Distributed Perception and Estimation in Multi-Robot Systems

Principles of Multi-Robot Systems - Workshop at RSS 2015

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Introduction

- Distributed perception and estimation – central problem in multi-robot systems

- Applications include
  - Localization & navigation
  - Tracking
  - Mapping
  - SLAM
  - ...

- Multi-robot collaboration provides key capabilities but introduces a number of challenges

Image courtesy of Y. Hidaka
This Talk

- Overview of main issues in distributed perception and estimation (focus on multi-robot SLAM and cooperative localization, as application)
- Address two key challenges
  - Consistent decentralized estimation
  - Robust decentralized perception
- Naturally, not all aspects are covered
- Only a few approaches/papers are mentioned – apologies!
Outline

- Introduction
- Probabilistic formulation
- Centralized framework
- Distributed framework
- Two particular challenges
  - Consistent distributed estimation
  - Robust distributed perception
Bayesian Inference

- State transition model
  \[ x_{k+1} = f(x_k, u_k) + w_k \]
  \[ p(x_{k+1}|x_k, u_k) \]

- Observation model
  \[ z_k = h(x_k) + v_k \]
  \[ p(z_k|x_k) \]

- A posteriori joint pdf:
  \[ p(x_{0:k}|u_{0:k-1}, z_{1:k}) = \eta p(x_0) \prod_{i=1}^{k} p(x_i|x_{i-1}, u_{i-1}) p(z_i|x_i) \]

- A posteriori pdf (marginalizing out past states):
  \[ p(x_k|u_{0:k-1}, z_{1:k}) \]
Bayesian Inference

- Objective - Maximum a posteriori (MAP) estimation:

\[ x^*_k = \arg \max_{x_k} p(x_k|u_{0:k-1}, z_{1:k}) \]

- Common approaches include
  - EKF, EIF
  - UKF
  - Incremental smoothing (iSAM)
  - PF
Multi-Robot Perception, Localization and SLAM

Centralized
Inference Over What?

- What are the variables of interest each robot aims to estimate?
- Depends on the problem at hand!
  - May be the same variables for all robots (e.g. tracking)
  - Different variables (e.g. localization)
  - Combination of both (e.g. SLAM)
Collaborative Estimation

- Key capability:
  - By sharing information between robots and formulating multi-robot constraints, performance of individuals in the group can be greatly improved
  - Additional advantages, according to application (e.g. mapping - extend sensing horizon)
(Direct) Multi-Robot Observations

- Multi-robot measurement equation (between robots $r$ and $r'$)

$$ z = h \left( x^r_k, x^{r'}_k \right) + v $$

- Common observation types (depends on available sensors)
  - Range
  - Bearing
  - Bearing + range
  - Relative pose
    (relative position, relative orientation)
Example

- Experiment setup
  - 3 Pioneer robots
  - Wheel-odometry based dead reckoning
  - Relative pose measurements of each other

Multi-Robot Perception and SLAM

- So far – direct multi-robot observations: robots observe and make measurements \textit{wrt each other}
- Instead, how about \textit{mutually observing the environment}?
  - Environment is known (map is given) – localization problem
  - Environment is unknown – mapping, SLAM
  - In a a key role
Multi-Robot Perception and SLAM

- Robots operate in and make observations of unknown environments
- The corresponding multi-robot constraints describe different robots observing a mutual scene, *not necessarily* at the same time
- Measurement equations either involve additional random variables (e.g. landmarks) or robot states from different time instances

Two common formulations
- Pose-SLAM, Collaborative localization
- Full-SLAM, Structure from Motion (SfM)
Multi-Robot Perception and SLAM

- **Multi-robot Pose-SLAM**
  - Estimate relative motion from raw observations (match images)
  - Formulate multi-robot constraints, e.g.:
    \[ z = h(x^r_k, x^r_j) + v \]
  
  - Joint pdf:
    \[
    p(X|Z) \propto \prod_r \left[ p(x_0^r) \prod_i p(x_i^r | x_{i-1}^r, u_{i-1}) \right] \prod_{(r,r',i,j)} p(z^r_{i,j} | x_i^r, x_j^{r'})
    \]
    
