Towards Cooperative Multi-Robot Belief Space Planning in Unknown Environments

Vadim Indelman

TECHNION
Israel Institute of Technology

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Introduction

- Key components for autonomous operation include
  - **Perception**: Where am I? What is the surrounding environment?
  - **Planning**: What to do next?

Integrated planning and perception

Perception ⇄ **Planning**

- Belief space planning - fundamental problem in robotics
- Computationally intractable to solve exactly (POMDP), approximate suboptimal approaches exist

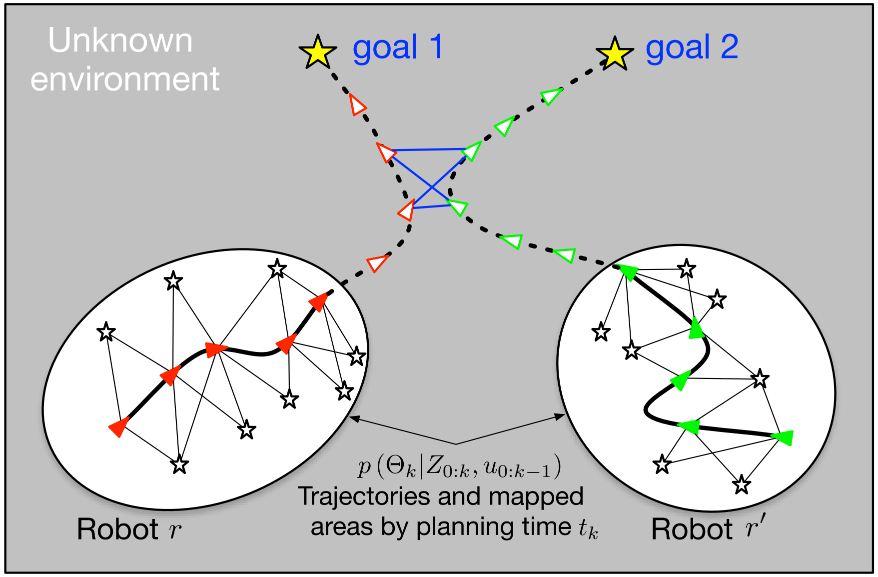
Related Work – Belief Space Planning

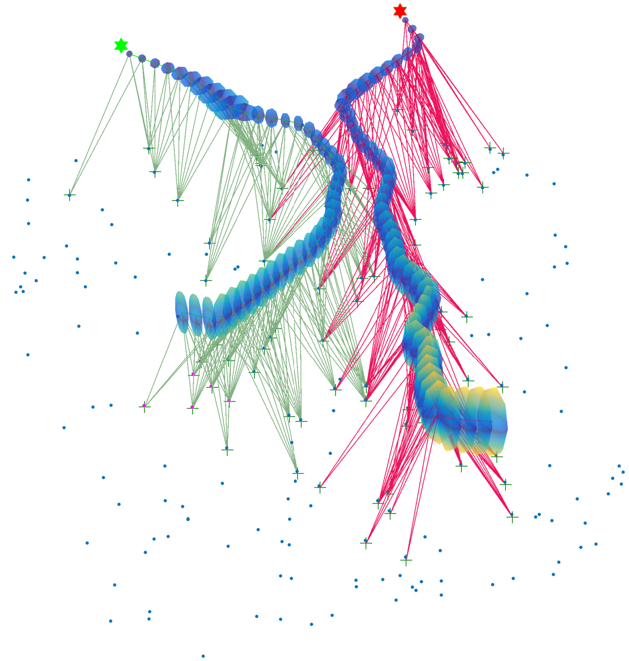
- Existing approaches typically assume **environment/map is known**
  [Prentice and Roy '09], [Miller et al. '09], [Platt et al. '10], [Van den Berg et al. '12], [Hollinger et al. '13]
- Recent research relaxes this assumption, **incorporates map uncertainty within the belief**
  [Valencia et al. '12], [Kim and Eustice '13], [Indelman et al. '15]
- Still, planning reasons only in terms of **environment observed thus far**

- **This work:**
  – Incorporate reasoning regarding environments **unknown at planning time** into belief space planning
  – Multi-robot centralized framework
  – Autonomous navigation in unknown environments, as application

