Topological Multi-Robot Belief Space Planning in Unknown Environments

[1]Technion - Israel Institute of Technology, Israel

Andrej Kitanov[1] and Vadim Indelman[1]

1. Introduction

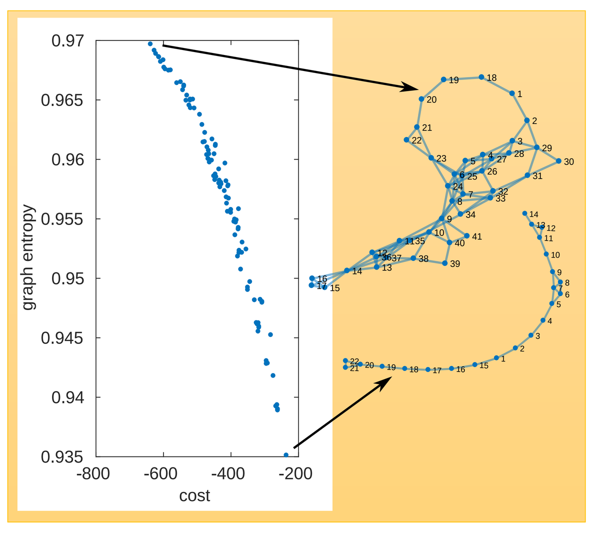
- Belief Space Planning (BSP) determines optimal control over the belief space with respect to a given objective, e.g. minimize state uncertainty
- instantiation of a Partially-Observable Markov Decision Process (POMDP)
- finding optimal solution to POMDP in the most general form is computationally intractable
- multi-robot (MR) BSP in unknown environments is such a case because of high dimensionality of the state space and exponential growth of the number of objective function evaluations with the number of robots

2. Contribution

- we introduce a novel concept, **topological belief space planning (tBSP)**, that uses topological properties of the underlying factor graph representation of future posterior beliefs to direct a search for an optimal solution
- no explicit inference required in optimization nor partial state covariance recovery
- enabling planning in high dimensional state spaces

3. Concept

- **Key observations**:
  - topological properties of factor graphs dominantly determine the estimation accuracy
  - topological space is often less dimensional than the embedded state space
- Computation of topological metric is much faster then explicit evaluation of an objective function



4. Technical approach

We consider R robots, each choosing between finite number of discrete actions/paths in each planning session and pose SLAM framework.

$X_k^r = \{x_0^r, x_1^r, \ldots, x_k^r\}$ robot's state (trajectory)

$\mathcal{H}_k^r \doteq \{\mathcal{Z}_{0:k}^r, \mathcal{U}_{0:k-1}^r\}$ observations and controls up to time k

Let a robot r select a candidate path

$$\{x_{k+1}^r, \ldots, x_{k+L}^r\} \Rightarrow$$

$$b[\mathcal{P}^r] \doteq \mathbb{P}(X_k^r, x_{k+1}^r, \ldots, x_{k+L}^r | \mathcal{H}_k^r, U(\mathcal{P}^r), Z(\mathcal{P}^r))$$

