Computationally Efficient Model-Based Decision Making in High-Dimensional State Spaces

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1. Introduction

- Decision making under **uncertainty** over high-dim. state spaces
- Belief space planning, active SLAM/sensing, graph sparsification
- **Objective:** Find action(s) that minimizes an information-theoretic objective function (e.g. entropy) of future belief
  - ***Unfocused* case:** future belief over all states
  - ***Focused* case:** future belief over only subset of states
- Standard approach to evaluate impact of a single action:
  - calculates determinant of nxn matrix, $O(n^3)$, with $X_k \in \mathbf{R}^n$
  - or, in square-root info form, updates posteriors
- Decision making over high-dim. state spaces is **Expensive**!
- Previously solved for specific cases via conservative info. space [1]

2. Problem Formulation

- Gaussian distribution: $p(X_k \mid Z_{0:k}, u_{0:k-1}) = N(\hat{X}_k, \Lambda_k^{-1})$
- Given action $a = u_{k:k+L-1}$ and new observations, future belief is:

$$b[X_{k+L}] = \eta p(X_k \mid Z_{0:k}, u_{0:k-1}) \prod_{l=k+1}^{k+L} p(x_l \mid x_{l-1}, u_{l-1}) p(Z_l \mid X_l^o)$$

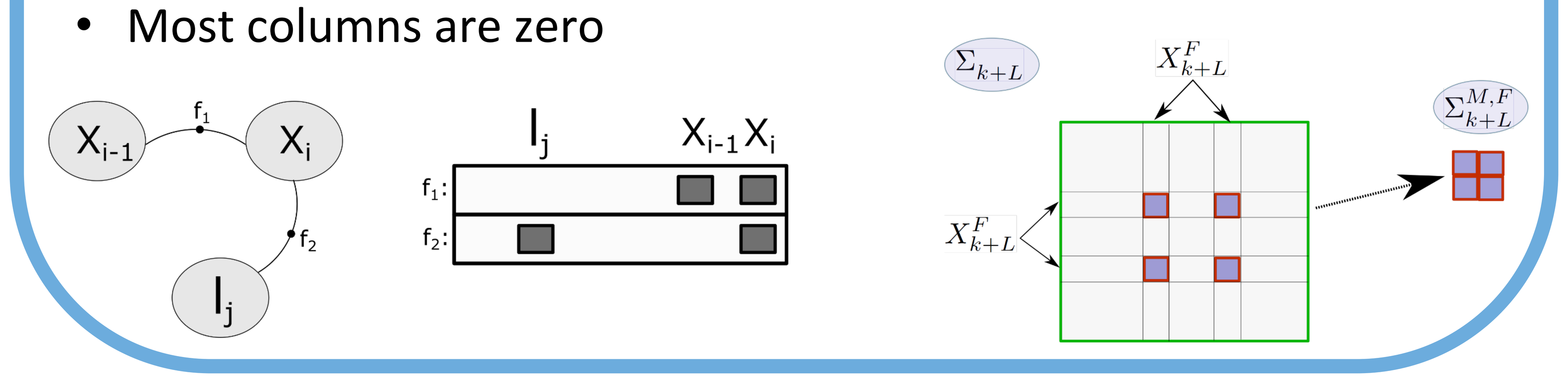
(New terms)

- A posteriori information matrix: $\Lambda_{k+L} = \Lambda_k + A^T A$ (Jacobian of action a)
- Final objective function:

Unfocused: $J_H(a) = H(b[X_{k+L}]) = \frac{n}{2} \cdot (1 + \ln(2\pi)) - \frac{1}{2} \ln|\Lambda_{k+L}|$

Focused: $J_H^F(a) = H(X_{k+L}^F) = \frac{n_F}{2} \cdot (1 + \ln(2\pi)) + \frac{1}{2} \ln|\Sigma_{k+L}^{M,F}|$, $X_{k+L}^F \subseteq X^{k+L}$

- Jacobian $A \in \mathbf{R}^{m \times n}$ is sparse • n is big, m is small
- Most columns are zero



3. Contributions

- Information Gain (IG) and Matrix Determinant Lemma
- Re-use of Calculation
- Focused Decision Making

3a. IG, Matrix Determinant Lemma

- **Key idea:** avoid calculating determinants of large matrices
- IG (suggested also in [4]) keeps same trend of action impacts:

$$J_{IG}(a) = H(b[X_k]) - H(b[X_{k+L}]), a^* = \arg\max_{a \in A} J_{IG}(a)$$

- Generalized matrix determinant lemma [2]:

$$|\Lambda_k + A^T \cdot A| = |\Lambda_k| \cdot |I_m + A \cdot \Sigma_k \cdot A^T|, \quad \Sigma_k \equiv \Lambda_k^{-1}$$

- Final Cost: $J_{IG}(a) = \frac{1}{2} \ln|I_m + A_C \cdot \Sigma_k^{M,C} \cdot A_C^T|$
  - C is set of involved variables in new terms
- Given $\Sigma_k^{M,C}$, complexity depends on m only