Contribution

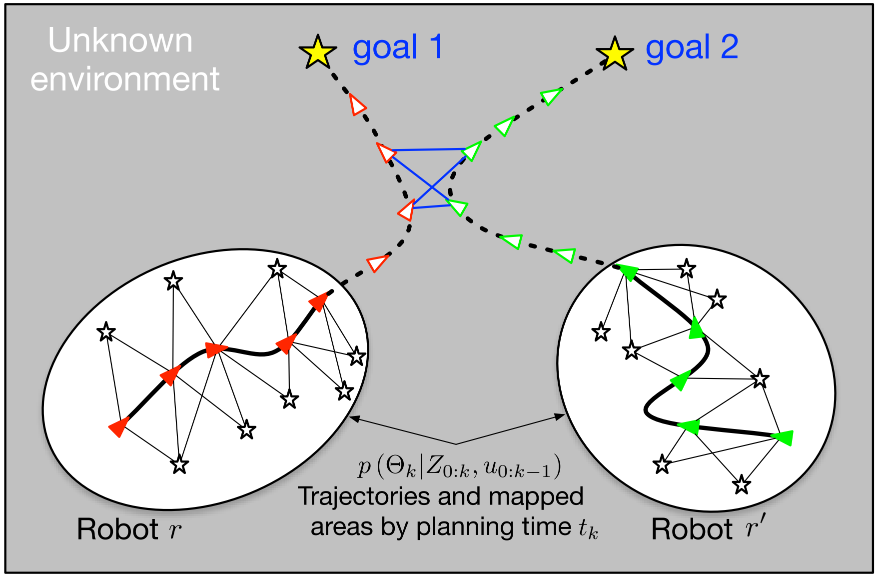
- Framework for active collaborative state estimation while operating in unknown environments
- Key idea
  - Reason about future observations of environments that are **unknown at planning time** within multi-robot belief space planning
  - Incorporate into belief the corresponding **indirect** multi-robot constraints

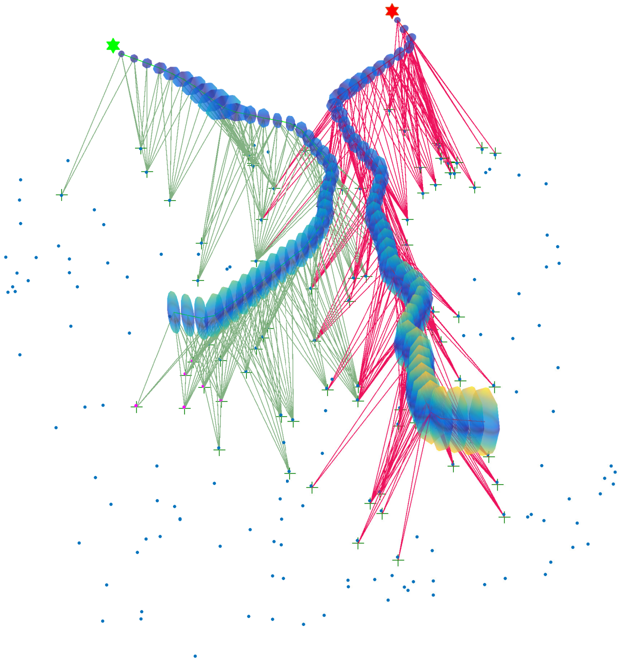




Contribution

- Approach can be used to
  - Identify best robot trajectories among candidates generated by existing motion planning approaches
  - Refine nominal trajectories into locally optimal trajectories using direct trajectory optimization techniques
- Approach **does not require rendezvous** between robots (enhanced flexibility for the group)





