Distributed Robust Localization from Arbitrary Initial Poses via EM and Model Selection

Vadim Indelman

Collaborators: Erik Nelson, Jing Dong, Nathan Michael and Frank Dellaert
Collaborative Localization and Mapping

- Important in a variety of scenarios
  - Exploration in unknown/uncertain, dangerous environments
  - Search and rescue
  - Surveillance, tracking …

- Cooperative inference requires
  - Sharing relevant information (observations, marginals over variables of interest)
  - Correct interpretation (data association)
  - Robustness to outliers
Motivating Scenario

- Robots are initially *unaware* of each others’ location
- How to establish collaboration and perform multi-robot localization?
  - *Unknown* multi-robot data association
  - *Unknown* initial relative poses between robots

* Slide adapted from Nelson14iser
Related Work

- **Known** data association and common reference frames
  - Full SLAM [Howard et al. 2006], [Andersson et al. 2008]
  - Pose SLAM (direct, indirect) [Roumeliotis et al. 2002], [Kim et al. 2010], [Indelman et al. 2012]
Related Work

- **Known** data association and common reference frames
  - Full SLAM [Howard et al. 2006], [Andersson et al. 2008]
  - Pose SLAM (direct, indirect) [Roumeliotis et al. 2002], [Kim et al. 2010], [Indelman et al. 2012]

- **Unknown** multi-robot data association and common reference frame
  - Full SLAM [Montijano et al. 2011], [Cunningham et al. 2012]
Related Work

- **Known** data association and common reference frames
  - Full SLAM [Howard et al. 2006], [Andersson et al. 2008]
  - Pose SLAM (direct, indirect) [Roumeliotis et al. 2002], [Kim et al. 2010], [Indelman et al. 2012]

- **Unknown** multi-robot data association and common reference frame
  - Full SLAM [Montijano et al. 2011], [Cunningham et al. 2012]

- **Robust** graph optimization ([single robot case – loop closures](#))
  - [Sunderhauf and Protzel 2012, 2013], [Latif et al. 2012], [Lee et al. 2013]

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![Image 1](Lee et al. 2013)

![Image 2](Latif et al. 2012)

![Image 3](Sunderhauf and Protzel 2012, 2013)
This Work

- Multi-robot framework with
  - Unknown multi-robot data association
  - Unknown initial relative poses between robots
  - Pose SLAM approach

- How to establish multi-robot data association when robots start operating from unknown locations?

- Outline:
  - Batch, centralized framework
  - Incremental, distributed framework
Multi-Robot Correspondences

- If **no** common reference frame is available, what information to share?
  - Robots share **informative** observations (e.g. laser scans)

Saliency is calculated based on [Nieto et al., Robot. Auton. Syst., 2007]
Multi-Robot Correspondences

- If **no** common reference frame is available, what information to share?
  - Robots share **informative** observations (e.g. laser scans)
  - Calculate **candidate** multi-robot relative pose constraints
    - Collect into set $\mathcal{F}$
    - Includes (many) outliers

*Arbitrary common reference frame*

*Ground truth*
Probabilistic Formulation

- **Notations:**
  - $\mathcal{F}$: Multi-robot correspondences set
  - $\mathcal{J}$: Latent variables to indicate inliers/outliers

- Joint pdf over robot trajectories **and** multi-robot data association:

\[
p(X, J|Z) \propto \prod_r p(X^r|Z^r) \prod_{(r_1, r_2, k, l) \in \mathcal{F}} p(j_{k,l}^{r_1, r_2}) p(u_{k,l}^{r_1, r_2}|x_{k}^{r_1}, x_{l}^{r_2}, j_{k,l}^{r_1, r_2})
\]

- Only local measurements
- Data association
- Multi-robot measurement likelihood, given data association
- Each multi-robot correspondence
Measurement likelihood

\[ p(u_{k,l}^{r_1,r_2} | x_k^{r_1}, x_l^{r_2}) \propto \exp \left( -\frac{1}{2} \left\| \text{err} \left( u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2} \right) \right\|_\Sigma^2 \right) \]

with

\[ \text{err} \left( u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2} \right) = \sum_{k,l} f \left( x_k^{r_1}, x_l^{r_2} \right) \]

measured \hspace{1cm} predicted

\[ \begin{align*}
\hat{p} & = x_k^{r_1} \oplus (T_{r_2}^{r_1} \oplus x_l^{r_2}) \\
\text{Unknown!!}
\end{align*} \]
Measurement likelihood

\[ p(u_{k,l}^{r_1,r_2} | x_{k}^{r_1}, x_{l}^{r_2}) \propto \exp \left( -\frac{1}{2} \| err \left( u_{k,l}^{r_1,r_2}, x_{k}^{r_1}, x_{l}^{r_2} \right) \|_2^2 \right) \]

with

\[ err \left( u_{k,l}^{r_1,r_2}, x_{k}^{r_1}, x_{l}^{r_2} \right) \triangleq u_{k,l}^{r_1,r_2} \ominus h(x_{k}^{r_1}, x_{l}^{r_2}) \]

measured predicted

\[ \triangleq x_{k}^{r_1} \ominus (T_{r_2}^{r_1} \oplus x_{l}^{r_2}) \]

Error distribution for all correspondences:

Must **first** infer a common reference frame \( T_{r_2}^{r_1} \)!