belief evolution over the future path of a single robot r

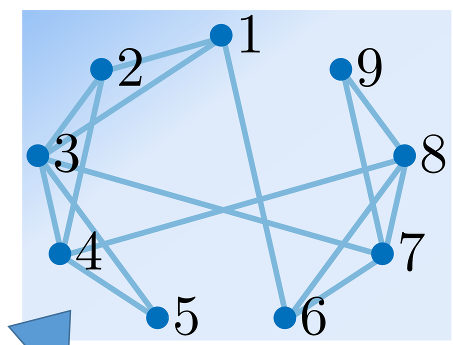
MR inference

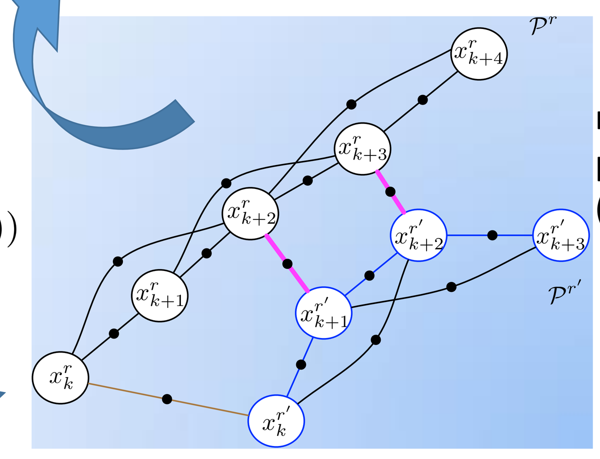
$X_k \doteq \{X_k^r\}_{r=1}^R$ multi-robot (joint) state at time k

$\mathcal{P} \doteq \{\mathcal{P}^r\}_{r=1}^R$ **action = single variation of robots' candidate paths**

$$b[\mathcal{P}] = \mathbb{P}(X_k|\mathcal{H}_k) \prod_{r=1}^{R} \left[\prod_{l=1}^{L(\mathcal{P}^r)} \mathbb{P}(x_{k+l}^r | x_{k+l-1}^r, u_{k+l-1}^r) \right.$$
$$\left. \cdot \mathbb{P}(Z_{k+l}^r | X_{k+l}^r) \prod_{\{i,j\}} \mathbb{P}(z_{i,j}^{r,r'} | x_i^r, x_j^{r'}) \right]$$

multi-robot pose SLAM posterior belief





Two graph signatures considered in tBSP:

$$s(G) = H_{VN}(G) = -\sum_{i=1}^{|\Gamma|} \frac{\hat{\lambda}_i}{|\Gamma|} \ln \frac{\hat{\lambda}_i}{|\Gamma|}$$
$$\approx 1 - \frac{1}{|\Gamma|} - \frac{1}{|\Gamma|^2} \sum_{(i,j)\in E} \frac{1}{d(i)d(j)}$$

Von Neumann entropy of G (VN) which is further simplified with a function of graph node degrees *d*

$$s(G) = \frac{3}{2}\tau(G) + \frac{|\Gamma|}{2}[\ln|\Omega_w| - \ln(2\pi e)]$$

signature based on the number of spanning trees of G (ST)

Optimization problem

$$J(\mathcal{U}) = \frac{n}{2}\ln(2\pi e) + \frac{1}{2}\ln|\Sigma(X_{k+L})|$$

$$\mathcal{U}^\star = \arg\min_{\mathcal{U}} J(\mathcal{U})$$

find control $\mathcal{U}^\star$ to improve estimation accuracy of the joint state X_{k+L} by minimizing the global entropy at the end of planning horizon of length L

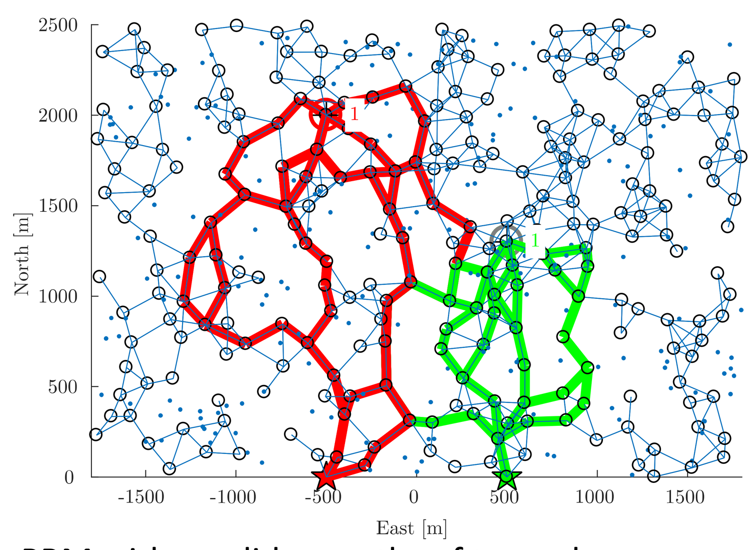
Approximate solution based only on topological properties of FG

$$\hat{\mathcal{U}}^\star = \arg\max_{\mathcal{U}} s[G(\mathcal{U})]$$

maximize function of the graph topology (a proxy for the true objective)