Notations and Probabilistic Formulation

Single robot $r \in \{1, \ldots, R\}$

- State transition and observation models:

$$x_{i+1}^r = f\left(x_i^r, u_i^r\right) + w_i^r$$

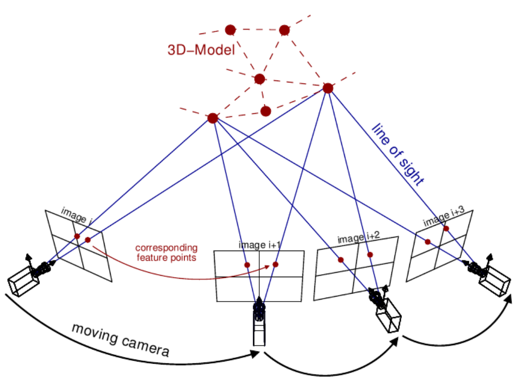
$$z_{i,j}^r = h\left(x_i^r, l_j\right) + v_i^r$$

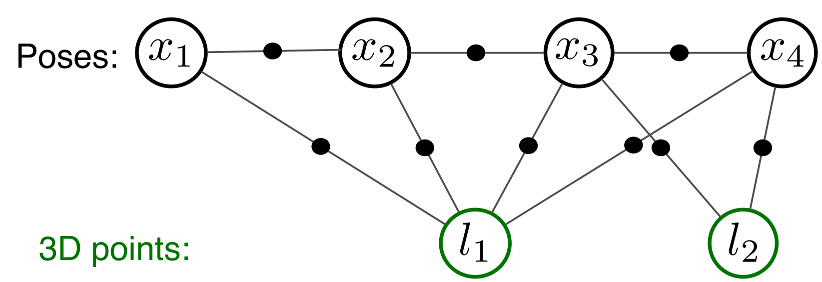
- Joint state: $\Theta_k^r \doteq X_k^r \cup L_k^r \ , \ X_k^r \doteq \{x_0^r, \ldots, x_k^r\}$

- Joint probability distribution function (pdf) at planning time t_k:

$$p\left(\Theta_k^r | Z_{0:k}^r, u_{0:k-1}^r\right) \propto p\left(x_0^r\right) \prod_{i=1}^{k} \left[p\left(x_i^r | x_{i-1}^r, u_{i-1}^r\right) p\left(Z_i^r | \Theta_i^{ro}\right) \right]$$

$$p\left(Z_i^r | \Theta_i^{ro}\right) \doteq \prod_j p\left(z_{i,j}^r | x_i^r, l_j\right)$$





Notations and Probabilistic Formulation

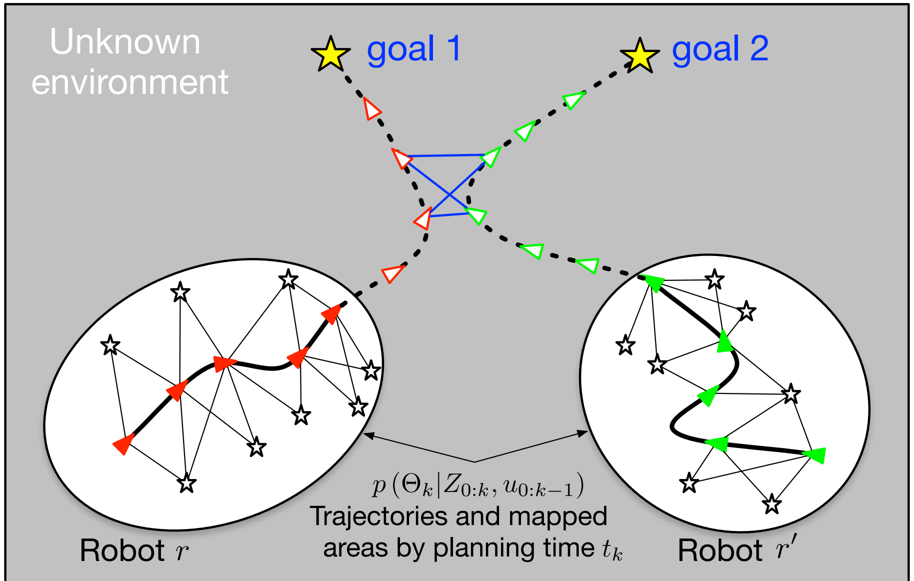
Multi-robot case

- Joint state for R robots:

