Towards Multi-Robot Active Collaborative State Estimation via Belief Space Planning

Vadim Indelman

TECHNION
Israel Institute of Technology

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Introduction

- Key components for autonomous operation include
  - **Perception**: Where am I? What is the surrounding environment?
  - **Planning**: What to do next?

Integrated planning and perception

Perception ⇄ **Planning**

- Belief space planning - fundamental problem in robotics
- Computationally intractable to solve exactly (POMDP), approximate suboptimal approaches exist

Related Work – Belief Space Planning

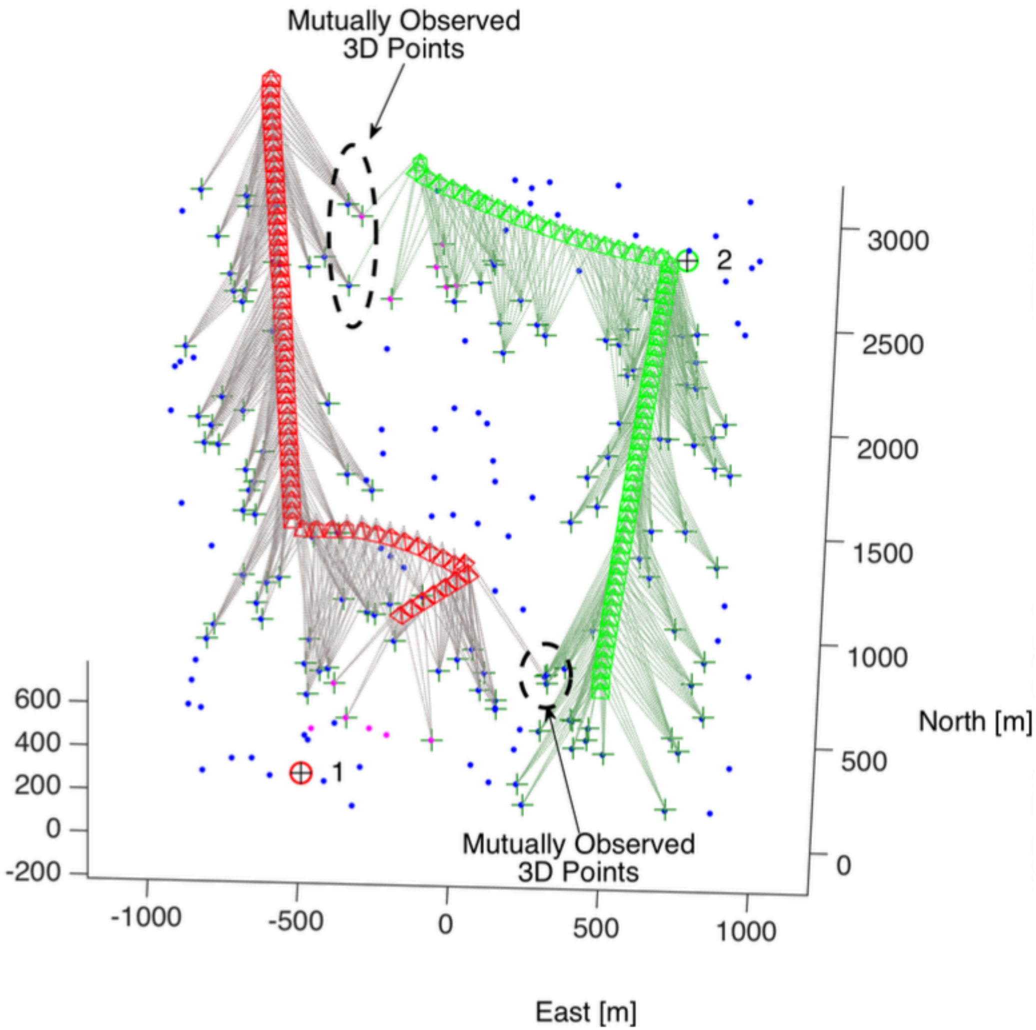
- Existing approaches typically assume **environment/map is known**
  [Prentice and Roy '09], [Miller et al. '09], [Platt et al. '10], [Van den Berg et al. '12], [Hollinger et al. '13]
- Recent research relaxes this assumption, **incorporates map uncertainty within the belief**
  [Valencia et al. '12], [Kim and Eustice '14], [Indelman et al. '15]

