Distributed Robust Localization from Arbitrary Initial Poses via EM and Model Selection

Vadim Indelman

Collaborators: Erik Nelson, Jing Dong, Nathan Michael and Frank Dellaert

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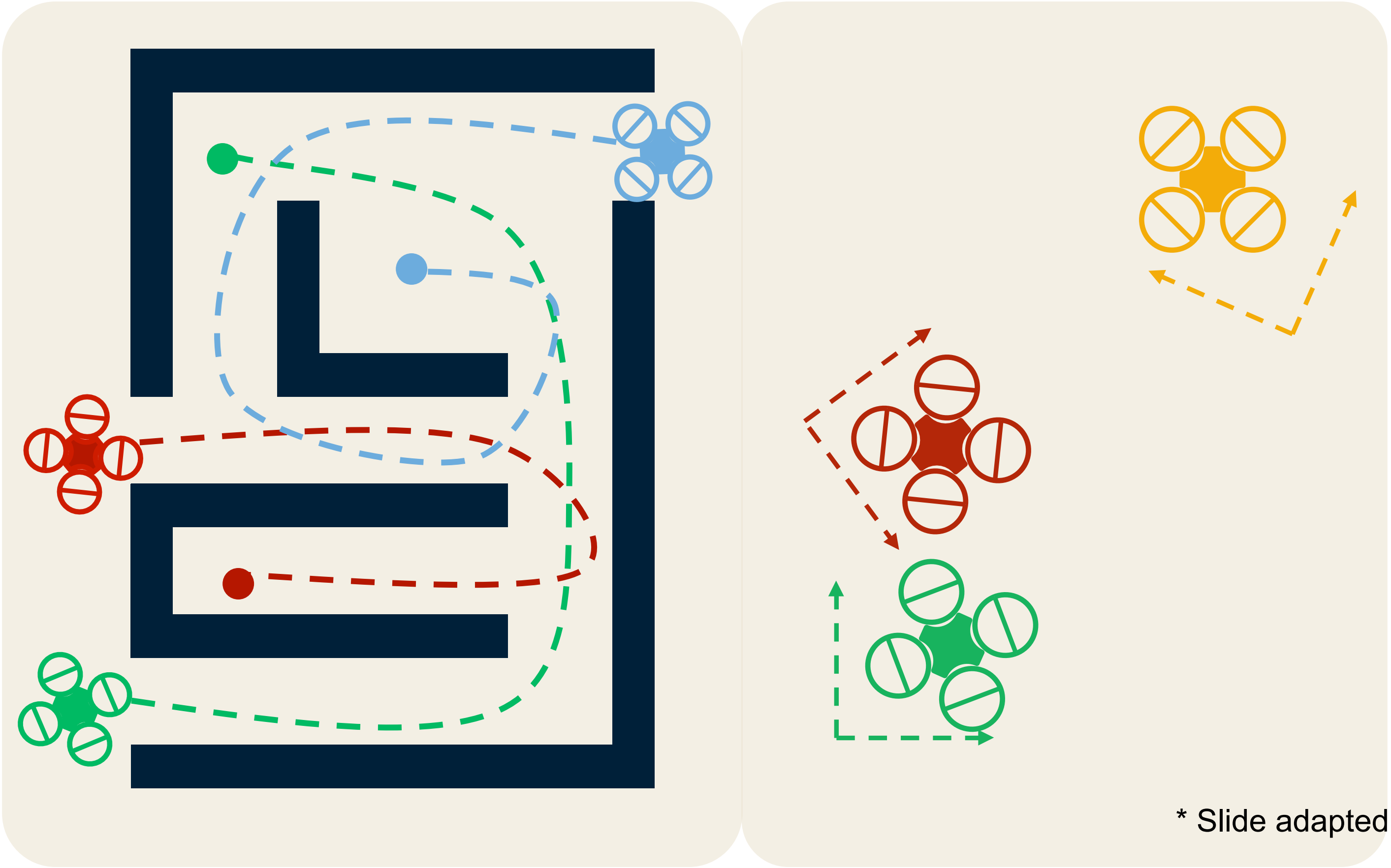
Collaborative Localization and Mapping

- Important in a variety of scenarios
  - Exploration in unknown/uncertain, dangerous environments
  - Search and rescue
  - Surveillance, tracking …

- Cooperative inference requires
  - Sharing relevant information (observations, marginals over variables of interest)
  - Correct interpretation (data association)
  - Robustness to outliers

Motivating Scenario

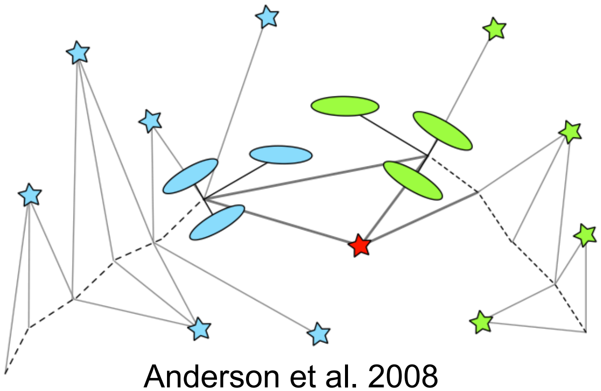
- Robots are initially unaware of each others' location
- How to establish collaboration and perform multi-robot localization?
  - Unknown multi-robot data association
  - Unknown initial relative poses between robots



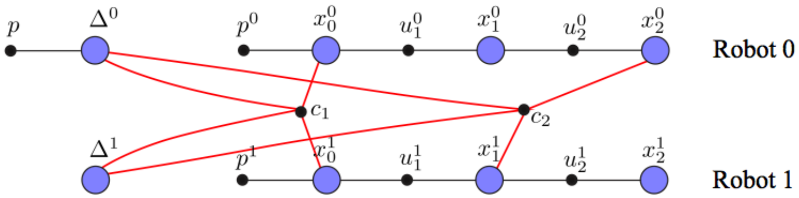
* Slide adapted from Nelson14iser

Related Work

- Known data association and common reference frames
  - Full SLAM [Howard et al. 2006], [Andersson et al. 2008]
  - Pose SLAM (direct, indirect) [Roumeliotis et al. 2002], [Kim et al. 2010], [Indelman et al. 2012]