$$\Theta_k \doteq X_k \cup L_k \ , \ X_k \doteq \{X_k^r\}_{r=1}^R$$

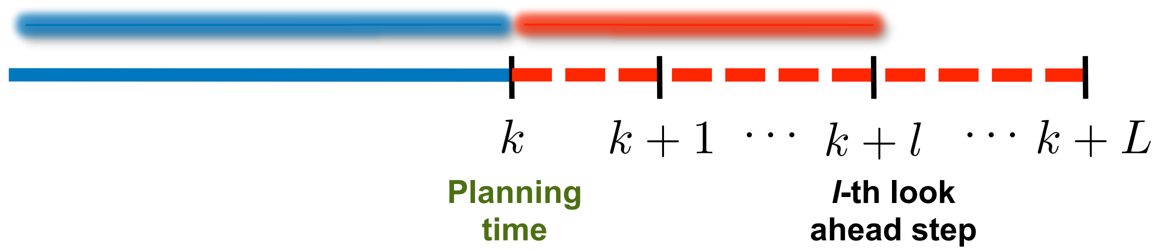
- Joint multi-robot pdf at planning time t_k

$$b\left(\Theta_k\right) \ \doteq p\left(\Theta_k | Z_{0:k}, u_{0:k-1}\right) \propto \prod_{r=1}^{R} p\left(\Theta_k^r | Z_{0:k}^r, u_{0:k-1}^r\right) ,$$



Multi-robot Belief Space Planning

- Multi-robot belief at a future time t_{k+l} : $b\left(\Theta_{k+l}\right) \doteq p\left(\Theta_{k+l} | Z_{0:k+l}, u_{0:k+l-1}\right)$



- Multi-robot objective function:

$$J\left(u_{k:k+L-1}\right) \doteq \mathbb{E}\left[\sum_{l=0}^{L} c_l\left(b\left(\Theta_{k+l}\right), u_{k+l}\right) + c_L\left(b\left(\Theta_{k+L}\right)\right)\right]$$

- Optimal controls for all R robots: $u^{\star}_{k:k+L-1} = \underset{u_{k:k+L-1}}{\arg\min}\, J\left(u_{k:k+L-1}\right)$

Evaluation of Candidate Paths

- Recall objective function $J\left(u_{k:k+L-1}\right) \doteq \mathbb{E}\left[\sum_{l=0}^{L} c_l\left(b\left(\Theta_{k+l}\right), u_{k+l}\right) + c_L\left(b\left(\Theta_{k+L}\right)\right)\right]$

- Need to evaluate objective function for each candidate path
- Involves inference over beliefs from different time instances

$$b\left(\Theta_{k+l}\right) \equiv p\left(\Theta_{k+l} | Z_{0:k+l}, \underline{u_{0:k+l-1}}\right) = N\left(\Theta^{\star}_{k+l}, I_{k+l}\right)$$

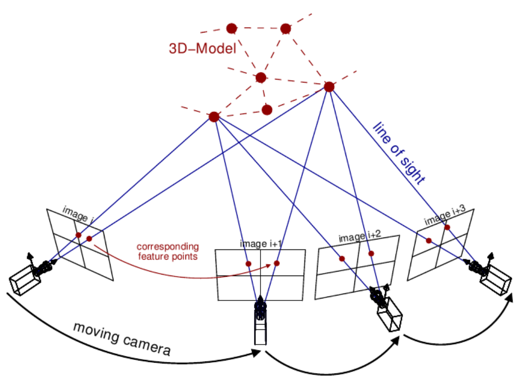
different for each path

Multi-Robot Belief

- Recall - Joint probability distribution function (pdf) at planning time t_k:

$$p\left(\Theta_k^r | Z_{0:k}^r, u_{0:k-1}^r\right) \propto p\left(x_0^r\right) \prod_{i=1}^{k} \left[p\left(x_i^r | x_{i-1}^r, u_{i-1}^r\right) \underline{p\left(Z_i^r | \Theta_i^{ro}\right)}\right]$$

