Inference over the belief:
\[
X_{k+l}^* = \arg \min_{X_{k+l}} \| X_k - X_k^* \|^2_k + \sum_{i=1}^{l} \| x_{k+i} - f (x_{k+i-1}, u_{k+i-1}) \|^2_{\Omega_w} \\
+ \sum_{i=1}^{l} \sum_{j=1}^{n_i} p (\gamma_{k+i,j} = 1 | \bar{X}_{k+l}) \| z_{k+i,j} - h (X_{k+i,j}^o) \|^2_{\Omega^i,j_v} 
\]
- Equivalent to weighting the measurement covariance matrix
\[
\tilde{\Omega}^i,j_v = p (\gamma_{k+i,j} = 1 | \bar{X}_{k+l}) \Omega^i,j_v \\
\| z_{k+i,j} - h (X_{k+i,j}^o) \|^2_{\Omega^i,j_v} 
\]

- E.g., probability to observe 3D points decreases with distance
- Contributes to \(\nabla J_k\), becomes dominant if information gain is substantial
Results

- **Autonomous navigation** to different goals in an unknown environment
  - **Objective function**: penalize control usage, uncertainty and distance to goal
  - No absolute information
  - Onboard sensors: camera and range sensor
  - Control: heading angle
Results

- **Autonomous navigation** to different goals in an unknown environment
  - **Objective function**: penalize control usage, uncertainty and distance to goal
  - Compared methods:
    - Planning in GBS
    - Planning in GBS, no uncertainty
    - Discrete planning - A*, adaptation of [Kim and Eustice 2013]

[Kim and Eustice 2013]

Discrete Planning (adaptation of [Kim and Eustice 2013])
Results

- **Autonomous navigation** to different goals in an unknown environment
  - **Objective function**: penalize control usage, uncertainty and distance to goal
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Results

- **Autonomous navigation** to different goals in an unknown environment
  - **Objective function**: penalize control usage, uncertainty and distance to goal
  - Compared methods:
    - Planning in GBS
    - Planning in GBS, no uncertainty
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Conclusions

- **Planning in the continuous domain - Generalized Belief Space**
  - General framework for planning under uncertainty (incl. uncertainty in environment)
  - Addresses 3 limitations of state of the art:
    
    | Discretization of state or control space | ✓ |
    | Maximum likelihood observations assumption | ✓ |
    | Exact prior knowledge regarding environment | ✓ |

- Limited sensing range
  - Latent variables to model acquisition probability of future observations
  - Allow planning loop closures outside sensing range
- Produces smooth trajectories with reduced control effort
Extras
Discrete Planning
Comparison Between Different Methods

![Graphs showing comparison between different methods.]