Anderson et al. 2008



Kim et al. 2010

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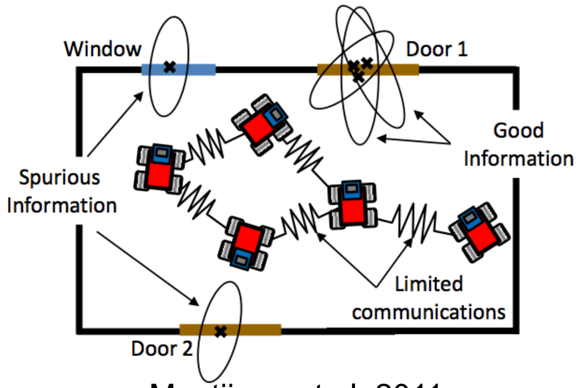
  - Pose SLAM (direct, indirect) [Roumeliotis et al. 2002], [Kim et al. 2010], [Indelman et al. 2012]
- Unknown multi-robot data association and common reference frame
  - Full SLAM [Montijano et al. 2011], [Cunningham et al. 2012]



Montijano et al. 2011

Related Work

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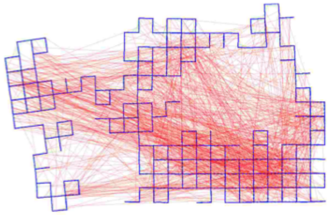
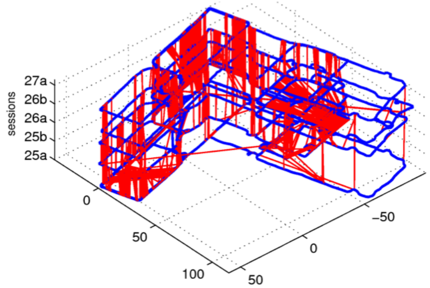
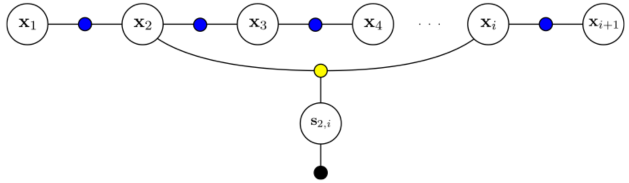
- Unknown multi-robot data association and common reference frame
  - Full SLAM [Montijano et al. 2011], [Cunningham et al. 2012]
- Robust graph optimization (**single** robot case – loop closures)
  - [Sunderhauf and Protzel 2012, 2013], [Latif et al. 2012], [Lee et al. 2013]



Lee et al. 2013



Latif et al. 2012



Sunderhauf and Protzel 2012, 2013

This Work

- Multi-robot framework with
  - **Unknown** multi-robot data association
  - **Unknown** initial relative poses between robots
  - **Pose SLAM** approach
- How to establish multi-robot data association when robots start operating from unknown locations?
- Outline:
  - **Batch, centralized framework**
  - **Incremental, distributed framework**

Multi-Robot Correspondences

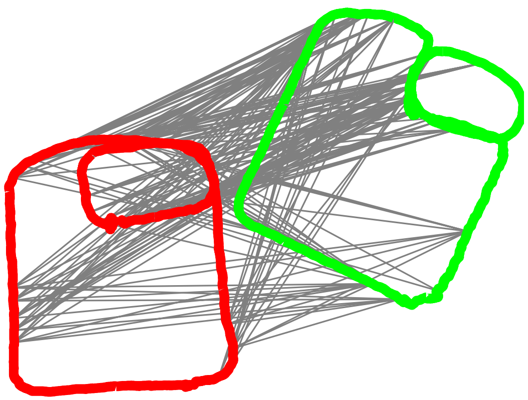
- If **no** common reference frame is available, what information to share?
  - Robots share informative observations (e.g. laser scans)

Saliency is calculated based on [Nieto et al., Robot. Auton. Syst., 2007]

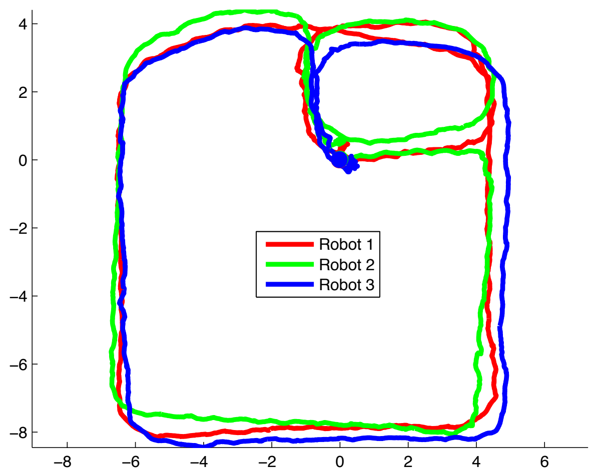
Multi-Robot Correspondences

- If **no** common reference frame is available, what information to share?
  - Robots share informative observations (e.g. laser scans)
  - Calculate candidate multi-robot relative pose constraints
    - Collect into set $\mathcal{F}$
    - Includes (many) outliers

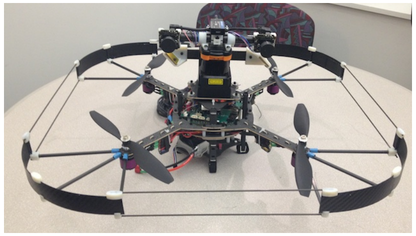
Arbitrary common reference frame

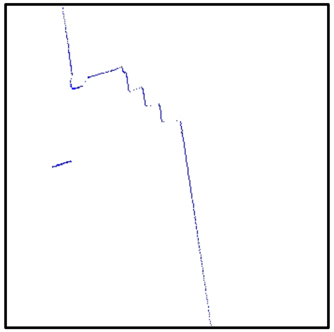


Ground truth



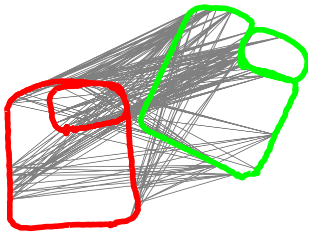
Indoor navigation





Probabilistic Formulation

- Notations:
  - $\mathcal{F}$: Multi-robot correspondences set
  - $\mathcal{J}$: Latent variables to indicate inliers/outliers



- Joint pdf over robot trajectories **and** multi-robot data association:

$$p\left(X,\mathcal{J}|Z\right)\propto\prod_{r}p\left(X^{r}|Z^{r}\right)\prod_{(r_1,r_2,k,l)\in\mathcal{F}}p\left(j_{k,l}^{r_1,r_2}\right)p\left(u_{k,l}^{r_1,r_2}|x_k^{r_1},x_l^{r_2},j_{k,l}^{r_1,r_2}\right)$$

Only local measurements

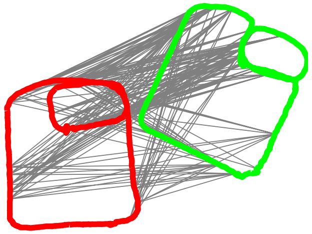
Each multi-robot correspondence

Data association

Multi-robot measurement likelihood, given data association

Measurement likelihood

$$p\left(u_{k,l}^{r_1,r_2} | x_k^{r_1}, x_l^{r_2}\right) \propto \exp\left(-\frac{1}{2}\left\| err\left(u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2}\right)\right\|_{\Sigma}^{2}\right)$$



with

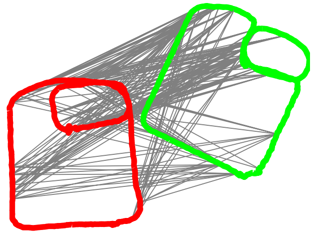
$$err\left(u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2}\right) \doteq \underbrace{u_{k,l}^{r_1,r_2}}_{\text{measured}} \ominus \underbrace{h\left(x_k^{r_1}, x_l^{r_2}\right)}_{\text{predicted}}$$

$$\doteq x_k^{r_1} \ominus \left(\underline{T_{r_2}^{r_1}} \oplus x_l^{r_2}\right)$$

Unknown!!