$$\doteq \prod_j p\left(z_{i,j}^r | x_i^r, l_j\right)$$

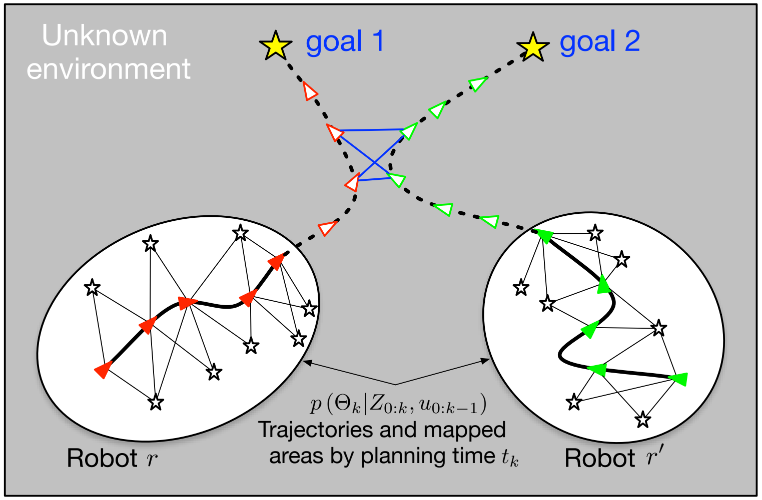


- Recursive formulation of a multi-robot belief at the l-th look ahead step:

$$b\left(\Theta_{k+l}\right) = \eta b\left(\Theta_{k+l-1}\right) \prod_{r=1}^{R} p\left(x_{k+l}^r | x_{k+l-1}^r, u_{k+l-1}^r\right) p\left(Z_{k+l}^r | \Theta_{k+l}^{ro}\right)$$

Incorporating Future Multi-Robot Constraints

- **Recall** - Key idea:
  - Reason about future observations of environments that are unknown at planning time within multi-robot belief space planning
  - Incorporate into belief indirect multi-robot constraints that correspond to these future observations
- **How?**



Incorporating Future Multi-Robot Constraints

- If we knew landmarks $L^{r,r'}_{k+l,k+j}$ will be observed from poses x^r_{k+l} and $x^{r'}_{k+j}$

$$p\left(x^r_{k+l}, x^{r'}_{k+j}, L^{r,r'}_{k+l,k+j} | z^r_{k+l}, z^{r'}_{k+j}\right) \propto p\left(z^r_{k+l} | x^r_{k+l}, L^{r,r'}_{k+l,k+j}\right) p\left(z^{r'}_{k+j} | x^{r'}_{k+j}, L^{r,r'}_{k+l,k+j}\right)$$

$$p\left(z^r_{k+l}, z^{r'}_{k+j} | x^r_{k+l}, x^{r'}_{k+j}\right) \propto \int p\left(x^r_{k+l}, x^{r'}_{k+j}, L^{r,r'}_{k+l,k+j} | z^r_{k+l}, z^{r'}_{k+j}\right) dL^{r,r'}_{k+l,k+j}$$

- But, what if environment is unknown?
- Recall passive framework (robot actions are determined):
  - Assume a mutual scene will be observed from some poses x^r_i and $x^{r'}_j$
  - Match between observations (image, laser scans) to get constraint $z^{r,r'}_{i,j}$
  - Measurement likelihood: $p(z^{r,r'}_{i,j} | x^r_i, x^{r'}_j) \propto \exp\left(-\frac{1}{2}\| z^{r,r'}_{i,j} - g\left(x^r_i, x^{r'}_j\right) \|^2_{\Sigma^{MR}_v}\right)$

Incorporating Future Multi-Robot Constraints

- Can we do the same in our (active) case?
  - Assume an unknown scene will be observed from future poses x^r_{k+l} and $x^{r'}_{k+j}$
  - Let $z^{r,r'}_{k+l,k+j}$ denote an unknown constraint calculated by matching the corresponding observations
  - Measurement likelihood is then

$$p\left(z^{r,r'}_{k+l,k+j}|x^r_{k+l},x^{r'}_{k+j}\right)\propto\exp\left(-\frac{1}{2}\|z^{r,r'}_{k+l,k+j}-g\left(x^r_{k+l},x^{r'}_{k+j}\right)\|^2_{\Sigma_v^{MR}}\right)$$

- $z^{r,r'}_{k+l,k+j}$ can be treated as random variable
- Taking a maximum-likelihood assumption, actual value is not important for belief inference

Incorporating Future Multi-Robot Constraints

- How to know two future poses mutually observe a scene?
  - Answer is scenario-specific
  - Can use
    - Indicator function $\gamma(x^r, x^{r'})$, as e.g. in [1]
    - Simpler criteria, e.g. if two poses are sufficiently nearby
    - Statistics (note: only used for planning, not for estimation)

[1] D. Levine, B. Luders, and J. P. How. Information-theoretic motion planning for constrained sensor networks. Journal of Aerospace Information Systems, 10(10):476–496, 2013.