Algorithm 1 Topological BSP
Require: set of factor graphs FG
Ensure: approximate solution to the BSP, $\hat{\mathcal{U}}$
1: represent each FG with a topological graph G
2: determine S_G, set of graph signatures of G
3: rank graphs according to their signatures
4: $U = \{$ top ranked candidates in $S_G\}$
5: $\hat{\mathcal{U}} = \arg\min J(U)$

5. Results



PRM with candidate paths of two robots (red and green) in a single planning session (S)

- tBSP applied to multi-robot collaborative active SLAM problem:
  - leads to significantly improved relative error convergence speed w.r.t. exhaustive (EX) undirected evaluation of candidate actions (Fig. 1)
  - not sensitive to initialization, as local methods are, e.g. announced path (AP) approach and with much less number of objective function evaluations until convergence (Table 1)

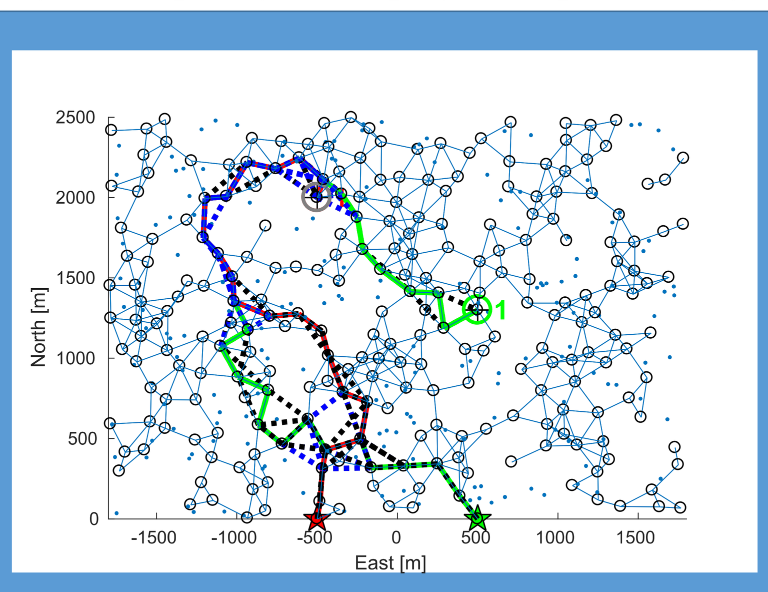
R = 2, |A| = 10, 25 planning sessions

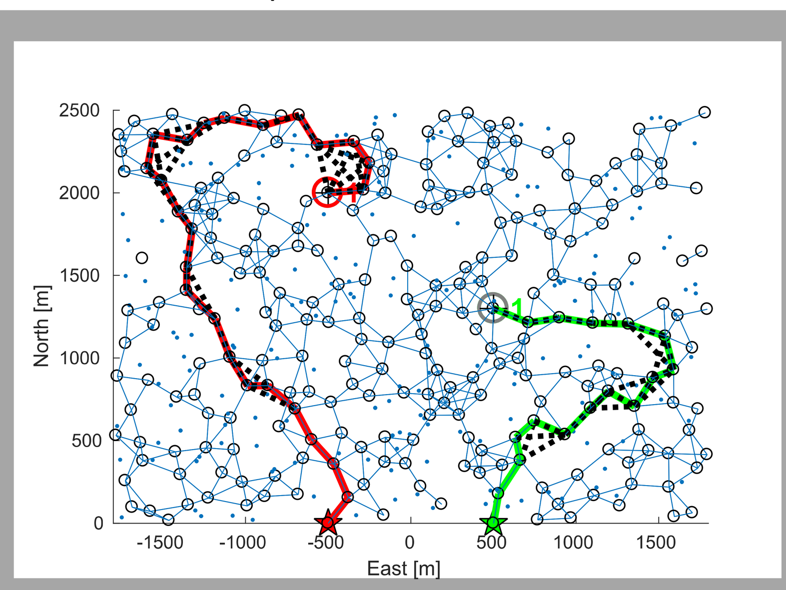
	VN	ST	AP	EX
mean	**2.12**	**1.52**	41.92	50
min	**1**	**1**	24	1
max	**12**	**5**	60	100