Measurement likelihood

$$p\left(u_{k,l}^{r_1,r_2} | x_k^{r_1}, x_l^{r_2}\right) \propto \exp\left(-\frac{1}{2}\left\| err\left(u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2}\right)\right\|_{\Sigma}^{2}\right)$$



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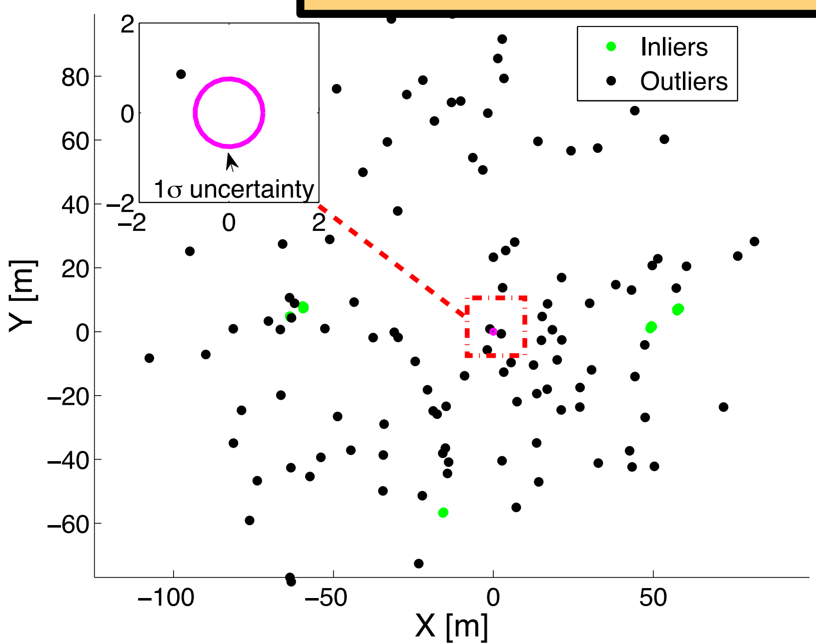
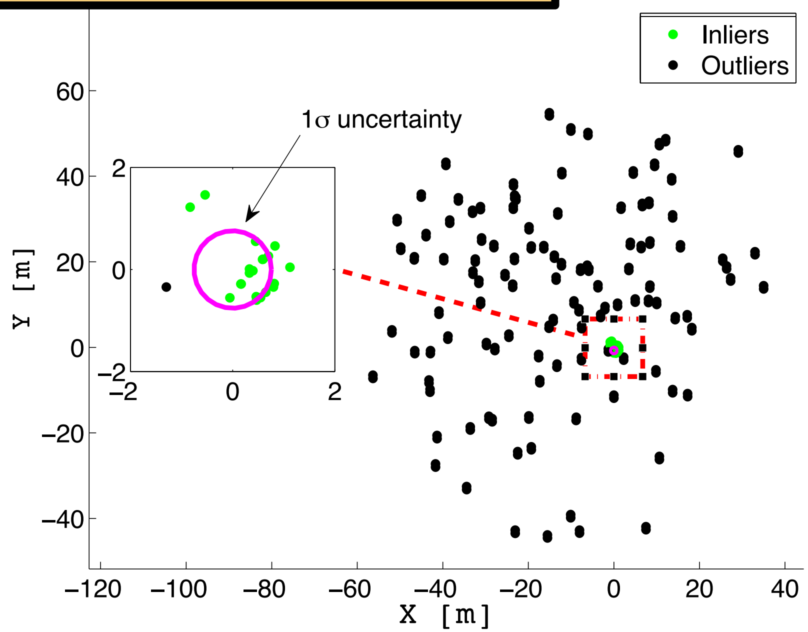
$$\doteq x_k^{r_1} \ominus \left(\underline{T_{r_2}^{r_1}} \oplus x_l^{r_2}\right)$$

Unknown!!

Error distribution for all correspondences:

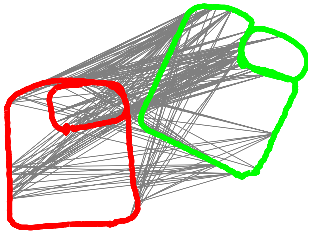
Must **first** infer a common reference frame $T_{r_2}^{r_1}$!



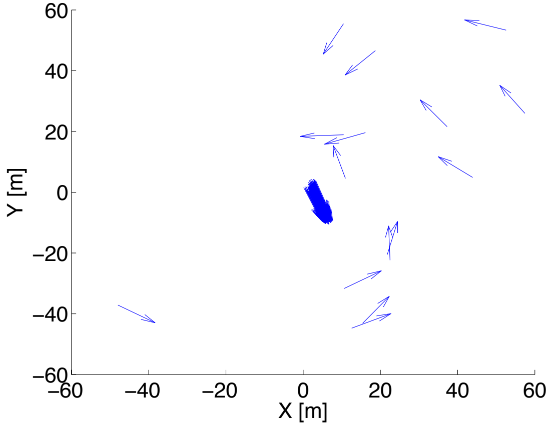


Key Observation

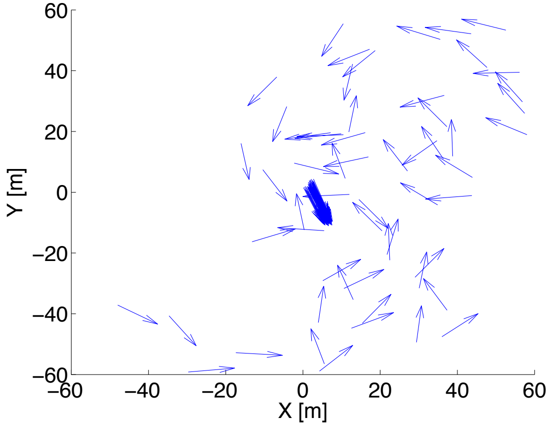
- Given robot local trajectories, relative initial pose can be calculated from each candidate multi-robot correspondence
  - **Only** inliers produce similar transformations
  - Objective: identify cluster