- **<u>This work:</u>**
  - Extension to multi-robot centralized framework
  - Opportunistic collaborative active state estimation in unknown environments
  - Direct trajectory optimization approach (starting from a nominal solution)

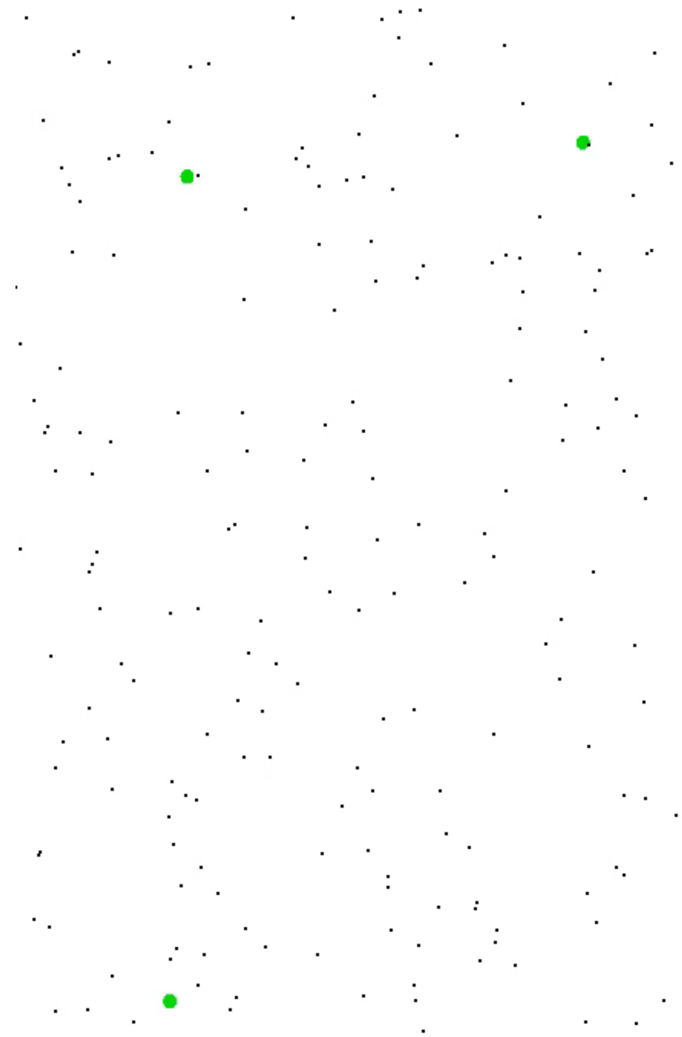
Contribution

- Framework for active collaborative state estimation while operating in unknown environments
- **Key idea**
  - Incorporate into belief **indirect** multi-robot constraints
  - To reduce uncertainty, each robot is guided to re-observe areas previously observed (only) by other robots.
- Approach **does not require rendezvous** between robots (enhanced flexibility for the group)

Spoiler



Mutually Observed
3D Points
Mutually Observed
3D Points
1
2
600
400
200
0
-200
-1000
-500
0
500
1000
East [m]
3000
2500
2000
1500
1000
500
0
North [m]



Notations and Probabilistic Formulation

Single robot $r \in \{1, \ldots, R\}$

- State transition and observation models:

$$x_{i+1}^r = f\left(x_i^r, u_i^r\right) + w_i^r$$

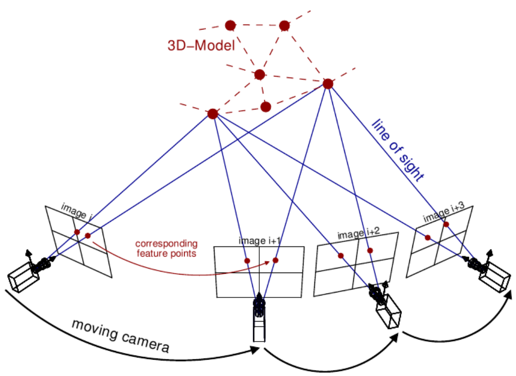
$$z_{i,j}^r = h\left(x_i^r, l_j\right) + v_i^r$$