Inliers
Outliers

\[ 1\sigma \text{ uncertainty} \]

\[ X [m] \]

\[ Y [m] \]

\[ X [m] \]

\[ Y [m] \]
Key Observation

- Given robot local trajectories, relative initial pose can be calculated from each candidate multi-robot correspondence
  - **Only** inliers produce similar transformations
  - Objective: identify cluster

Initial relative pose between two robots (planar case: $x, y, \theta$) [synthetic data]

10% outliers

40% outliers

85% outliers
Key Observation

- Given robot local trajectories, relative initial pose can be calculated from each candidate multi-robot correspondence
  - **Only** inliers produce similar transformations
  - Objective: identify cluster

Initial relative pose between two robots (planar case: $x, y, \theta$)  
[real data]
Inference Over Common Reference Frame via EM

- MAP estimate of $T^r_{r_2}$ given robot local trajectories ($\hat{X}^{SR}$):

$$\hat{T}^r_{r_2} = \arg \max_{T^r_{r_2}} p \left( T^r_{r_2} | \hat{X}^{SR}, Z \right) = \arg \max_{T^r_{r_2}} \sum_{J} p \left( T^r_{r_2}, J | \hat{X}^{SR}, Z \right)$$

- $J$: Latent binary variables to indicate inliers/outliers

- EM formulation ($T \doteq T^r_{r_2}$):

$$\hat{T}^{(i)} = \arg \max_{T} \sum_{J} p \left( J | \hat{T}^{(i-1)}, \hat{X}^{SR}, Z \right) \log \left[ p \left( T, J | \hat{X}^{SR}, Z \right) \right]$$

Local trajectories

$$\hat{X}^r = \arg \max_{X^r} p \left( X^r | Z^r \right)$$

$$\hat{X}^{SR} \doteq \left\{ \hat{X}^r \right\}_{r=1}^R$$

E step M step
Convergence only to **local** minima

Therefore:
- Start process from several initial guesses of $T_{r_2}^{r_1}$
- Results in several locally-optimal **hypotheses** (inliers/outliers, estimated $T_{r_2}^{r_1}$)
- Which one to choose? (next)
Inference Over Robot Trajectories

- Once a common reference frame is established:
  - Multi-robot localization becomes possible
  - Robot trajectories can be expressed in the same frame

- Infer robot trajectories via EM:

\[
\hat{X} = \arg \max_X \sum_J p(J|\hat{X}, Z) \log p(X, J|Z)
\]

- Identified common reference frame is used as \textit{initial guess} within measurement likelihood

\[
p(u_{r1}^{k,l}, u_{r2}^{l}, x_{r1}^{k}, x_{r2}^{l}) \propto \exp \left( -\frac{1}{2} \left\| \text{err} \left( u_{r1}^{k,l}, x_{r1}^{k}, x_{r2}^{l} \right) \right\|_{\Sigma}^2 \right)
\]
Results (Batch, Centralized)

Local trajectories; Arbitrary common reference frame

Indoor navigation

Estimated

Ground truth

Inliers
Outliers

Robot 1
Robot 2
Robot 3

[Diagram showing local trajectories for different robots and an indoor navigation scenario with estimated and ground truth trajectories]
Incremental Framework

- Challenges
  - Multiple hypotheses
  - How to know when to make a decision?
    - Robot trajectories and observed environments may initially not overlap
  - Perceptual aliasing

Candidate correspondences:

Arbitrary common reference frame

Ground truth trajectory
Incremental Framework (Cont.)

- Choosing an incorrect hypothesis:
Incremental Framework (Cont.)

- Approach
  - Hypothesis model-based selection
  - Chinese restaurant process hypothesis prior
Hypothesis Model-Based Selection

- Calculate probability of each hypothesis $h \in \mathcal{H}$

\[ p\left(h|Z, \hat{X}^{SR}\right) \]

- Explicitly:

\[ p\left(h|Z, \hat{X}^{SR}\right) \]

- Measurement likelihood
  - Prioritizes hypotheses
  - Does **not** address:
    - Is sufficient data available to choose a hypothesis?
    - Perceptual aliasing
Hypothesis Prior

- Introduce **null-hypothesis** – corresponds to perceptual aliasing
  - All correspondences are actually outliers

- Chinese restaurant process, assuming:
  - Robots operate in closed indoor environment
  - Eventually, will observe common places (not necessarily concurrently)
Hypothesis Prior (Cont.)

- **Chinese restaurant process**
  - Probability of observing a new place reduces over time
  - Use to discriminate between different hypotheses
    - As more data comes in – hypotheses priors become distinguishable

- **Example**
Results

As opposed to
Results - Experiments @ CMU

Trial T1

Trial T2

Trial T3

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<td>Error $|x,y|$ (m)</td>
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<td>0.15</td>
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<td>Error $\theta$ (rad)</td>
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<td>0.00</td>
<td>0.11</td>
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Distributed Real-time Cooperative Localization and Mapping using an Uncertainty-Aware Expectation Maximization Approach

Jing Dong, Erik Nelson, Vadim Indelman, Nathan Michael, Frank Dellaert

Georgia Institute for Robotics and Intelligent Machines
Carnegie Mellon University
Technion Israel Institute of Technology
Conclusions

- **Collaborative inference from unknown initial poses and data association**
  - Key observation (clusters for inlier correspondences)
  - EM approach to infer common reference frames and data association
  - Once established, joint inference over robot poses

- **Distributed and incremental framework:**
  - Challenges: How to know when to make a decision? Perceptual aliasing
  - Model-based selection + hypothesis prior