# Topological aspects in Information-Theoretic Belief Space Planning

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## Belief Space Planning (BSP)

- BSP determines optimal non-myopic control action $\mathcal{U}^{\star}_{k:k+L-1} = \arg\min_{\mathcal{U}} J_k(\mathcal{U})$ over the prediction horizon $L$ at planning time $k$ with respect to a given objective function $J_k$ related to the design task

$$J_k(\mathcal{U}) = \mathop{\mathbb{E}}_{\mathcal{Z}} \left\{ \sum_{l=0}^{L-1} c_l \left[ b(X_{k+l}), \mathcal{U}_{k+l} \right] + c_L \left[ b(X_{k+L}) \right] \right\}$$

$b(X_{k+l})$ future posterior belief at time $t_{k+l}$ based on observations $\mathcal{Z}_{k+l} \subseteq \mathcal{Z}$ until that time

$\mathcal{U}_{k+l}$ control applied at time $t_{k+l}$

- instantiation of a Partially-Observable Markov Decision Process (POMDP)
- finding optimal solution to POMDP in the most general form is computationally intractable
- in information-theoretic BSP, $J_k$ is a function of state uncertainty

## Topological BSP (t-BSP)

- we introduce a novel concept, topological belief space planning (t-BSP), that uses topological properties of the underlying factor graph representation of future posterior beliefs to direct a search for an optimal BSP solution
- topological space is often less dimensional then the embedded state space
- we look for topological representation of the belief and a metric that is highly correlated to $J_k$ but much easier to calculate
- no explicit inference required in optimization nor partial state covariance recovery
- enables planning in high dimensional state spaces

Key insight: Any topological representation $\mathcal{T}$ and derived metric which preserves action ordering (the best action) can be used to solve BSP. Exact value of the objective function is not neccessary.

## Topology of a belief in active SLAM

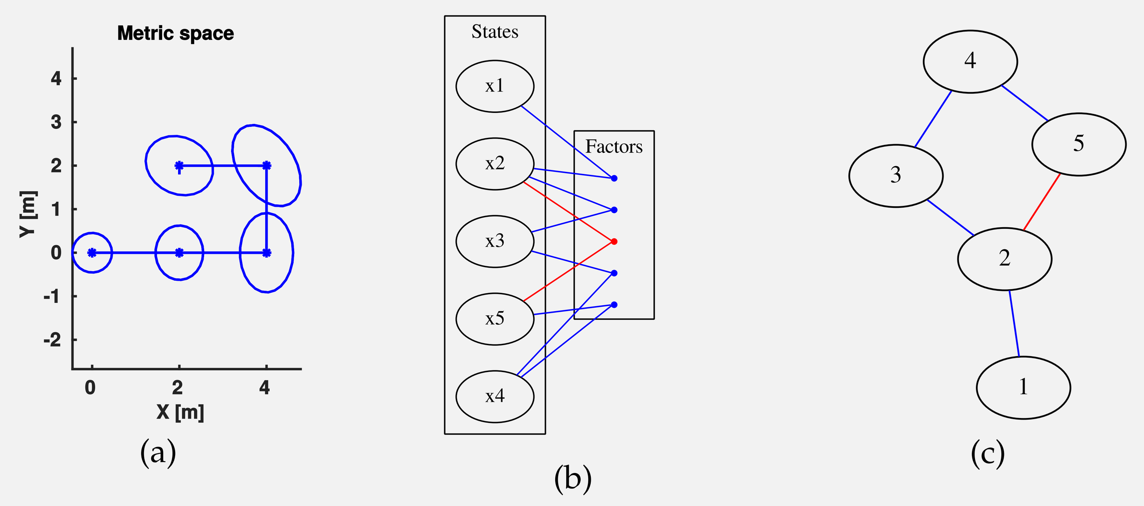
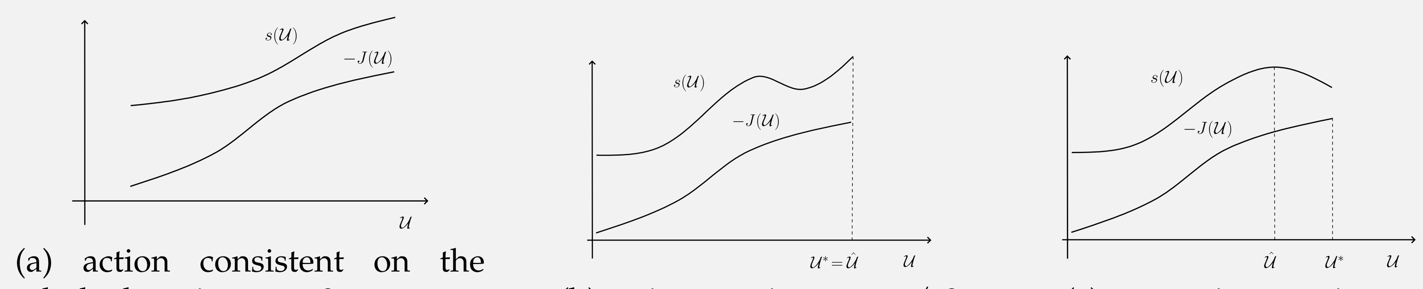