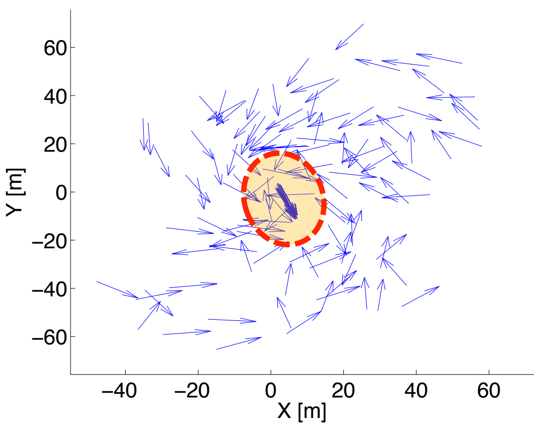
Initial relative pose between two robots (planar case: x, y, θ) [synthetic data]



10% outliers

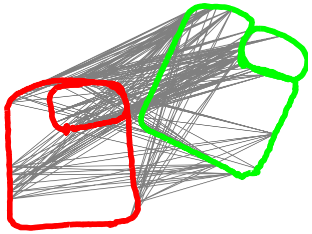


40% outliers



85% outliers

Key Observation



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  - **Only** inliers produce similar transformations
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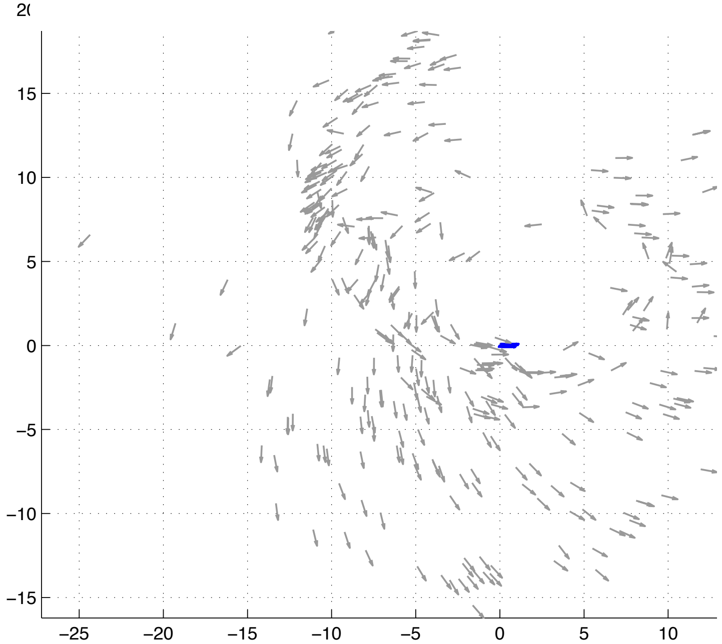
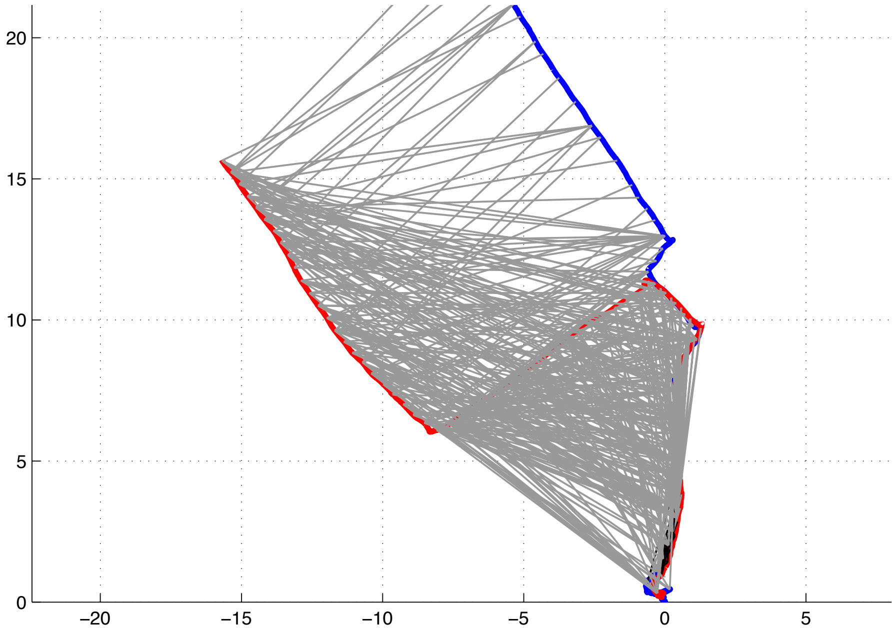
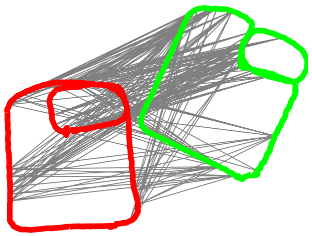
Initial relative pose between two robots (planar case: x, y, θ)
[real data]





Inference Over Common Reference Frame via EM



- MAP estimate of $T_{r_2}^{r_1}$ given robot local trajectories ($\hat{X}^{SR}$):

$$\hat{T}_{r_2}^{r_1} = \arg\max_{T_{r_2}^{r_1}} p\left(T_{r_2}^{r_1} | \hat{X}^{SR}, Z\right) = \arg\max_{T_{r_2}^{r_1}} \sum_{\mathcal{J}} p\left(T_{r_2}^{r_1}, \mathcal{J} | \hat{X}^{SR}, Z\right)$$

  – $\mathcal{J}$: Latent binary variables to indicate inliers/outliers

Local trajectories

$$\hat{X}^r = \arg\max_{X^r} p\left(X^r | Z^r\right)$$

$$\hat{X}^{SR} \doteq \left\{\hat{X}^r\right\}_{r=1}^{R}$$

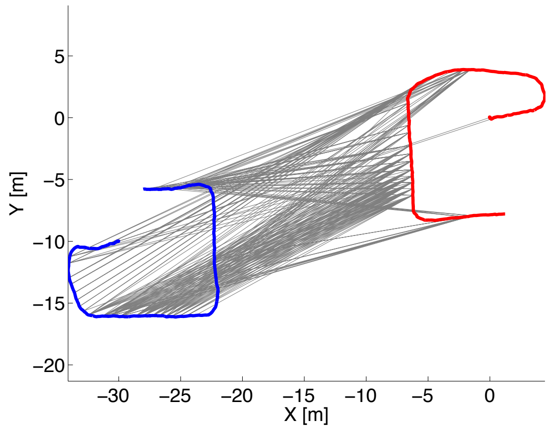
- EM formulation ($T \doteq T_{r_2}^{r_1}$) :