    Multi-robot constraints

- Efficient MAP inference (sparsity, re-use calculations)
Multi-Robot Perception and SLAM

- **Multi-robot Full-SLAM**
  - Both robot states and the map are inferred
  - e.g.: robots $r$ and $r'$ observe the same landmark $l_j$:
    \[
    z_{k,j}^r = h(x_{k}^r, l_j) + v \\
    z_{i,j}^{r'} = h(x_{i}^{r'}, l_j) + v \\
    p(z_{k,j}^r | x_{k}^r, l_j) p(z_{i,j}^{r'} | x_{i}^{r'}, l_j) 
    \]
  - Joint pdf:
    \[
    p(X, L | Z) \propto \prod_i p(x_0^r) \prod_i p(x_i^r | x_{i-1}^r, u_i^r) \prod_{j \in \mathcal{M}_i} p(z_{i,j}^{r'} | x_{i}^{r'}, l_j) 
    \]
  - Similar approaches to recover MAP estimate

Image from “C-SAM: Multi-Robot SLAM using Square Root Information Smoothing”, ICRA 2008
Multi-Robot Perception and SLAM

Notes:

- All methods require multi-robot data association
- Common reference frame
- Thus far – centralized framework
Multi-Robot Perception, Localization and SLAM

Distributed

- Decentralized EKF, Decentralized EIF
- DDF
- Consensus
Cooperative Localization - Decentralized EKF

- Simultaneous localization of robots capable of sensing each other
  [Roumeliotis and Bekey ‘02]

- A single EKF estimator for the entire group

- Equations can be written in a decentralized form
  - Each robot maintains an augmented covariance matrix
  - Each robot calculates its own update

\[
P(t_k) = \begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{12}^T & P_{22} & P_{23} \\
P_{13}^T & P_{23}^T & P_{33}
\end{bmatrix}
\]

\[
K(t_k) = \begin{pmatrix}
K_1 \\
K_2 \\
K_3
\end{pmatrix}
\]

Decentralized EIF

- Designed for single-beacon cooperative (acoustic) navigation of multiple client underwater vehicles [Webster et al. ‘13]

- Ranges and state information from a single vehicle (server) are used to improve estimation of other vehicles (clients)

- Calculations in information form

- Algorithm yields identical results compared to a centralized version

Decentralized Data Fusion (DDF)

- DDF framework [Durrant-Whyte and Stevens ‘01]
  - Robots infer variables of interest based on local measurements and information communicated by nearby robots
  - No central computational unit
  - Numerous advantages over a centralized framework (scalability, robustness to failure)

Image from “Data Fusion in Decentralised Sensing Networks ”, Fusion, 2001
DDF – Calculations in Information Form

- Information vector and matrix  \( \eta \triangleq \Sigma^{-1} x \quad \Lambda \triangleq \Sigma^{-1} \)

- For simplicity, consider linear observation model:  \( z_i = H_i x + v_i \quad , \quad v_i \sim N(0, \Sigma_{v_i}) \)

- Prior information  \( p(x) = N(\hat{x}_0, \Sigma_0) \)

- Posterior given observations from other sensors/robots:
  \[
  p(x|Z) \propto p(x) \prod_i p(z_i|x) = N(\hat{x}, \Sigma)
  \]

- Inference can be efficiently performed in information form:
  \[
  \Lambda = \Lambda_0 + \sum_i H_i^T \Sigma_{v_i}^{-1} H_i \quad \eta = \eta_0 + \sum_i H_i^T \Sigma_{v_i}^{-1} z_i \quad \rightarrow \hat{x}, \Sigma
  \]

- Avoid double counting information via information down-dating (more soon)
DDF-SAM

- Extension of DDF to multi-robot smoothing and mapping (SAM) [Cunningham et al. ‘12, ’13]

- Each robot
  - Communicates only with its neighbors
  - Calculates and sends marginal distributions over variable of interest (e.g. landmarks)
  - Consistent estimation by explicitly avoiding double-counting information (discussed next)
Average Consensus Algorithms

- Distributed algorithms to integrate information across network
  [Olfati-Saber and Murray, IEEE TAC ’04]