3b. Re-use of Calculation

- Re-use expensive computation of $\Sigma_k^{M,C}$ for all candidate actions
- C_{All} - is set of all involved variables in all actions
- Compute prior marginal covariance $\Sigma_k^{M,C_{All}}$
- Using it, calculate $J_{IG}(a)$ for each action
- One-time calculation depends on n
- Per-candidate calculation depends only on m

3c. Focused Decision Making

- Same ideas (IG, re-use) are applicable for *focused* scenario
- Matrix Partitions:

$$\Sigma_k = \begin{bmatrix} \Sigma_k^{M,R} & \Sigma_k^{M,RF} \\ (\Sigma_k^{M,RF})^T & \Sigma_k^{M,F} \end{bmatrix}, \Lambda_k = \begin{bmatrix} \Lambda_k^R & \Lambda_k^{R,F} \\ (\Lambda_k^{R,F})^T & \Lambda_k^F \end{bmatrix}$$

- **Schur** complement:

$$\Lambda_k^{M,F} \equiv (\Sigma_k^{M,F})^{-1} = \Lambda_k^F - (\Lambda_k^{RF})^T \cdot (\Lambda_k^R)^{-1} \cdot \Lambda_k^{RF}$$

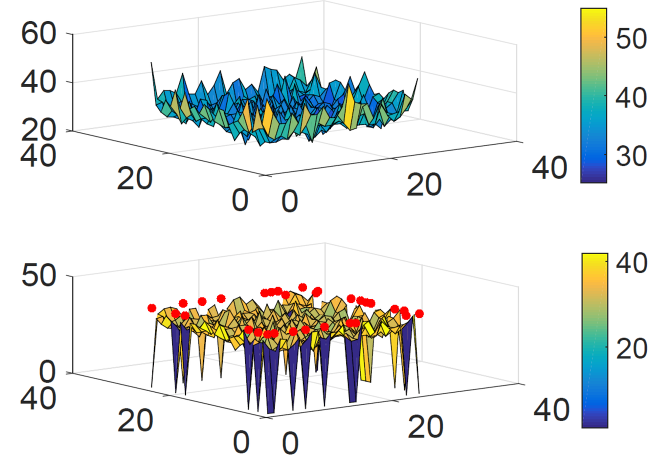
- Determinant Lemma of **Schur** complement [3]: $|\Lambda_k| = |\Lambda_k^{M,F}| \cdot |\Lambda_k^R|$
- Final Cost:

$$J_{IG}^F(a) = \frac{1}{2} \ln \frac{|I_m + A \cdot \Sigma_k \cdot A^T|}{|I_m + A_R \cdot \Sigma_k^{R|F} \cdot A_R^T|}, \quad \Sigma_k^{R|F} \equiv (\Lambda_k^R)^{-1}$$

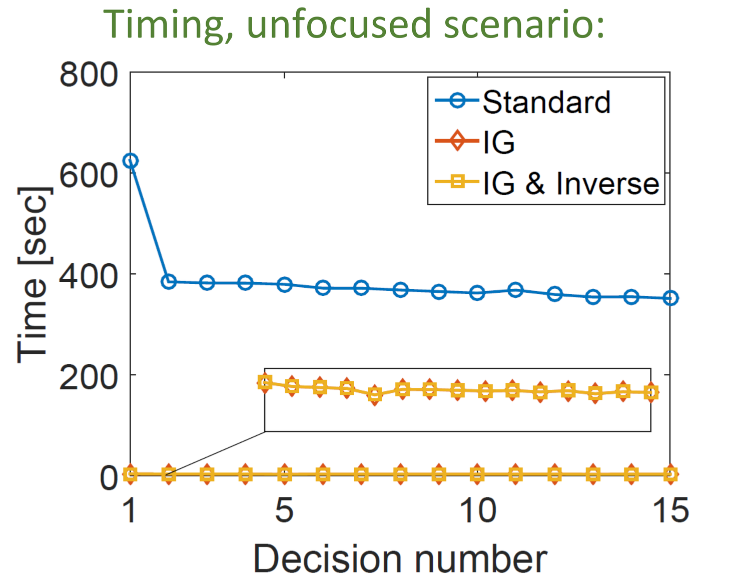
- Calculation re-use can be applied here in the same way
- Also here, per-candidate calculation depends only on m