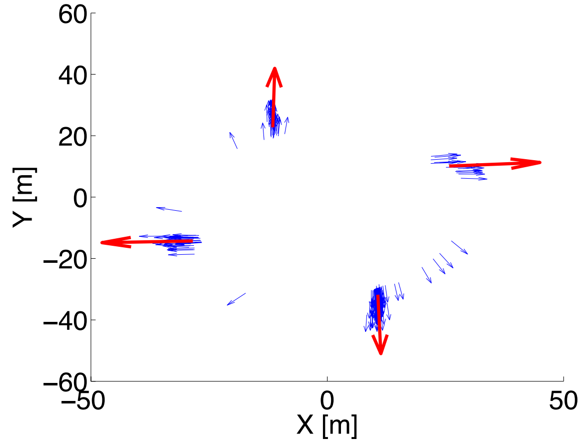
$$\hat{T}^{(i)} = \arg\max_{T} \sum_{\mathcal{J}} p\left(\mathcal{J} | \hat{T}^{(i-1)}, \hat{X}^{SR}, Z\right) \log\left[p\left(T, \mathcal{J} | \hat{X}^{SR}, Z\right)\right]$$

E step **M step**

Inference Over Common Reference Frame via EM (Cont.)

- Convergence only to **local** minima
- Therefore:
  - Start process from several initial guesses of $T_{r_2}^{r_1}$
  - Results in several locally-optimal **hypotheses** (inliers/outliers, estimated $T_{r_2}^{r_1}$)
  - Which one to choose? (next)





Inference Over Robot Trajectories

- Once a common reference frame is established:
  - Multi-robot localization becomes possible
  - Robot trajectories can be expressed in the same frame
- Infer robot trajectories via EM:

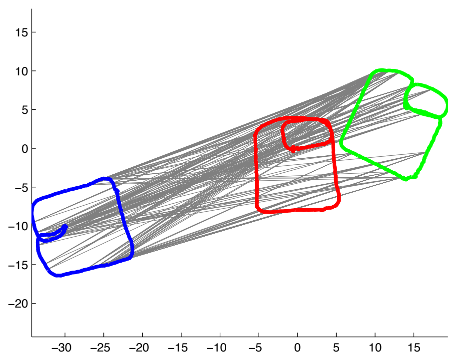
$$\hat{X} = \arg\max_{X} \sum_{\mathcal{J}} p\left(\mathcal{J} | \hat{X}, Z\right) \log p\left(X, \mathcal{J} | Z\right)$$

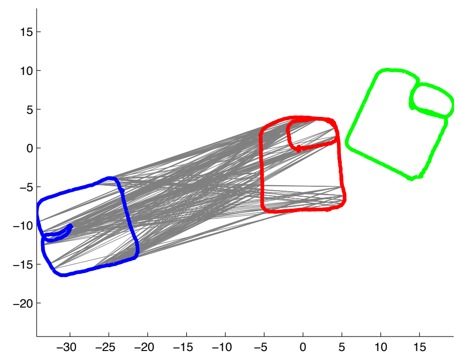
  - Identified common reference frame is used as **initial guess** within measurement likelihood

$$p\left(u_{k,l}^{r_1,r_2} | x_k^{r_1}, x_l^{r_2}\right) \propto \exp\left(-\frac{1}{2}\left\| err\left(u_{k,l}^{r_1,r_2}, x_k^{r_1}, x_l^{r_2}\right)\right\|_{\Sigma}^{2}\right)$$

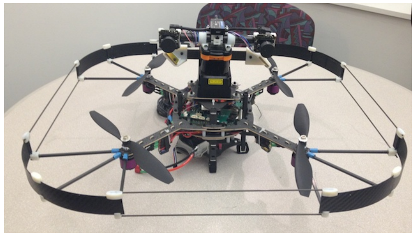
Results (Batch, Centralized)

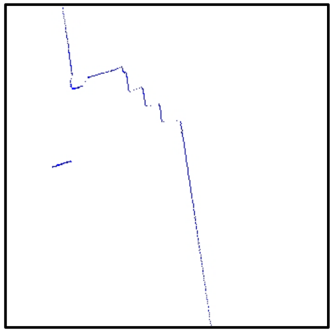
Local trajectories; Arbitrary common reference frame