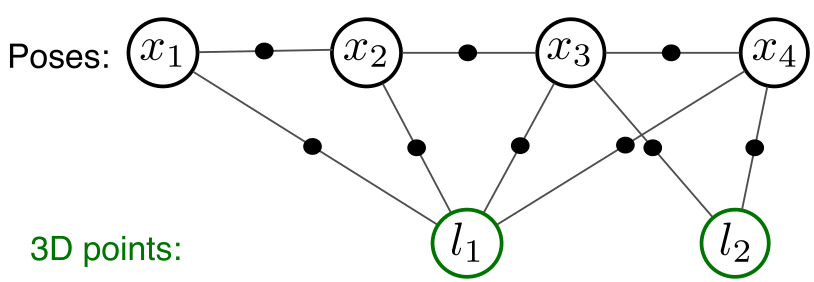
- Joint state: $\Theta_k^r \doteq X_k^r \cup L_k^r \;,\; X_k^r \doteq \{x_0^r, \ldots, x_k^r\}$

- Joint probability distribution function (pdf) at planning time t_k:

$$p\left(\Theta_k^r | Z_{0:k}^r, u_{0:k-1}^r\right) \propto p\left(x_0^r\right) \prod_{i=1}^{k} \left[p\left(x_i^r | x_{i-1}^r, u_{i-1}^r\right) \underline{p\left(Z_i^r | \Theta_i^{ro}\right)} \right]$$

$$\doteq \prod_j p\left(z_{i,j}^r | x_i^r, l_j\right)$$





Notations and Probabilistic Formulation

Multi-robot case

- Joint state for R robots:

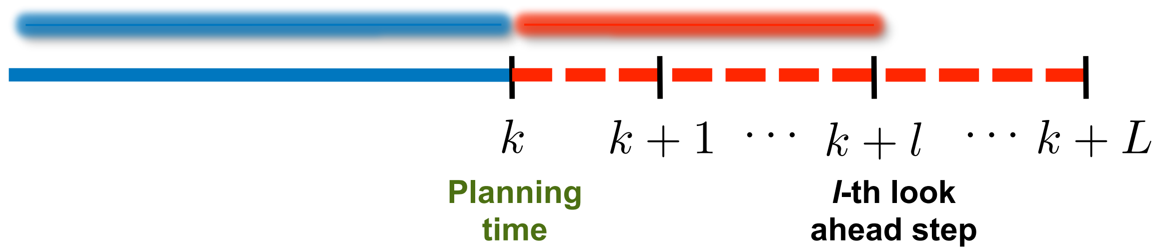
$$\Theta_k \doteq X_k \cup L_k \ , \ X_k \doteq \{X_k^r\}_{r=1}^R$$

- Joint multi-robot pdf at planning time t_k

$$b(\Theta_k) \ \doteq p(\Theta_k | Z_{0:k}, u_{0:k-1}) \propto \prod_{r=1}^{R} p\left(\Theta_k^r | Z_{0:k}^r, u_{0:k-1}^r\right),$$

Multi-robot Belief Space Planning

- Multi-robot belief at a future time t_{k+l} : $b\left(\Theta_{k+l}\right) \doteq p\left(\Theta_{k+l} | Z_{0:k+l}, u_{0:k+l-1}\right)$



- Multi-robot objective function:

$$J\left(u_{k:k+L-1}\right) \doteq \mathbb{E}\left[\sum_{l=0}^{L} c_l\left(b\left(\Theta_{k+l}\right), u_{k+l}\right) + c_L\left(b\left(\Theta_{k+L}\right)\right)\right]$$

- Optimal controls for all R robots: $u^{\star}_{k:k+L-1} = \underset{u_{k:k+L-1}}{\arg\min}\, J\left(u_{k:k+L-1}\right)$