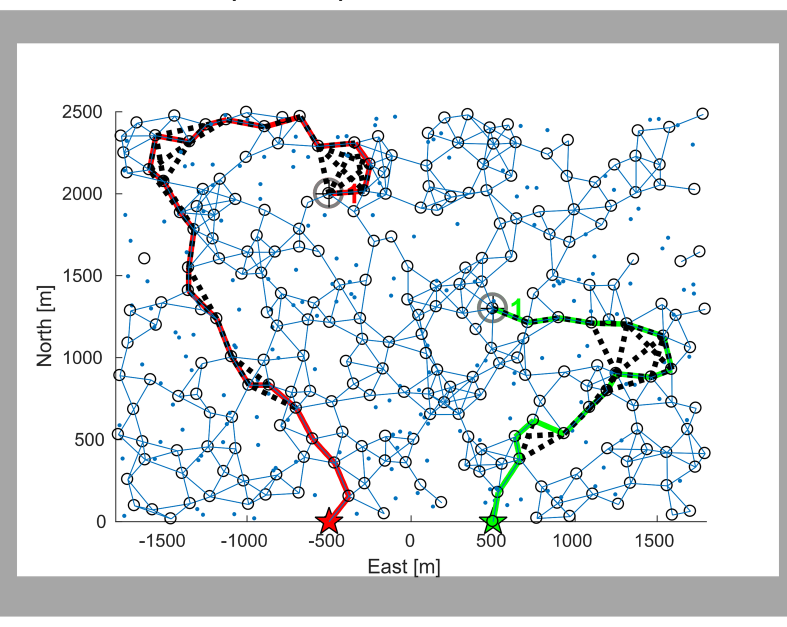
Table. 1. number of objective function evaluations until convergence