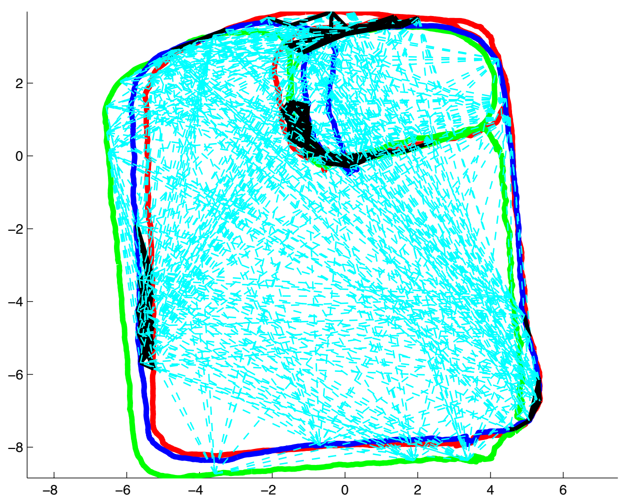
Indoor navigation



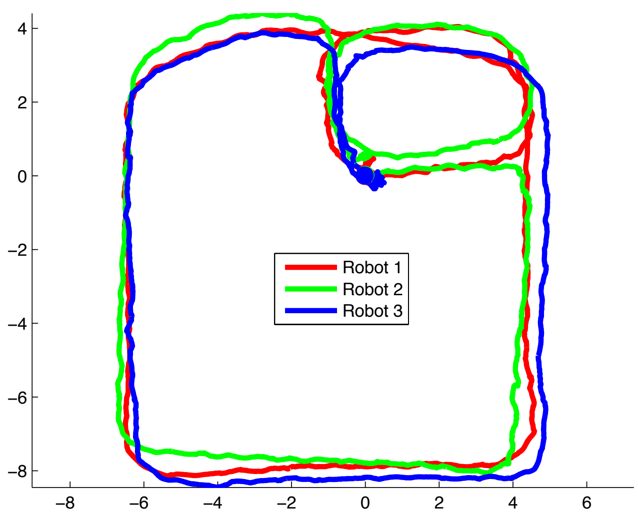


Inliers
Outliers

Estimated



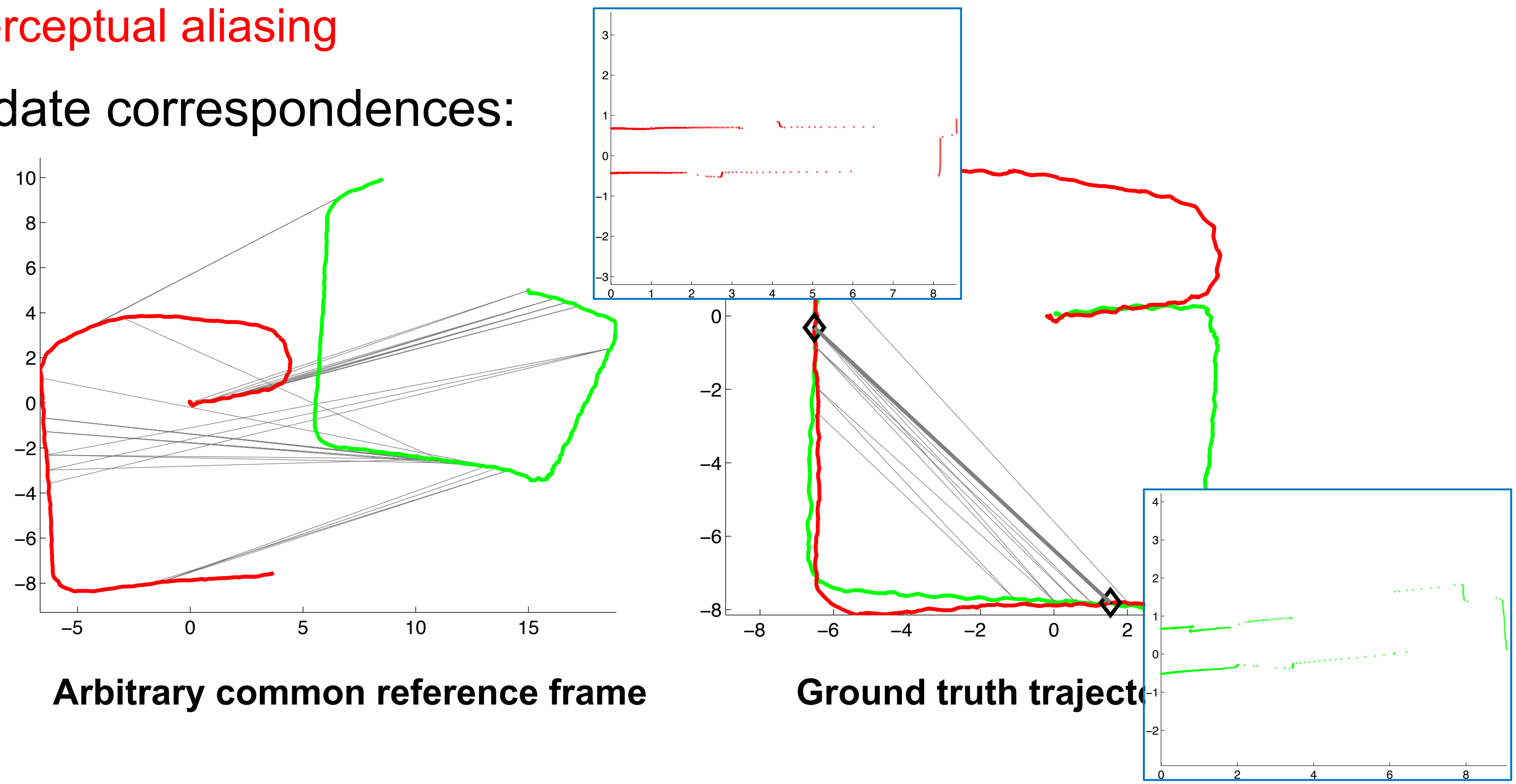
Ground truth



Incremental Framework

- **Challenges**
  - Multiple hypotheses
  - How to know when to make a decision?
    - Robot trajectories and observed environments may initially not overlap
  - Perceptual aliasing

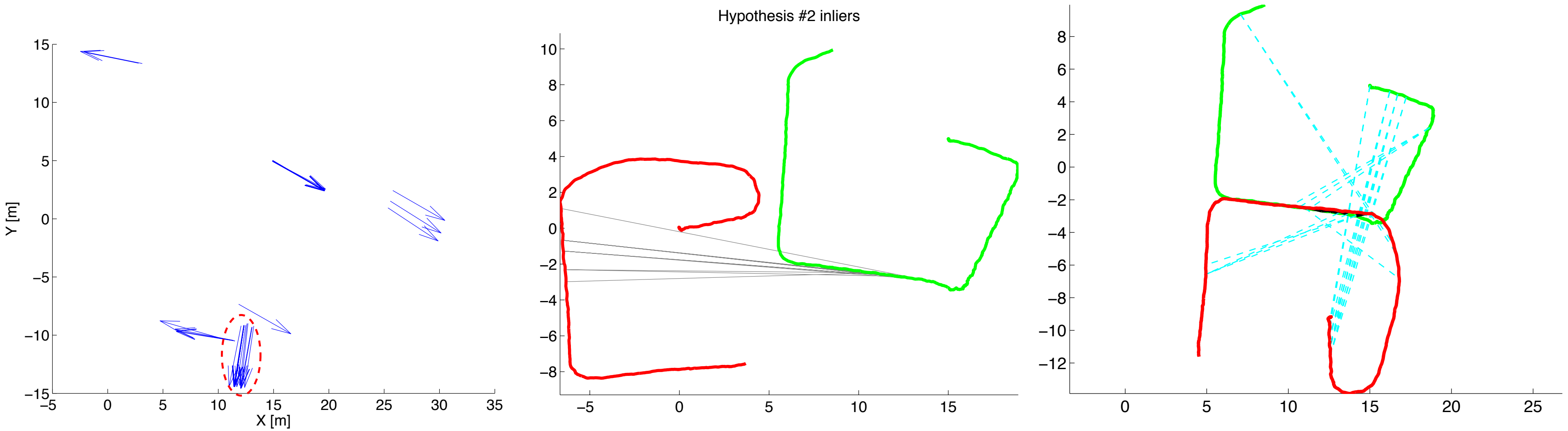
Candidate correspondences:



Arbitrary common reference frame **Ground truth traject**

Incremental Framework (Cont.)

- Choosing an **incorrect** hypothesis:



Incremental Framework (Cont.)