Direct Trajectory Optimization Approach

- Recall objective function $J\left(u_{k:k+L-1}\right) \doteq \mathbb{E}\left[\sum_{l=0}^{L} c_l\left(b\left(\Theta_{k+l}\right), u_{k+l}\right) + c_L\left(b\left(\Theta_{k+L}\right)\right)\right]$
- Calculate locally-optimal controls $u_{k:k+L-1}$ for all robots
- Starting from a nominal trajectory or controls

Dual-layer Inference (as in [Indelman et al. IJRR'15])

Outer layer: iterative optimization over the controls

$$u_{k:k+L-1} \leftarrow u_{k:k+L-1} + \Delta u_{k:k+L-1}$$

$$u^r_{k:k+L-1} \leftarrow u^r_{k:k+L-1} - \lambda \nabla_r J$$

Inner layer: inference over beliefs from different time instances

$$b\left(\Theta_{k+l}\right) \sim N\left(\Theta^{\star}_{k+l}, \Sigma_{k+l}\right)$$

Incorporating Future Multi-Robot Constraints

- Joint belief at the l-th look ahead step:

$$b\left(\Theta_{k+l}\right) = p\left(\Theta_{k+l} | Z_{0:k+l}, u_{0:k+l-1}\right)$$

- Recall it is a function of the (unknown) future observations $Z_{k+1:k+l}$ and controls $u_{k:k+l-1}$ of all robots
- Model probabilistically if a future observation will be indeed acquired (similar to [Kim and Eustice IJRR'14, Indelman et al. IJRR15])
- Similar treatment for future multi-robot indirect constraints (mutual observations of the same landmark(s) possibly at different times)

Incorporating Future Multi-Robot Constraints

- Introduce two types of latent binary variables
  - $\gamma_{k+l,j}^{r}$ - loop closure event to happen at time t_{k+l}: robot r re-observes 3D points l_j (previously seen by robot r or by other robots)
  - $\psi_{k+l,m}^{r}$ - observations, to be made by robot r, of 3D points l_m that were **only** observed by **other** robots
- Collect variables of both types for all robots:

$$\Gamma_{k+l} \doteq \cup_{r=1}^{R} \Gamma_{k+l}^{r} \ , \ \Psi_{k+l} \doteq \cup_{r=1}^{R} \Psi_{k+l}^{r}$$

- Incorporate latent variables into the joint belief:

$$p\left(\Theta_{k+l}, \Gamma_{k+l}, \Psi_{k+l} | Z_{0:k+l}, u_{0:k+l-1}\right) \propto b\left(\Theta_{k+1-1}\right) \cdot \prod_{r=1}^{R} p\left(x_{k+1}^{r} | x_{k}^{r}, u_{k}^{r}\right) p\left(Z_{k+l}^{r}, \Gamma_{k+l}^{r}, \Psi_{k+l}^{r} | \Theta_{k+l}^{ro}\right)$$

Inference over the Belief

- Recall – inner layer performs inference over the belief:

$$b\left(\Theta_{k+l}\right) \sim N\left(\Theta^{\star}_{k+l}, \Sigma_{k+l}\right)$$

- The MAP estimate can be written recursively as (see paper)

$$\Theta^{\star}_{k+l} = \underset{\Theta_{k+l}}{\arg\min} \left\|\Theta_{k+l-1} - \Theta^{\star}_{k+l-1}\right\|^2_{\Sigma_{k+l-1}} + \sum_{r=1}^{R} \left\|x^r_{k+l} - f\left(x^r_{k+l-1}, u^r_{k+l-1}\right)\right\|^2_{\Sigma^r_w} +$$

$$+ \sum_{r=1}^{R} \sum_{l_j \in L^r_k} \underline{p\left(\gamma^r_{k+l,j} = 1 | \hat{x}^r_{k+l}, \hat{l}_j, z^r_{k+l,j}\right)} \left\|z^r_{k+l,j} - h\left(x^r_{k+l}, l_j\right)\right\|^2_{\Sigma_v} +$$

$$+ \sum_{r=1}^{R} \sum_{l_m \in L_k \setminus L^r_k} \underline{p\left(\psi^r_{k+l,m} = 1 | \hat{x}^r_{k+l}, \hat{l}_m, z^r_{k+l,m}\right)} \left\|z^r_{k+l,m} - h\left(x^r_{k+l}, l_m\right)\right\|^2_{\Sigma_v}$$