Recursive Formulation of a Multi-Robot Belief

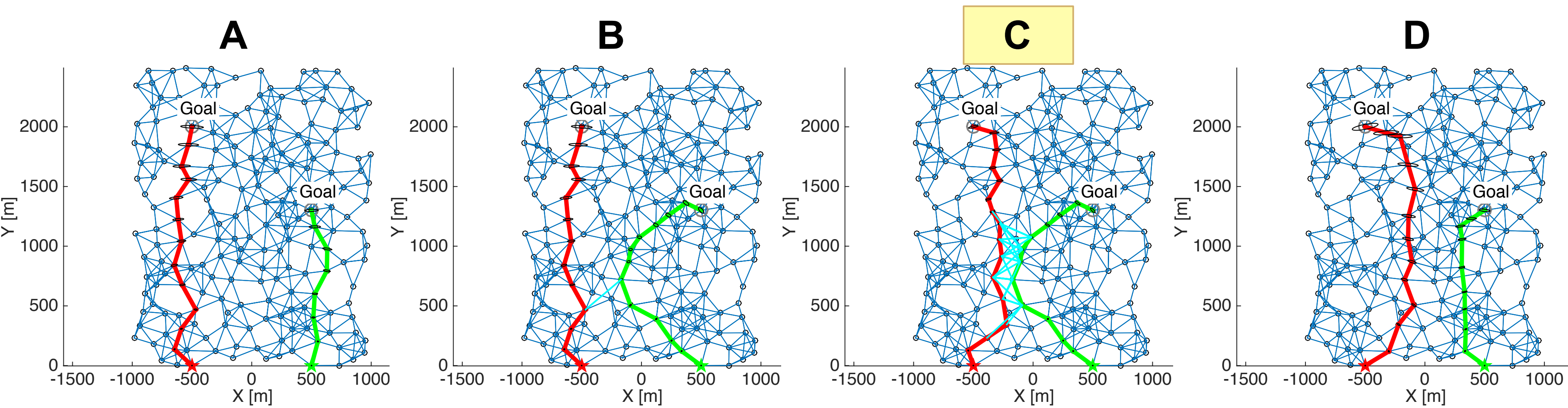
- Plugging-in, final expression for a future time t_{k+l}