- Have been applied to **distributed estimation** [Xiao et al. ’05]
  
  - Centralized: \( \theta_{\text{ML}} = (\sum_{i=1}^{N} \Lambda_i^{-1})^{-1} \sum_{i=1}^{N} \Lambda_i^{-1} x_i \)

  - Distributed (information form):
    
    Initialization: \( P_i(0) = \Lambda_i^{-1}, \ q_i(0) = \Lambda_i^{-1} x_i \)
    
    Each iteration: \[
    P_i(t+1) = P_i(t) + \sum_{j \in N_i(t)} a_{ij}(t)(P_j(t) - P_i(t)), \\
    q_i(t+1) = q_i(t) + \sum_{j \in N_i(t)} a_{ij}(t)(q_j(t) - q_i(t)),
    \]

Average Consensus Algorithms

- Have been recently extended for **distributed map merging** [Aragues et al. ‘12]
  - Exploit additive operations in information form
  - Robots execute in parallel consensus algorithm on each entry of the information matrix and information vector

Image from “Distributed Consensus on Robot Networks for Dynamically Merging Feature-Based Maps”, TRO 2012
Outline

- Introduction
- Probabilistic formulation
- Centralized
- Decentralized/Distributed
- Two particular challenges
  - Consistent distributed estimation
  - Robust distributed perception (data association)
Consistent Decentralized Estimation
Consistent Decentralized Estimation

- Intuitive example:
  - Consider 3 robots: A, B, and C, and a cyclic communication
  - Each robot estimates the variable x based on available data
  - Assume A transmits to B message $p(x|Z_A)$
  - B then passes to C the message $p(x|Z_B, Z_A)$
  - C sends to A the message $p(x|Z_C, Z_B, Z_A)$

- If A treats $p(x|Z_C, Z_B, Z_A)$ as independent wrt its local belief - it will double count information
Consistent Decentralized Estimation

- Problem becomes more complicated if additional variables are involved, as common in multi-robot perception & SLAM

- Key difficulty:
  - Robots share with each other distributions over landmarks or past poses
  - Need to track common information
  - Typically, the identity of involved variables is unknown ahead of time

Image from “Distributed vision-aided cooperative localization and navigation based on three-view geometry“, RAS 2012
Consistent Decentralized Estimation

- Main approaches include:

  - Maintain a bank of filters [Bahr et al. ‘09]
  
  - Conservative info fusion via covariance intersection [Julier et al. ‘97, Carrillo-Arce et al. ’13]
  
  - Calculate required correlation terms on demand [Indelman et al. ‘12]
  
  - Use information down-dating to prevent double counting [Durrant-Whyte et al. ‘01, Cunningham et al. ‘13]
Robust Distributed Perception
Robust Distributed Perception

- Data association problems in **distributed** robot systems
  - **Objective**: determine association between local measurements of the world (e.g. images) and measurements communicated by other robots
  - Extensively investigated by the computer vision community (e.g. RANSAC), typically assuming a centralized framework
  - In the distributed case, each robot has access to only partial information
Robust Distributed Perception

Image from “Consistent data association in multi-robot systems with limited communications“, RSS 2010
Robust Distributed Perception

- Main approaches include:
  - Distributed RANSAC with distributed averaging via consensus
    [Montijano et al. ‘11, ‘15]
  - Multi-robot data association within DDF-SAM framework
    [Cunningham et al. ‘12]
  - Robust inference - introduce latent variables modeling outlier/inlier correspondences
    [Latif et al. ‘12, Sunderhauf and Protzel ‘12, Lee et al. ‘13, Indelman et al. ‘14]
Robust Distributed Perception

- In particular challenging:
  - When information is obtained \textit{incrementally} (as the robots move and explore the environment)
  - In presence of \textit{perceptual aliasing} (e.g. two buildings/corridors that look alike)
  - Need to decide \textit{when} sufficient information has been accumulated for reliable data association
Summary

- High-level overview of distributed perception and estimation
  - **Centralized** framework
  - **Distributed** framework
  - **Consistent** distributed inference
  - **Robust** distributed perception & inference