Inference over the Belief

$$\Theta^{\star}_{k+l} = \underset{\Theta_{k+l}}{\arg\min} \left\| \Theta_{k+l-1} - \Theta^{\star}_{k+l-1} \right\|^2_{\Sigma_{k+l-1}} + \sum_{r=1}^{R} \left\| x^r_{k+l} - f\left(x^r_{k+l-1}, u^r_{k+l-1}\right) \right\|^2_{\Sigma^r_w} +$$

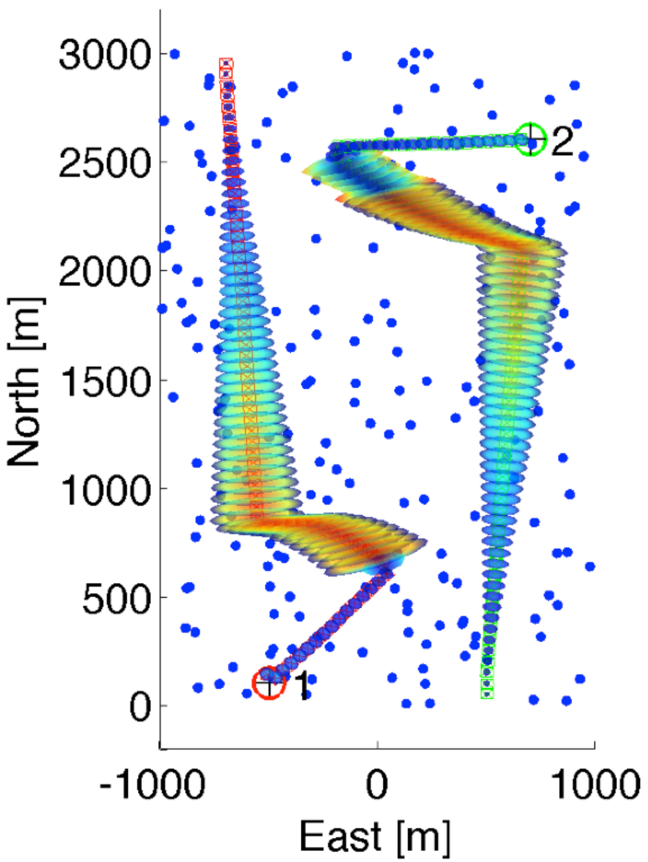
$$+ \sum_{r=1}^{R} \sum_{l_j \in L^r_k} p\left(\gamma^r_{k+l,j} = 1 | \hat{x}^r_{k+l}, \hat{l}_j, z^r_{k+l,j}\right) \left\| z^r_{k+l,j} - h\left(x^r_{k+l}, l_j\right) \right\|^2_{\Sigma_v} +$$

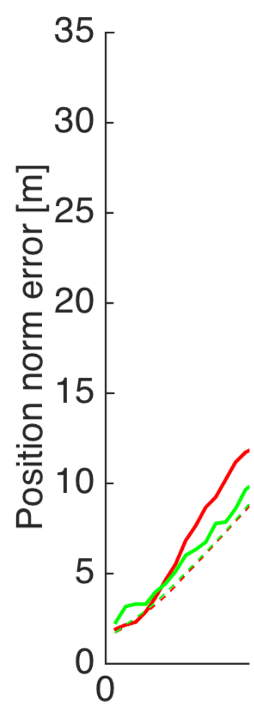
$$+ \sum_{r=1}^{R} \sum_{l_m \in L_k \setminus L^r_k} \underline{p\left(\psi^r_{k+l,m} = 1 | \hat{x}^r_{k+l}, \hat{l}_m, z^r_{k+l,m}\right)} \left\| z^r_{k+l,m} - h\left(x^r_{k+l}, l_m\right) \right\|^2_{\Sigma_v}$$