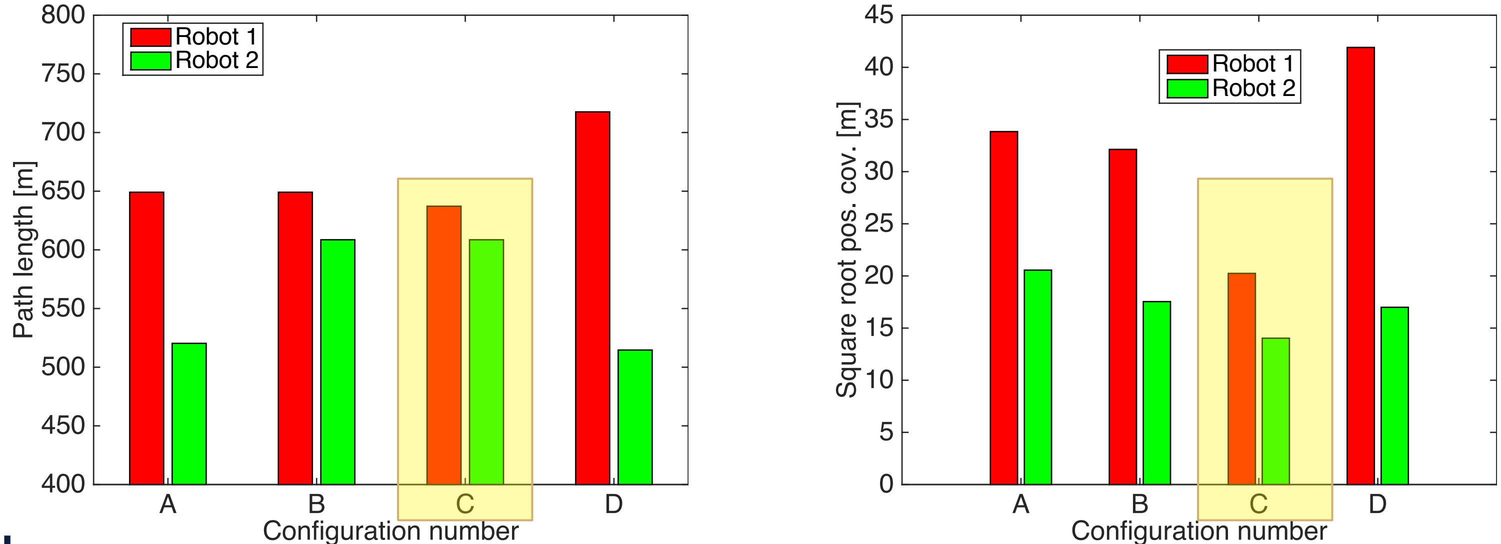
$$b\left(\Theta_{k+l}\right) = \eta b\left(\Theta_{k+l-1}\right)\prod_{r=1}^{R}\Bigg[p\left(x^r_{k+l}|x^r_{k+l-1},u^r_{k+l-1}\right)\prod_{l_j\in\Theta^{ro}_{k+l}} p\left(z^r_{k+l,j}|x^r_{k+l},l_j\right)$$

$$p\left(z^r_{k+l,k+l-1}|x^r_{k+l},x^r_{k+l-1}\right)\cdot\underline{\prod_j p\left(z^{r,r'}_{k+l,k+j}|x^r_{k+l},x^{r'}_{k+j}\right)}\Bigg].$$

Preliminary Evaluation

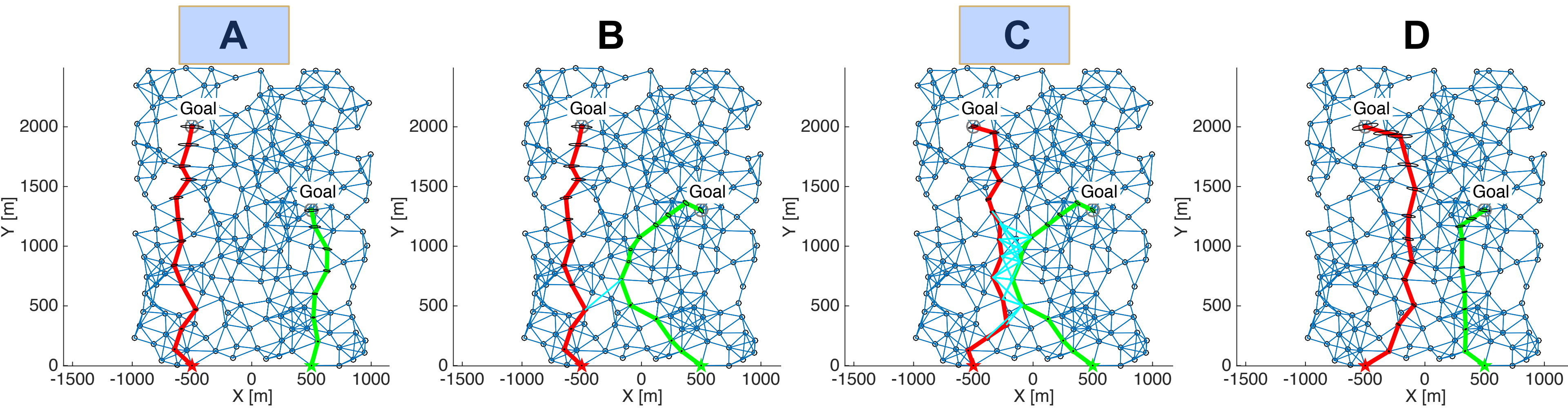
- Objective function $J = \sum_{r=1}^{R} \left[\kappa^r t^r_{goal} + (1-\kappa^r) tr\left(\Sigma^r_{goal}\right)\right]$
- PRM