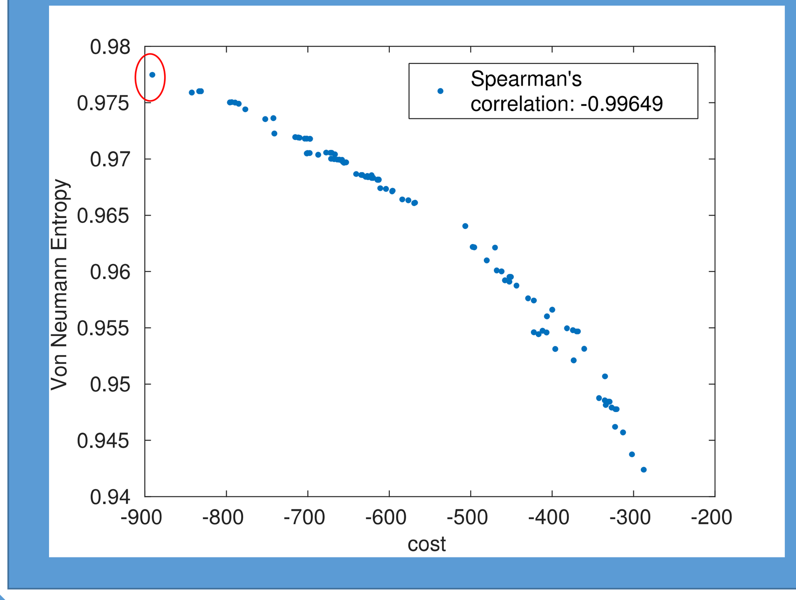
S2: chosen paths

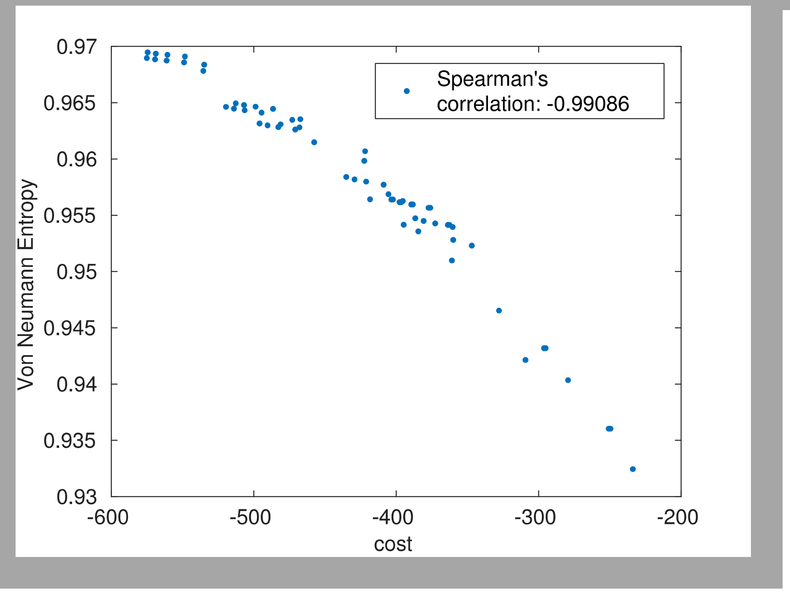
S2: optimal paths











S1: optimal solution found after one sample

S2: relative error 2.5% after the first sample

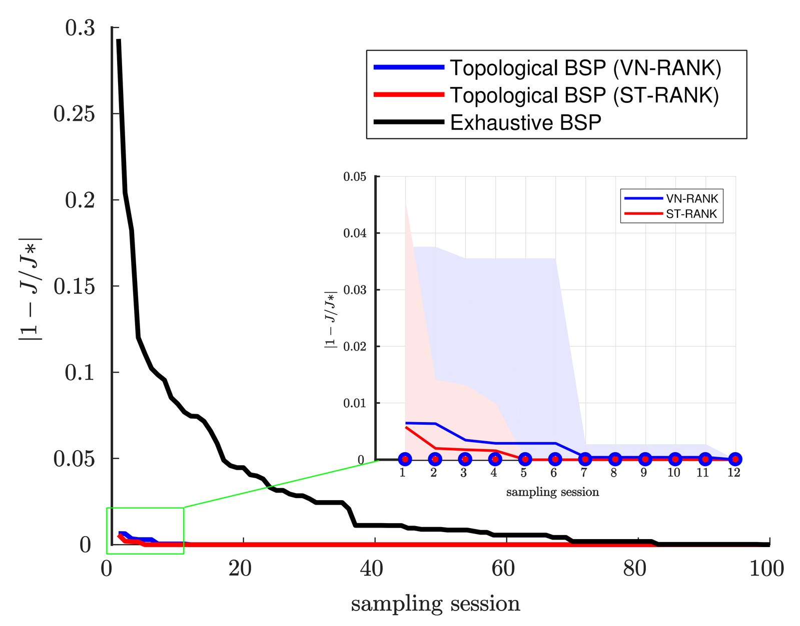


Fig. 1. relative error and its 95 % confidence region

6. Conclusions

- a novel concept introduced, topological belief space planning to tackle computational complexity aspects of belief space planning (BSP) in high dimensional state spaces
- this general concept can be applied in multi-robot BSP to overcome main drawbacks of the state-of-the art exhaustive search and announced paths approaches