Focus on What Matters: Topological Aspects in Information-Theoretic Belief Space Planning

Andrej Kitanov and Vadim Indelman

I. Motivation and Problem formulation

Decision making under uncertainty is one of the fundamental problems in robotics and artificial intelligence and appears in any system that accounts for uncertainty during planning before task execution to improve the quality of computed plans. It can be formulated as a solution to Belief Space Planning (BSP) or stochastic control problem where optimal non-myopic control action $\mathcal{U}^{\star}_{k:k+L-1} = \arg\min_{\mathcal{U}} J_k(\mathcal{U})$ over the prediction horizon L at planning time k needs to be found with respect to a given objective function J_k related to the design task

$$J_k(\mathcal{U}) = \mathop{\mathbb{E}}_{\mathcal{Z}} \left\{ \sum_{l=0}^{L-1} c_l \left[b(X_{k+l}), \mathcal{U}_{k+l} \right] + c_L \left[b(X_{k+L}) \right] \right\}. \quad (1)$$

As can be seen, the BSP problem is an instance of Partially Observable Markov Decision Process (POMDP) because the state of the system X is not directly observable by the controller, but through a set of stochastic measurements $\mathcal{Z}$ from which the future posterior beliefs $b(X_{k+l})$ must be inferred upon optimization. Expectation in (1) therefore is taken with respect to future (unknown) observations $\mathcal{Z}_{k+1:k+L}$. The objective function in its general form reflects the design task through immediate cost functions c_l, which depend on the belief evolution $b(X_{k+l})$ and a control action $\mathcal{U}_{k+l}$ applied at time t_{k+l}, and through a final cost c_L. For example, the cost functions can be chosen to minimize trajectory uncertainty, time or energy required to reach a goal, state uncertainty of variables of interest at some specific time instant etc. In information-theoretic BSP, one is interested in state uncertainty minimization which can be expressed through some information-theoretic cost $c(.)$. This type of cost functions is usually computationally the most expensive to optimize in many BSP problems in robotics, especially in high dimensional state spaces when finding globally optimal solutions to the POMDP problem becomes computationally intractable. Existing BSP solutions for continuous action spaces calculate only locally optimal trajectories and control policies using trajectory optimization methods (e.g. dynamic programming, gradient descent), starting from a given nominal path. In discrete action spaces, state of the art BSP approaches prior to evaluating objective function calculate the marginal posterior covariance matrix for each candidate action, which involves belief propagation on the planning horizon for each candidate action and additionally, in the focused BSP, where a subset of system states is of interest in optimization, expensive computation of a Schur complement. Moreover, state of the art approaches typically perform these calculations from scratch, or in the best scenario, reuse some calculations by a one-time computation of marginal covariances of involved variables, the complexity of which still depends on state-dimensionality and system sparsity.

We developed a new method that performs planning in continuous state and discrete action space, *topological belief space planning* (`t-BSP`), which uses topological properties of the underlying factor graph representation of future posterior beliefs to direct the search for a globally optimal solution [1]. Our approach does not require explicit inference in optimization, nor partial state covariance recovery for all actions compared to existing solutions. Recently, we established error bounds of `t-BSP` to provide global optimality guarantees or uncertainty margin of its solution [2].

We formulate the optimization problem in a topological space where it is more easily solved, yet its solution is close to a solution of the original BSP optimization problem (1). Our method uses the following key ideas: i) topological properties of factor graphs dominantly determine actions ranking by the information-theoretic cost; ii) topological space is often less dimensional then the embedded state space; and iii) computation of a topological metric can be done much faster than explicit evaluation of an objective function. In the design of topological metrics for BSP, we were motivated by the results of [3] in the context of measurement selection and pose-graph pruning problems in SLAM that characterizes the impact of the graph topology of SLAM (described by weighted tree-connectivity metric) on the estimation reliability. We extended these results to BSP problem in [1] and introduced new topological metrics $s(\mathcal{U})$ to approximate the solution to information-theoretic BSP. In particular, one `t-BSP` metric we introduced for minimizing the joint state Shannon entropy is based on the number of spanning trees of the topological posterior factor graph representation s_{ST} and the other on its von Neumann entropy approximated by a function of graph node degrees $\hat{s}_{VN}$. We derived error bounds of `t-BSP` which can be calculated online, i.e. with a small additional cost to the topological metric. One can then resort to `t-BSP` to drastically reduce computational cost while carefully monitoring a conservative estimate on the sacrifice in performance, that would be provided by these bounds, or to guarantee global optimality of `t-BSP` by evaluating generally a much smaller number of candidate actions.