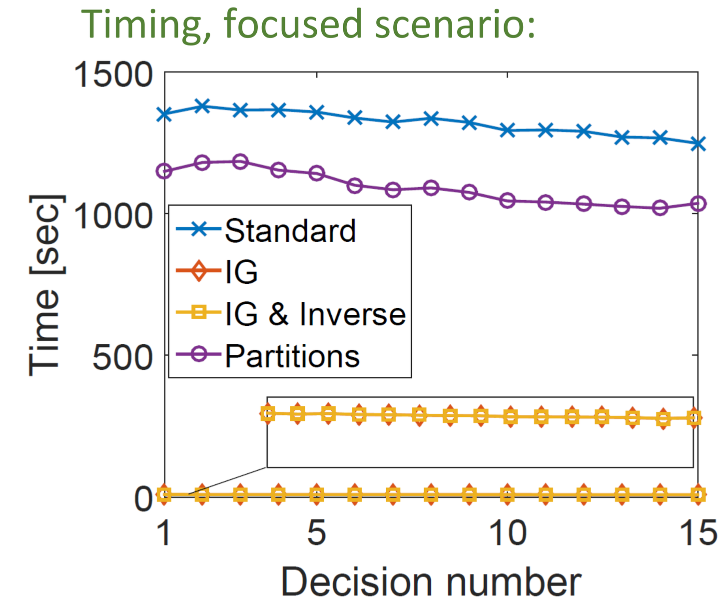
4. Experiments I: Sensor Deployment

- Select most informative locations to deploy sensors:



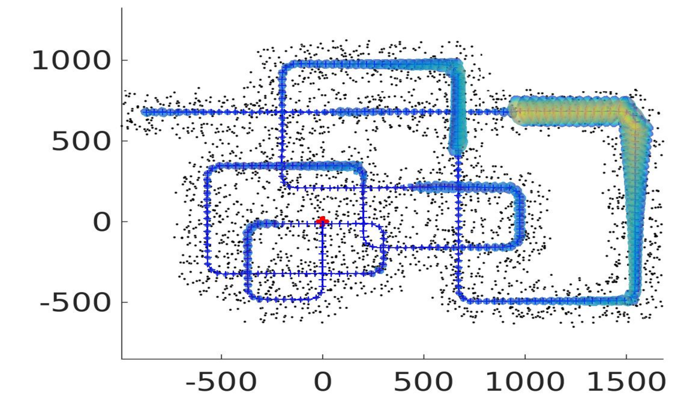
- Comparing standard and presented techniques:



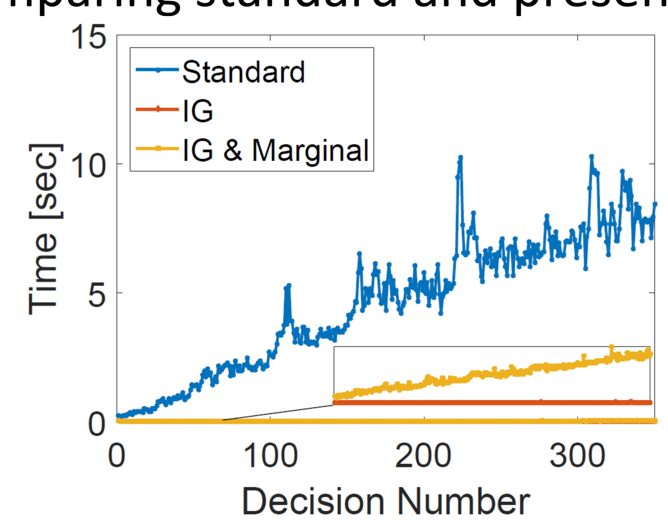


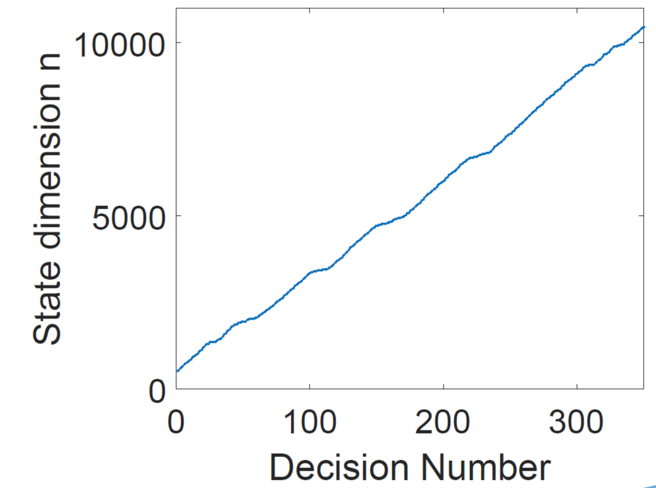
5. Experiments II: Measurement Selection

- Selecting most informative measurements in SLAM:



- Comparing standard and presented techniques:





6. Conclusions

- Decision making (*unfocused* and *focused*) through calculation re-use
- Determinant Lemma – reduces problem dimension
- Calc. re-use – combine and compute expensive computation only once
- Per-candidate complexity doesn't depend on state dimension
- Exact (no approximations applied)
- General (any measurement model)
- Applicable to many domains

7. References

[1] "No Correlations Involved: Decision Making Under Uncertainty in a Conservative Sparse Information Space", V. Indelman, IEEE Robotics And Automation, 2016

[2] "Matrix algebra from a statistician's perspective", D. Harville, Technometrics, 1998.

[3] "Schur complements and statistics", D. V. Ouellette, Linear Algebra and its Applications, 1981

[4] "Information-based compact Pose SLAM", V. Ila et al., IEEE Trans. Robotics, 2010

8. Funding

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