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On Decision Making and Planning in the *Conservative* Information Space - Is the Concept Applicable to Active SLAM?

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1. Introduction

- Decision making under uncertainty - fundamental problem in autonomous systems and artificial intelligence
- Objective: find action(s) that minimizes an information-theoretic objective function (e.g. entropy)
- Decision making over **high-dim.** state spaces is **expensive**!
- Evaluating impact of a candidate action - $O(n^3)$ with $x \in \mathbb{R}^n$

2. Introduction – Motivating Example

- Active SLAM, belief space planning
  - State: $x_{0:k} \doteq \{ \ x_0 \ \cdots \ x_k \ \ L_k \ \}$
  - pdf: $p\left(x_{0:k}|z_{0:k}, u_{0:k-1}\right)$
- How to autonomously determine future action(s)?
- Involves reasoning, for different candidate actions, about $p\left(x_{0:k+L}|z_{0:k}, u_{0:k-1}, u_{k:k+L-1}, z_{k+1:k+L}\right)$

3. Concept

- Resort to **conservative** information fusion techniques for information-theoretic decision making
- Conservative information fusion approaches (e.g. [2]) - Allow to *consistently* fuse information from multiple correlated sources, *without* knowing the correlation
- **Key idea**: Reduce computational complexity in decision making by (appropriately) dropping correlations
- Extreme case - drop all correlations: $O(n^3) \Rightarrow O(n)$
- **Do we get the same performance??**

References

[1] "Towards Information-Theoretic Decision Making in a Conservative Information Space", V. Indelman, ACC, 2015.
[2] "A non-divergent Estimation Algorithm in the Presence of Unknown Correlations", S. Julier, J. Uhlmann, ACC, 1979
[3]" Sensor Selection in High-Dimensional Gaussian Trees with Nuisances", D. Levine, J. How, NIPS, 2013

4. Theorem – 1D Case

- Consider some two actions **a** and **b** with measurement models $z_a = h_a(x) + v_a$ and $z_b = h_b(x) + v_b$
- Theorem:

$$\left|I^{a+}\right| \leq \left|I^{b+}\right| \iff \left|I_c^{a+}\right| \leq \left|I_c^{b+}\right|$$

Action **a**

$$I^{a+} = I + H_a^T \Sigma_v^{-1} H_a$$

$$I_c^{a+} = I_c + H_a^T \Sigma_v^{-1} H_a$$

Action **b**

$$I^{b+} = I + H_b^T \Sigma_v^{-1} H_b$$

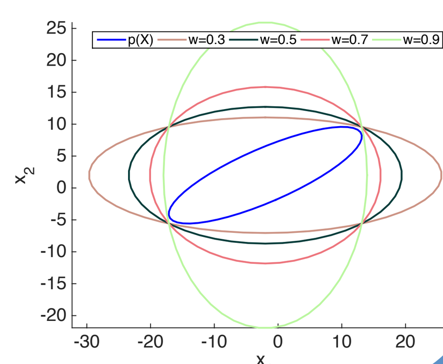
$$I_c^{b+} = I_c + H_b^T \Sigma_v^{-1} H_b$$

- In words: the impact of any two candidate actions has **the same trend** regardless if it is calculated based on the original or conservative information space
- Therefore: decision making can be done considering a conservative information space

5. High-Dimensional State Space

- Is the concept valid?
- Going to the extreme: appropriately **drop all correlation terms.**
- **Decoupled conservative pdf:**

$$p_c(X) = \eta \prod p_i^{w_i}(x_i) \qquad \forall x_i \in X \qquad \sum_i w_i = 1$$



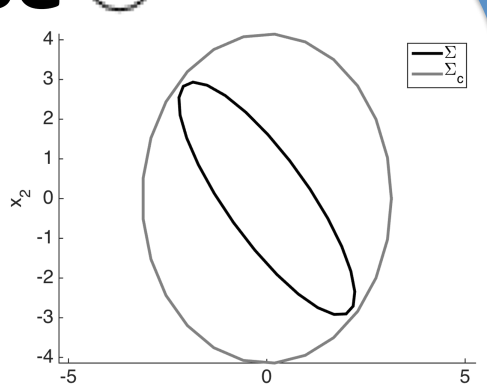
6. Concept is Valid (at Least) for

1. Observation models that include the same arbitrary states, possibly with different measurement noise covariance $z_i = h(X') + v_i \quad X' \subseteq X$
2. Unary observation models, possibly involving different states $z_i = h_i(x_i) + v_i \quad x_i \in X$
3. Pairwise observation models with the same uncorrelated state x: $z_i = h_i(x_i, x) + v_i \quad x, x_i \in X$

7. 2D Example for Case 2

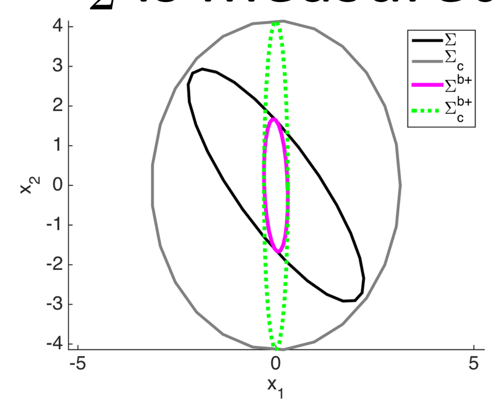
a priori covariance:

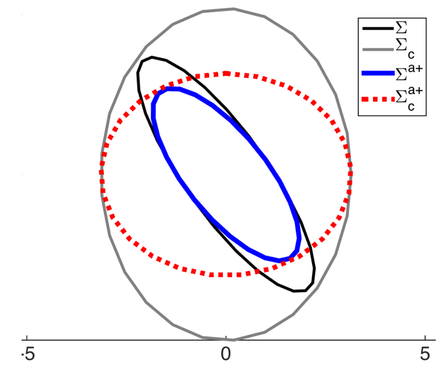
$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix} \quad \Sigma_c = \begin{bmatrix} \Sigma_{c,11} & 0 \\ 0 & \Sigma_{c,22} \end{bmatrix}$$

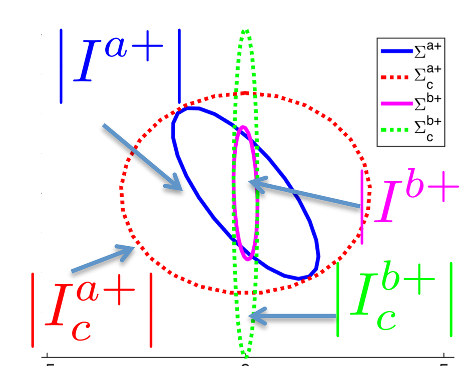


Action **a**: x_2 is measured

Action **b**: x_1 is measured







Do **not** need correlations to decide which action is better!

8. Applicable to Active SLAM? – Key Questions

The following aspects are suggested for future investigation:

- More general observation models, e.g. binary models involving robot poses and landmarks (addressed in [1] for specific cases).
- Examine concept while incorporating control terms.
- Active focused inference [3] (objective function involves only part of the variables).
- Multi-step planning horizon to support planning in the *conservative* belief space