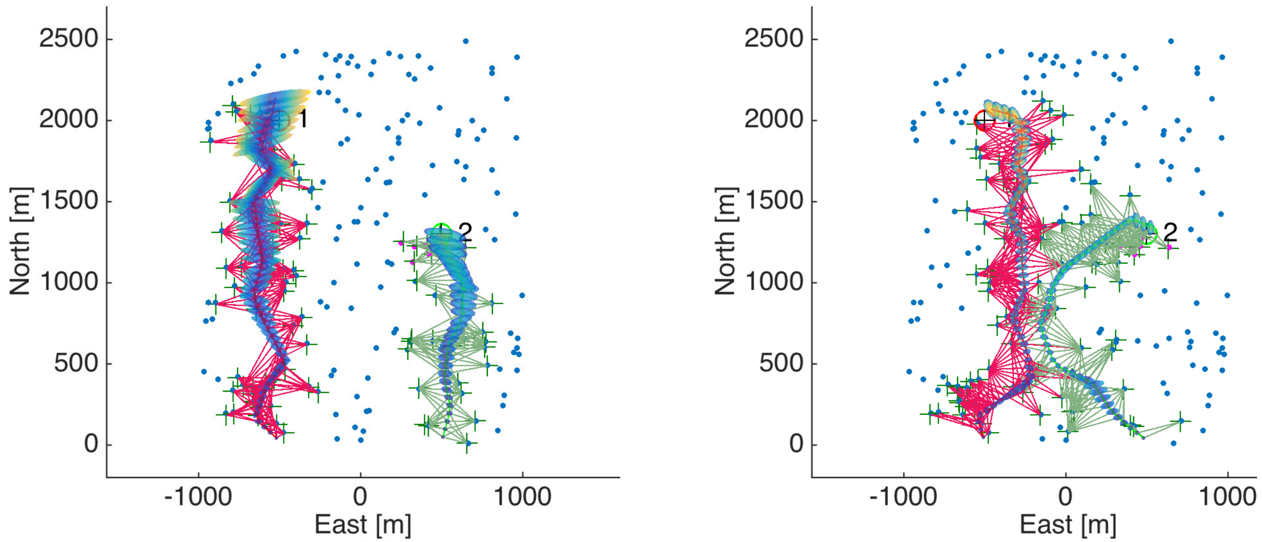


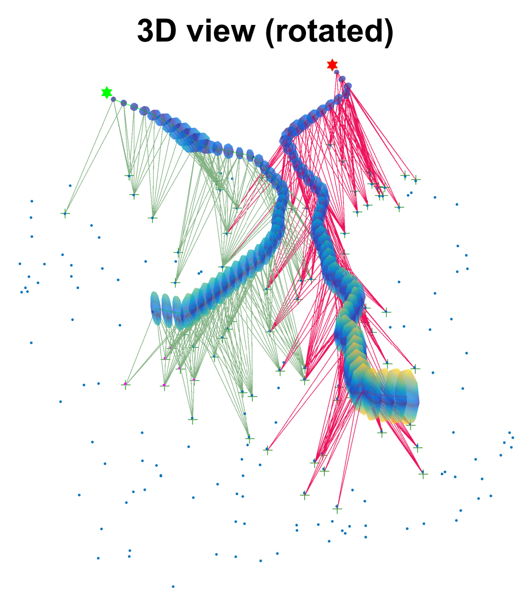
Preliminary Evaluation

- Objective function $J = \sum_{r=1}^{R} \left[\kappa^r t_{goal}^r + (1-\kappa^r) tr\left(\Sigma_{goal}^r\right) \right]$
- PRM



Path execution:





Conclusions and Future Work

- Collaborative multi-robot belief space planning in unknown environments
- Contribution:
  - Incorporate within the belief constraints that represent multi-robot mutual observations of unknown environments
  - Enhanced flexibility to the group - rendezvous are no longer necessary
  - Concept can be used both within sampling based and direct trajectory optimization approaches
- Future work includes:
  - Extensive evaluation
  - Distributed/decentralized framework