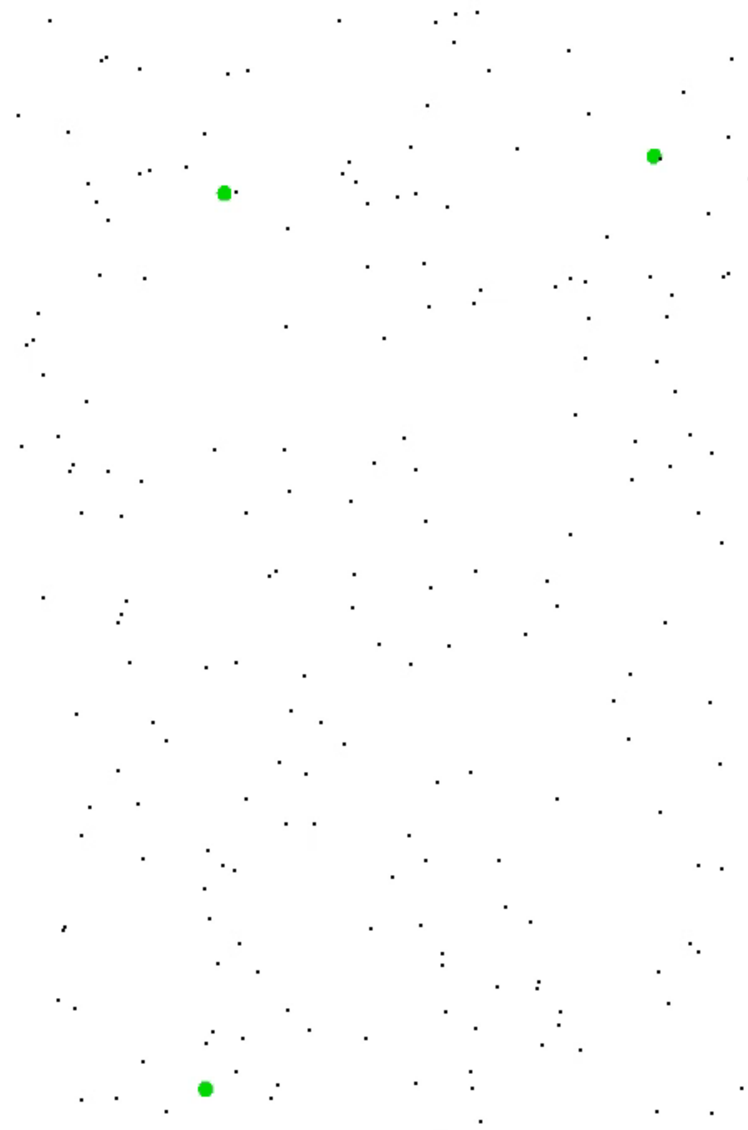
- Models robot r's observations of landmarks that were **only** previously observed by **other** robots
- Intuition: much to gain (for robot r) by observing **informative** such landmarks
- How to accomplish? Model these probabilistic terms to induce non-zero contribution to the gradient $\nabla_r J$
(more details in the paper)

Recall outer layer: $u^r_{k:k+L-1} \leftarrow u^r_{k:k+L-1} - \lambda \nabla_r J$