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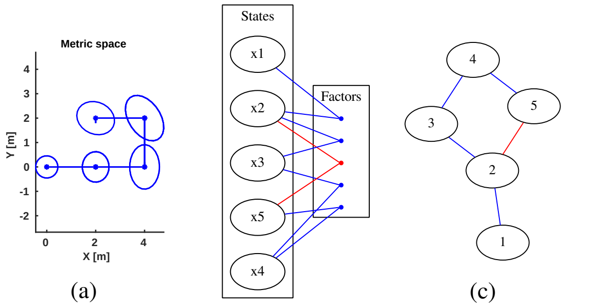
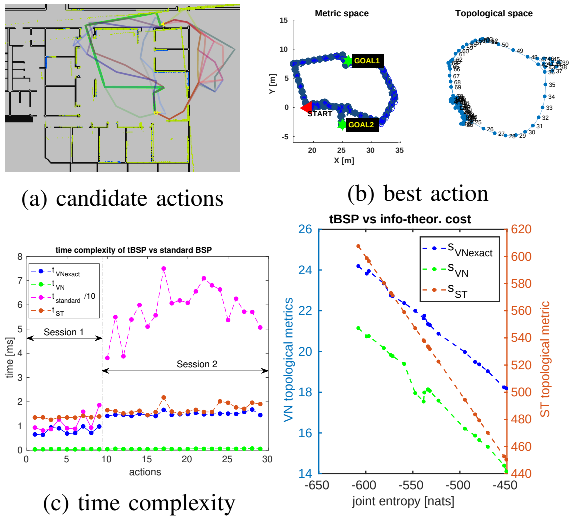
The authors are with the Department of Aerospace Engineering, Technion - Israel Institute of Technology, Haifa 32000, Israel. {`andrej`, `vadim.indelman`}`@technion.ac.il`.



(a) (b) (c)

Fig. 1: Each candidate action corresponds to posterior belief $b[X_{k+L}]$, i.e. trajectory uncertainty (a) which can be represented with a factor graph (b) and assigned a topology. In the case of active pose SLAM, topology can be defined with a simple undirected graph $G = (V, E)$ such that graph nodes V represent robot's poses and edges E pose constraints between them (c). Topological BSP aims to determine a graph invariant topological metric $s : G \to \mathbb{R}_0^+$ which is highly correlated with the information-theoretic cost and maintains action consistent decision making.



(a) candidate actions

(b) best action

(c) time complexity

(d) actions consistency

Fig. 2: Second planning session in Gazebo/ROS simulation of active pose SLAM after which both exploration and exploitation actions are available (a). Topological BSP is able to determine the least uncertain path/action, corresponding to a big loop closure (b), and requires much less computation time then standard BSP. s_{ST} and s_{VN} (exact and approximated) are highly correlated with the joint entropy.

II. Decision making by t-BSP in the context of active pose SLAM

To show the main concept of t-BSP we study an active pose SLAM problem and minimizing path uncertainty (quantified by joint entropy). Extension to a feature-based SLAM and map entropy is possible under the proposed framework as long as the measurements can be expressed in the form of pairwise relations between states. This is quite common since robot's landmark measurements are often given relative to the robot's pose, e.g. as range or bearing measurements.

Let the belief $b[X_k] = \mathbb{P}(X_k|\mathcal{H}_k)$ represent the posterior probability density function (pdf) at planning time t_k over states of interest X_k of the robot. In the pose SLAM framework states of interest are robot's current and past poses, i.e. $X_k = \{x_0, x_1, \ldots, x_k\}$. History $\mathcal{H}_k \doteq \{\mathcal{Z}_{1:k}, \mathcal{U}_{0:k-1}\}$ contains all observations $\mathcal{Z}_{1:k}$ and controls $\mathcal{U}_{0:k-1}$ by time t_k. Given a set of candidate control actions (e.g. see Fig. 2a), we predict which future observations will be available to the robot. The pair of action and observations set determines a posterior belief $\mathbb{P}(X_{k+L}|\mathcal{H}_{k+L})$ whose different representations are shown in Fig. 1. In t-BSP we need to maintain only topological graphs $G(\mathcal{U})$ from the set of input factor graphs and, ideally, find an appropriate graph invariant metric to keep the same action ranking as would be obtained when evaluating the objective (1) exactly. The best control action obtained by solving the decision making problem using either of the topological metrics $s \in \{s_{ST}, \hat{s}_{VN}\}$ is given by $\hat{\mathcal{U}} = \arg\max_{\mathcal{U}} s[G(\mathcal{U})]$.

While the proposed topological metrics exhibit strong correlation with the information theoretic cost [1], in the general case, the obtained best action $\hat{\mathcal{U}}$ may be somewhat different than the optimal action $\mathcal{U}^\star$ from (1), leading to some error in the quality of solution. In [2] we provide bounds on this error $|J_k(\hat{\mathcal{U}}) - J_k(\mathcal{U}^\star)|$ which can be efficiently calculated with a small overhead to the topological metric.

One of the biggest advantages of using the topological metric $\hat{s}_{VN}$ in BSP, being a function of graph node degrees, is its possible incremental update: as the robot explores the environment and acquires measurements, only a small number of graph node degrees should be updated. This substantially improves computational complexity of the t-BSP algorithm to nearly $O(1)$ (in the worst case $O(k)$) since the number of measurements between two planning sessions is limited and the maximal node degree is usually bounded given that SLAM graphs are sparse. On the other hand, the topological metric s_{ST} is more accurate and preserves better action consistency (Fig. 2d). Nevertheless, realistic active SLAM simulations demonstrate that t-BSP based on both metrics outperforms standard BSP with respect to time performance (Fig. 2c). They also maintain very good action consistency which proves that topology of beliefs is indeed an important factor in BSP.

References

[1] A. Kitanov and V. Indelman, "Topological multi-robot belief space planning in unknown environments," in *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2018.

[2] ——, "Topological information-theoretic belief space planning with optimality guarantees," *ArXiv*, March 2019. [Online]. Available: https://arxiv.org/abs/1903.00927

[3] K. Khosoussi, M. Giamou, G. S. Sukhatme, S. Huang, G. Dissanayake, and J. P. How, "Reliable graph topologies for SLAM," *Intl. J. of Robotics Research*, 2018.