- **Approach**
  - **Hypothesis model-based selection**
  - **Chinese restaurant process hypothesis prior**

Hypothesis Model-Based Selection

- Calculate probability of each hypothesis $h \in \mathcal{H}$

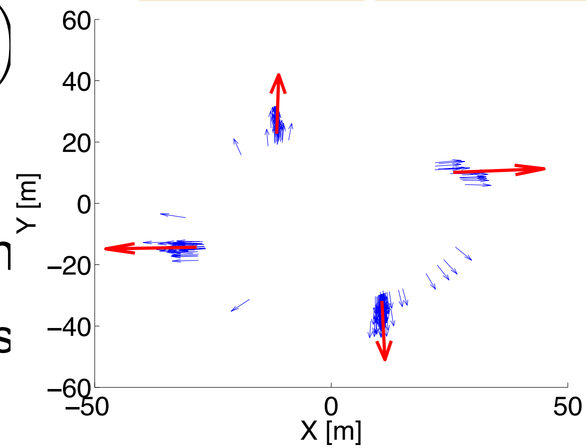
$$h = \{I, O\}$$

Inliers **Outliers**

$$p\left(h | Z, \hat{X}^{SR}\right)$$

- Explicitly:

$$p\left(h | Z, \hat{X}^{SR}\right)$$



$^{SR})$

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- Measurement likelih
  - Prioritizes hypothes
  - Does **not** address:
    - Is sufficient data available to choose a hypothesis?
    - Perceptual aliasing

Hypothesis Prior

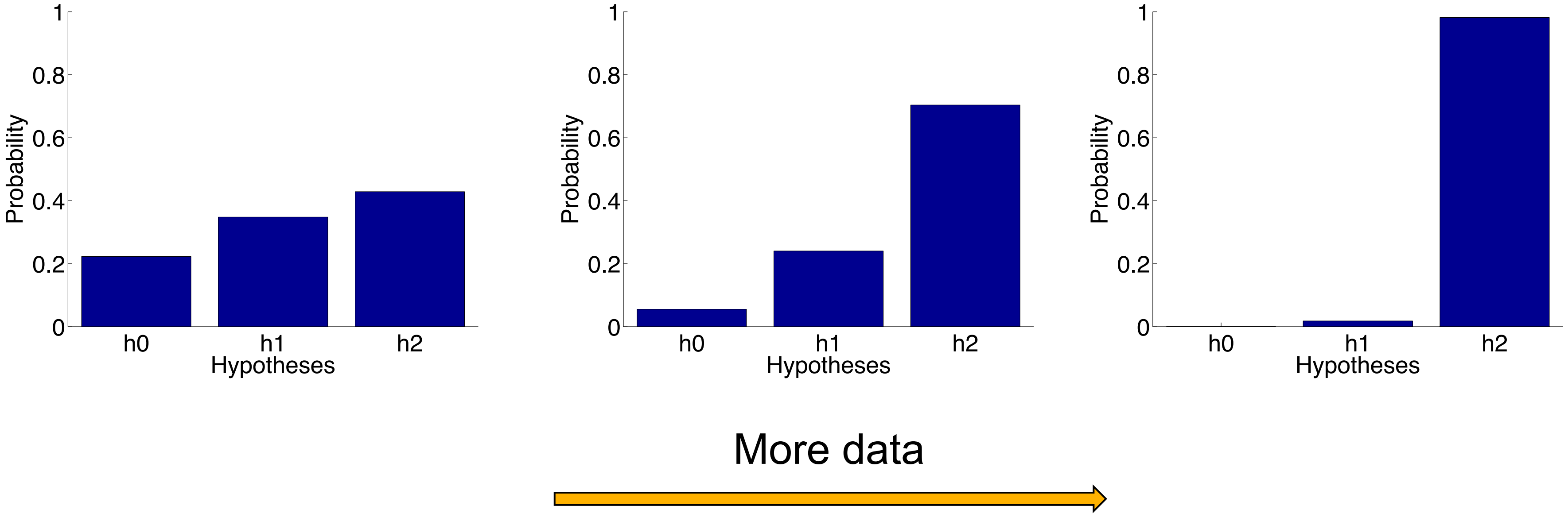
- Introduce **null-hypothesis** – corresponds to perceptual aliasing
  - All correspondences are actually outliers

- Chinese restaurant process, assuming:
  - Robots operate in closed indoor environment
  - Eventually, will observe common places (not necessarily concurrently)

Hypothesis Prior (Cont.)

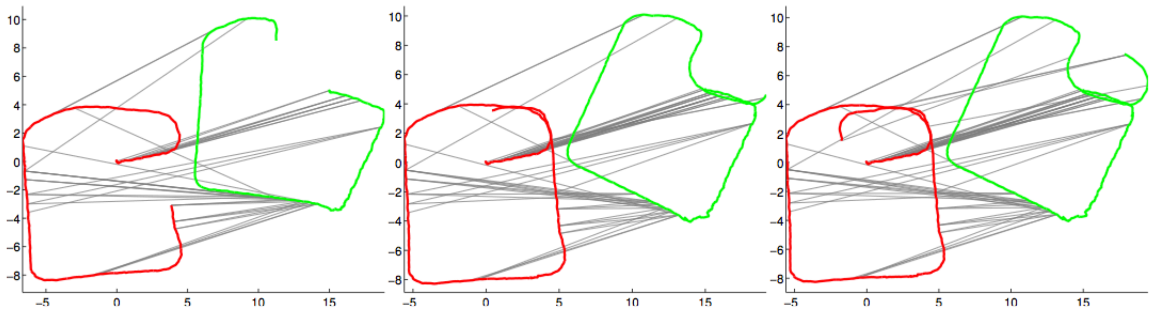
- Chinese restaurant process
  - Probability of observing a new place reduces over time
  - Use to discriminate between different hypotheses
    - As more data comes in – hypotheses priors become distinguishable

- Example

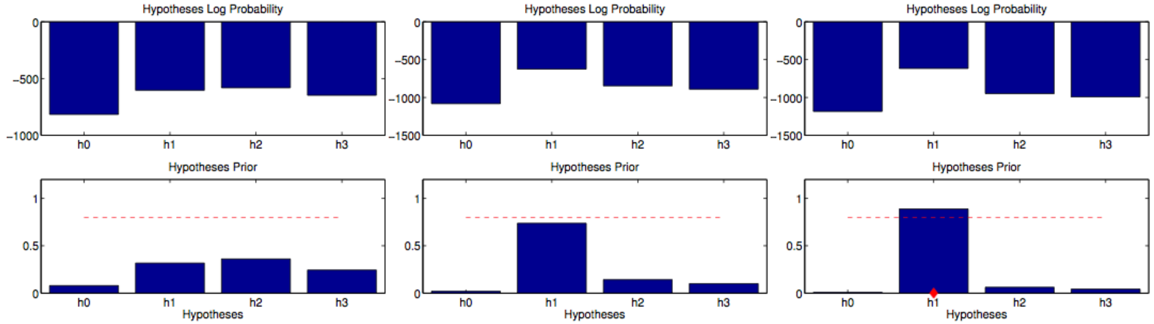


Results

time

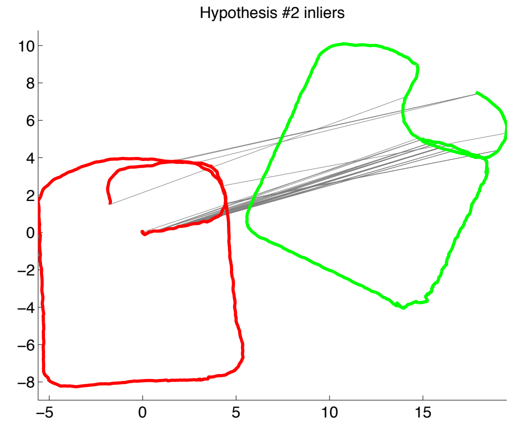


(a)