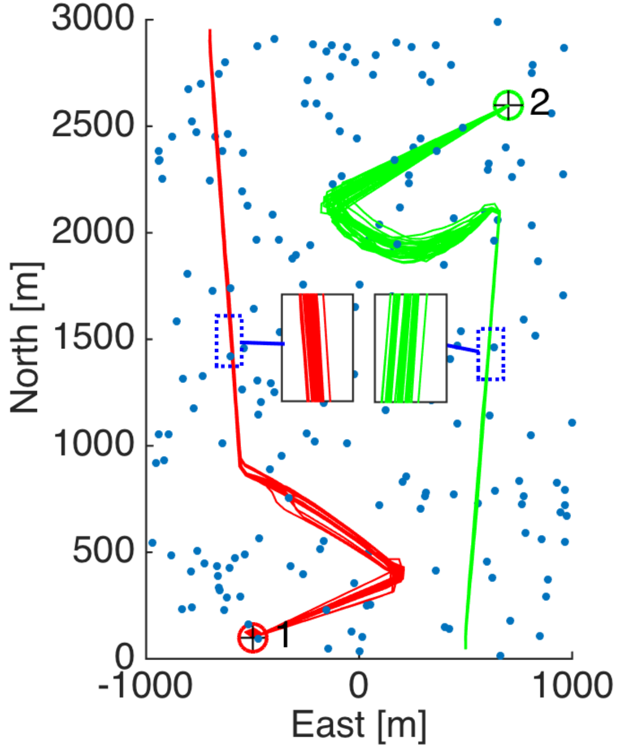
Experiments

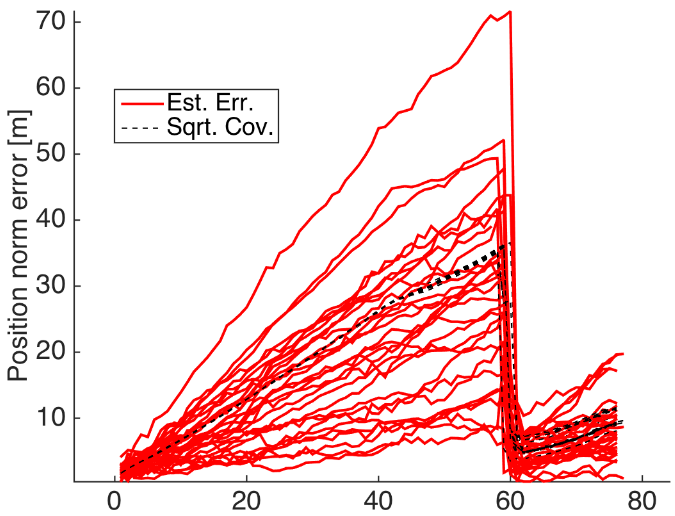


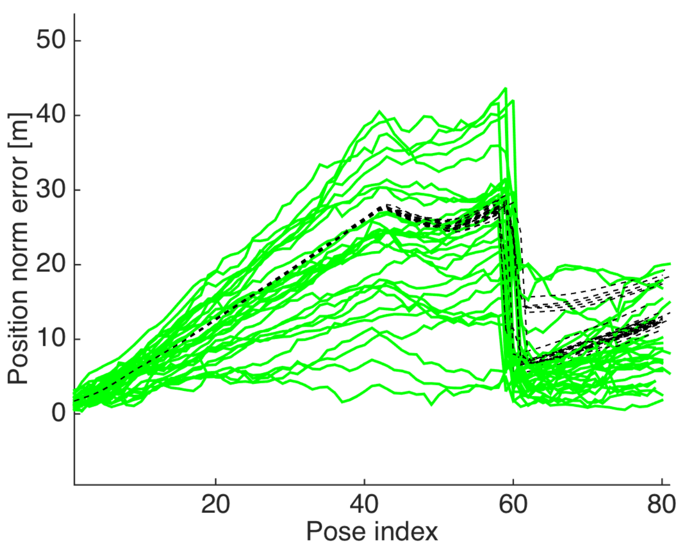




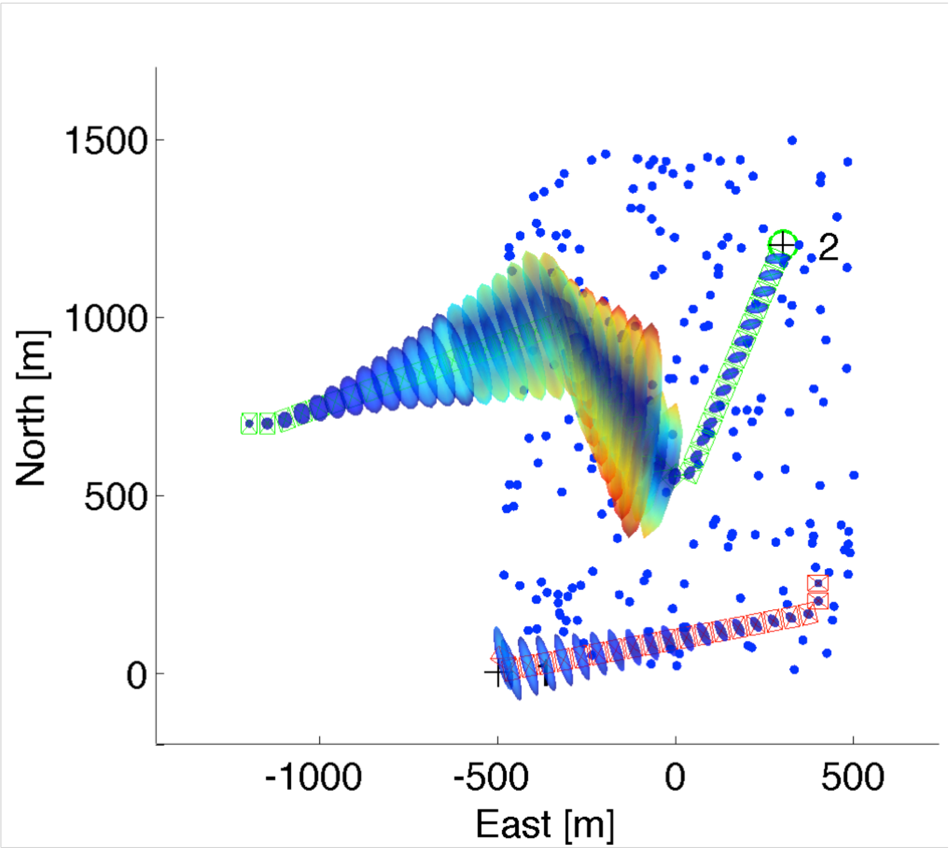
Experiments – Monte Carlo Runs

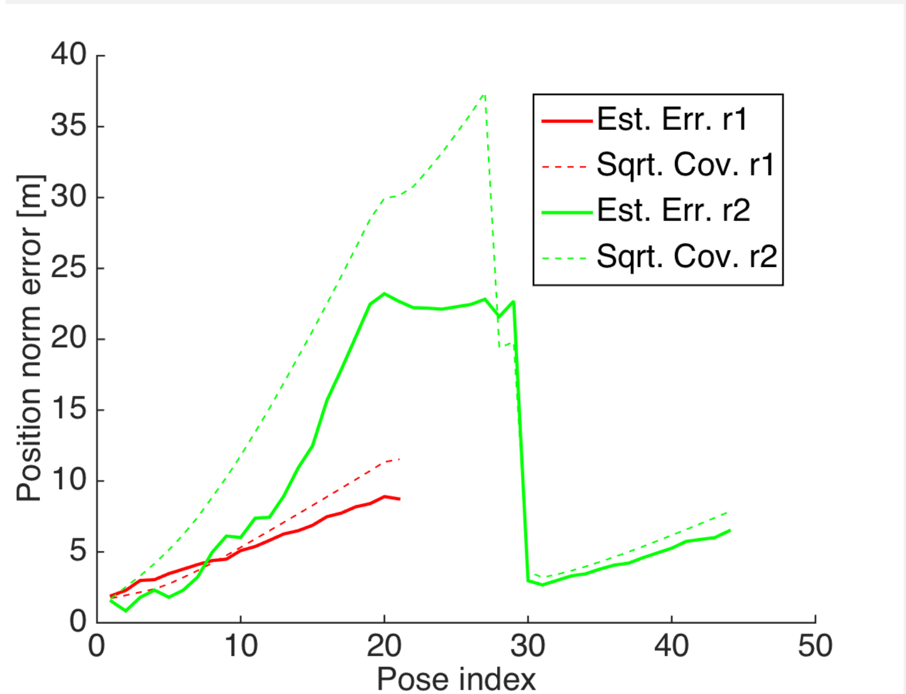






Experiments





Conclusions

- Active collaborative state estimation in unknown environments via belief space planning
  - Incorporate into belief **indirect** multi-robot constraints
  - To reduce uncertainty, each robot is guided to re-observe areas previously observed (only) by other robots
  - Enhanced flexibility to the group - **rendezvous are no longer necessary**
  - Centralized framework