Figure 1: Each candidate action corresponds to posterior belief, i.e. trajectory uncertainty (a) which can be represented with a factor graph (b) and assigned a topology. In the case of active pose SLAM, topology can be defined with a simple undirected graph such that graph nodes represent robot's poses, and edges pose constraints between them (c). Topological BSP aims to determine a graph invariant topological metric which is highly correlated with the information-theoretic cost and maintains action consistent decision making.

Topological metric

$s : \mathcal{T} \to \mathbb{R}$

## Action consistency and t-BSP error



(a) action consistent on the whole domain, $\gamma_f = 0$ (b) action consistent, $\gamma_f \neq 0$ (c) non action consistent

Figure 2: Different topological metrics and their influence on decision making.

Decision making via t-BSP

$$\hat{\mathcal{U}} = \arg\max_{\mathcal{U}} s(\mathcal{U}).$$

The error of t-BSP is

$$\epsilon(J, s) \doteq |J(\mathcal{U}^{\star}) - f[s(\hat{\mathcal{U}})]|$$

where $\hat{\mathcal{U}} = \arg\max_{\mathcal{U}} s(\mathcal{U})$ and $f$ is a monotonic function such that $f[s(\hat{\mathcal{U}})] = J(\hat{\mathcal{U}})$ and $f = \arg\min_f \gamma_f$, where $\gamma_f = \max_{\mathcal{U}} |J(\mathcal{U}) - f[s(\mathcal{U})]|$.

## Information-theoretic objective

### Minimizing Shannon joint entropy of the posterior Gaussian belief

$$J(\mathcal{U}) = N/2 \ln(2\pi e) + 1/2 \ln|\Sigma(X_{k+L})|,$$

where $\Sigma(X_{k+L})$ denotes the estimated covariance of the robot's belief $b[X_{k+L}]$, and $N$ dimension of the state $X_{k+L}$.

### Proposed topological metrics

- Von Neuman graph entropy and its approximation by a function of node degrees $d$(see [1])⇒ **faster to calculate, effectively $\mathbf{O}(1)$, worst $\mathbf{O}(n)$**

$$s_{VN}(G) = -\sum_{i=1}^{n} \hat{\lambda}_i / 2 \ln(\hat{\lambda}_i / 2) \approx$$

$$\hat{s}_{VN}(G) = n/2 \ln 2 - 1/2 \sum_{(i,j) \in E} 1/[d(i)d(j)]$$

- Function of the number of spanning trees $t(G)$ of a graph motivated by [2]⇒ **more accurate, computational complexity depends on the graph sparsity and the number of states**

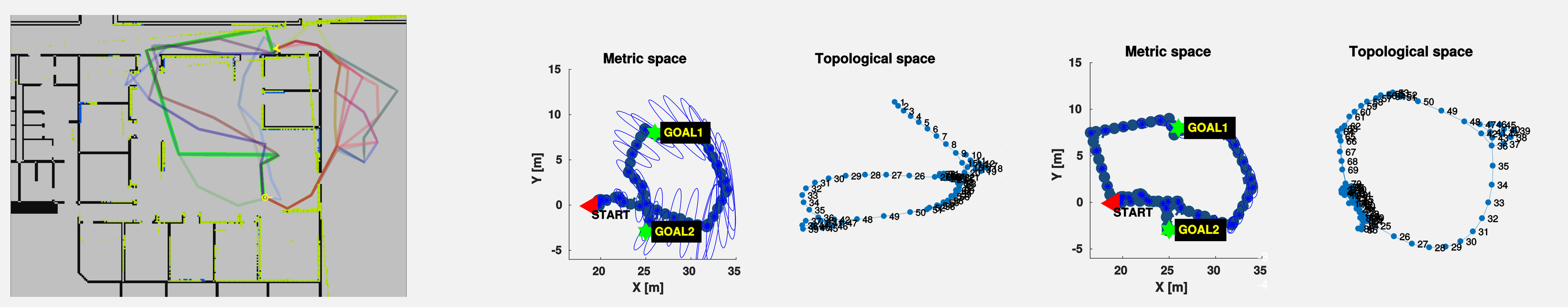
$$s_{ST}(G) = 3/2 \ln t(G) + n/2 [\ln|\Omega_{vij}| - \ln(2\pi e)^{\kappa}]$$