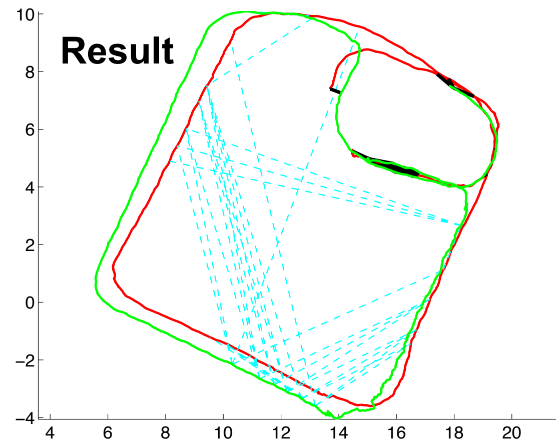


(b)

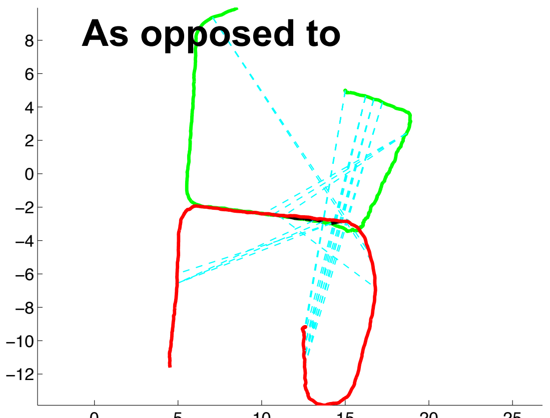
Inliers of chosen hypothesis



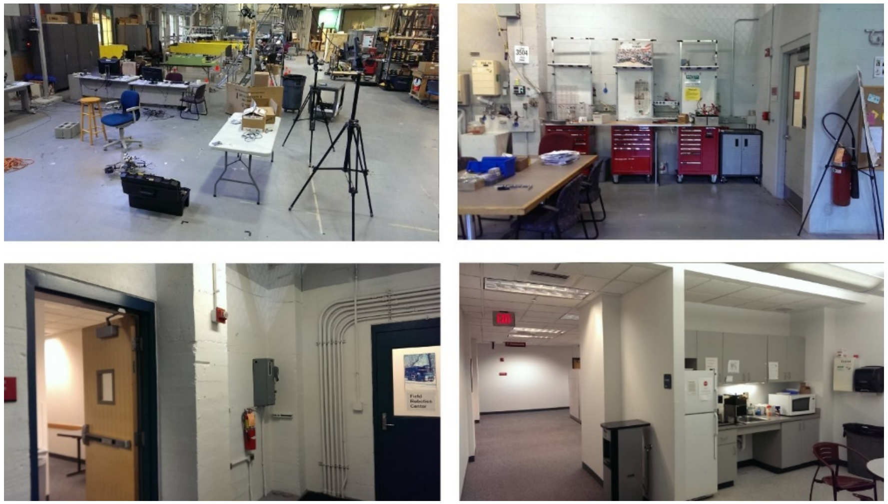
Result



As opposed to

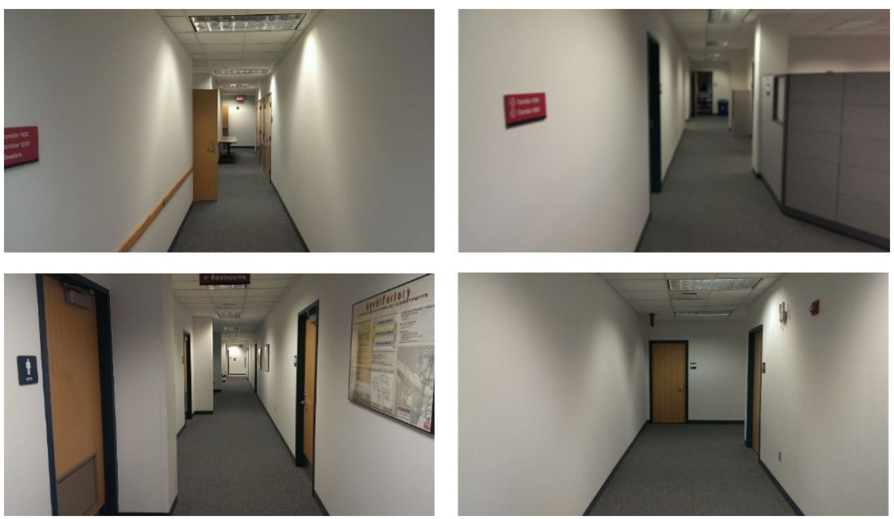


Results - Experiments @ CMU



Trial **T1**

			Trial **T1**		Trial **T2**		Trial **T3**	
			$T_{r_2}^{r_1}$	$T_{r_3}^{r_1}$	$T_{r_2}^{r_1}$	$T_{r_3}^{r_1}$	$T_{r_2}^{r_1}$	$T_{r_3}^{r_1}$
Computed	x	(m)	-0.12	0.15	2.62	-4.53	1.41	-13.59
	y	(m)	-0.03	-0.27	7.45	-4.09	-3.99	-1.24
	θ	(rad)	-0.02	0.03	-1.57	0.00	0.97	2.05
Measured	x	(m)	0.00	0.00	2.48	-4.60	1.42	-13.63
	y	(m)	0.00	0.00	7.50	-3.99	-3.90	-1.02
	θ	(rad)	0.00	0.00	-1.57	0.00	1.08	2.01
Error	$\|\|x, y\|\|$	(m)	0.12	0.31	0.15	0.12	0.09	0.22
	θ	(rad)	0.02	0.03	0.00	0.00	0.11	0.04



Trial **T2**

Trial **T3**

Results - Experiments @ CMU

[ICRA 2015]

Conclusions

- Collaborative inference from unknown initial poses and data association
  - Key observation (clusters for inlier correspondences)
  - EM approach to infer common reference frames and data association
  - Once established, joint inference over robot poses
- Distributed and incremental framework:
  - Challenges: How to know when to make a decision? Perceptual aliasing
  - Model-based selection + hypothesis prior