**t-BSP error $\epsilon(J, s)$ can be calculated from topological metric $s_{ST}$ and prior maximum likelihood estimate**[3]

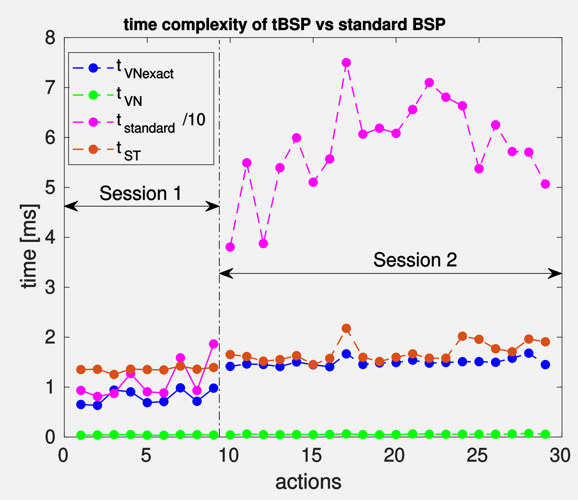
$$\epsilon(J, s) \leqslant \Delta J_{max}, \text{ where } \Delta J_{max} = \mathcal{UB}[J(\hat{\mathcal{U}})] - \min_{\mathcal{U}} \mathcal{LB}[J(\mathcal{U})], \text{ and}$$

$$\mathcal{UB}[J(\hat{\mathcal{U}})] = -s_{ST}(\hat{\mathcal{U}}) \text{ and } \mathcal{LB}[J(\mathcal{U})] = -s_{ST}(\mathcal{U}) + 1/2(\tau(\mathcal{U}) - \prod_{i=2}^{n} [d_{\mathcal{U}}(i) + \Psi(\mathcal{U})])$$

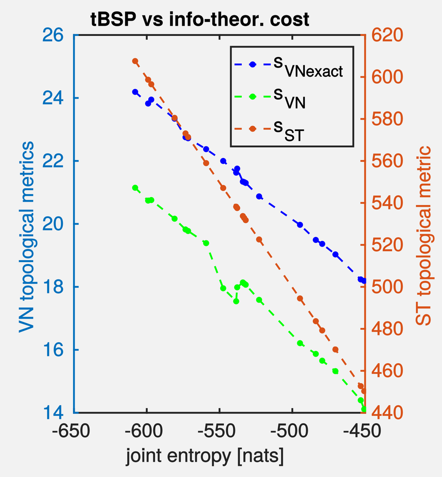
## Results




(a) candidate actions (b) trajectory uncertainty and topology of the worst (left) and best (right) action




(c) time complexity




(d) actions consistency

Figure 3: Second planning session in Gazebo/ROS simulation of active pose SLAM after which both exploration and exploitation actions are available (a). Topological BSP is able to determine the least uncertain path/action, corresponding to a big loop closure (b), and requires much less computation time then standard BSP (c). $s_{ST}$ and $s_{VN}$ (exact and approximated) are highly correlated with the joint entropy (d).

## Conclusion

- topological properties of factor graphs dominantly determine estimation accuracy and enable efficient information-theoretic BSP
- decision making under some conditions (e.g. linear observation models, large diversity among candidate actions, certain noise properties) is action consistent
- in other cases, t-BSP enables eliminating sub-optimal actions

## References

[1] Andrej Kitanov and Vadim Indelman. Topological multi-robot belief space planning in unknown environments. In *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2018.

[2] Kasra Khosoussi, Matthew Giamou, Gaurav S Sukhatme, Shoudong Huang, Gamini Dissanayake, and Jonathan P How. Reliable graph topologies for SLAM. *Intl. J. of Robotics Research*, 2018.

[3] Andrej Kitanov and Vadim Indelman. Topological information-theoretic belief space planning with optimality guarantees. *ArXiv*, March 2019.

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