Efficient Belief Space Planning in High-dimensional State Spaces by Exploiting Sparsity and Calculation Re-use

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Introduction

- Belief Space Planning - fundamental problem in autonomous systems and artificial intelligence, where states are beliefs

- Examples
  - **Active simultaneous localization and mapping** (SLAM)
  - **Informative planning, active sensing**
  - **Sensor selection, sensor deployment**
  - **Multi-agent** informative planning and active SLAM
  - **Graph sparsification** for long-term autonomy
  - **Autonomous navigation**

Introduction

- Information-theoretic belief space planning
  - **Objective**: find action that minimizes an information-theoretic metric (e.g. entropy, information gain, mutual information)

- Decision making over **high-dimensional** state spaces is expensive!

$$X \in \mathbb{R}^n \qquad \Lambda \equiv \Sigma^{-1} \in \mathbb{R}^{n \times n}$$

- Evaluating action impact typically involves determinant calculation: $O(n^3)$

- Existing approaches typically calculate posterior information (covariance) matrix for **each** candidate action, and then its determinant

BSP Problem Types

- By objective's goal:
  - **Unfocused** – reduce uncertainty of all variables
  - **Focused** – reduce uncertainty of only specific variable subset
- By state dimensionality:
  - **Not-Augmented** – state vector is unchanged by action
  - **Augmented** – new state variables are introduced by action (e.g. new robot poses)

BSP cases	Non-Augmented	Augmented
Unfocused	✓	✓
Focused	✓	✓

Motivating Example I – Belief Space Planning

- **Joint** state vector

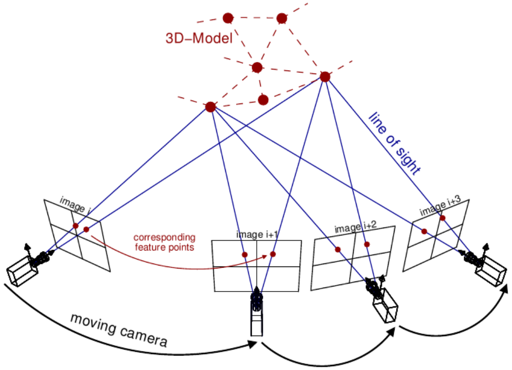
$$X_k \doteq \{\underbrace{x_0, \ldots, x_k}_{\text{Past \& current robot states}}, \underbrace{L_k}_{\text{Mapped environment}}\}$$

- **Joint** probability distribution function $p\left(X_k | \mathcal{Z}_k, \mathcal{U}_{k-1}\right)$

$$p\left(X_k | \mathcal{Z}_k, \mathcal{U}_{k-1}\right) = priors \cdot \prod_{i=1}^{k} p\left(x_i | x_{i-1}, u_{i-1}\right) \underline{p\left(z_i | X_i^o\right)}$$

General observation model $X_i^o \subseteq X_i$

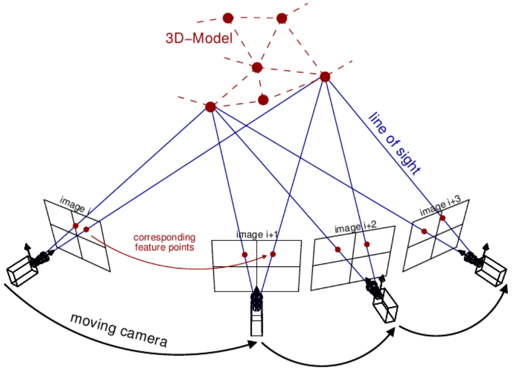
- Computationally-efficient maximum a posteriori inference e.g. [Kaess et al. 2012]



$$p\left(X_k | \mathcal{Z}_k, \mathcal{U}_{k-1}\right) \sim N\left(X_k^*, \Sigma_k\right)$$

Motivating Example I – Belief Space Planning

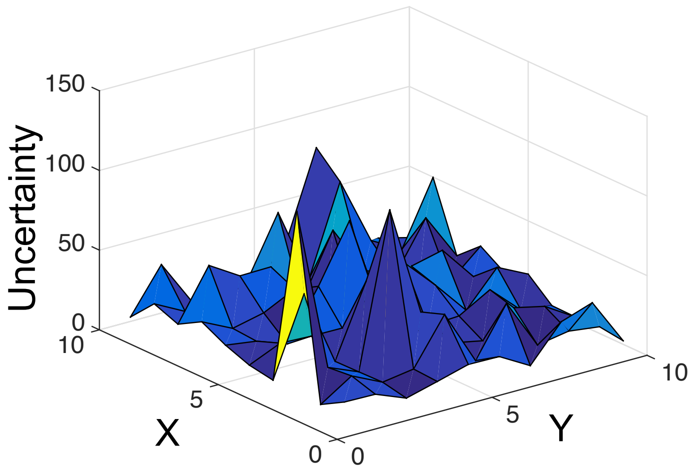
- How to autonomously determine future action(s)?
- Involves reasoning, for different candidate actions, about belief evolution
- Problem is to find trajectory with minimal posterior uncertainty:
  - Augmented BSP
  - Objective can be Unfocused / Focused



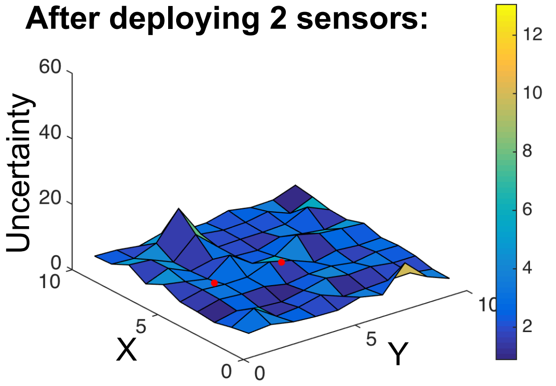
$$p\left(X_k | \mathcal{Z}_k, \mathcal{U}_{k-1}\right) \sim N\left(X_k^*, \Sigma_k\right)$$

Motivating Example II - Sensor Deployment

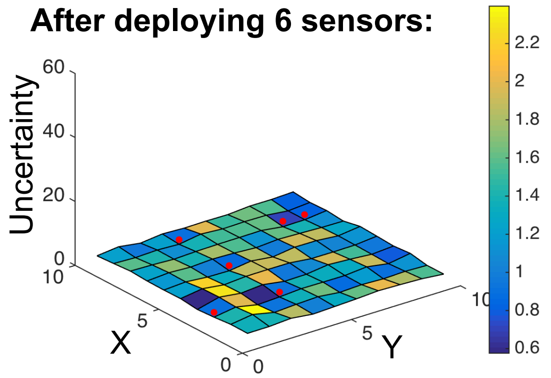
- **Objective**: deploy k sensors in an $N \times N$ area
  - provide localization
  - monitor spatial-temporal field (e.g. temperature)
  - Not-Augmented BSP



Prior uncertainty field:



After deploying 2 sensors:



After deploying 6 sensors:

Related Work

- Existing approaches often
  - Propagate **posterior belief for each action**
  - Compute determinants of **huge** matrices
  - Assume known environment (e.g. map)
  - Consider small state space

Contributions

- Computationally-efficient **information-theoretic BSP** approach
  - **Without** posterior propagation for each candidate action
  - **Avoid** calculating determinants of large matrices
  - Calculation **Re-use**
- Per-action evaluation **does not depend on state** dimension
- **Exact** and **general** solution
- Approach addresses all cases of BSP problem:

BSP cases	Non-Augmented	Augmented
Unfocused	✓	✓
Focused	✓	✓

Problem Formulation

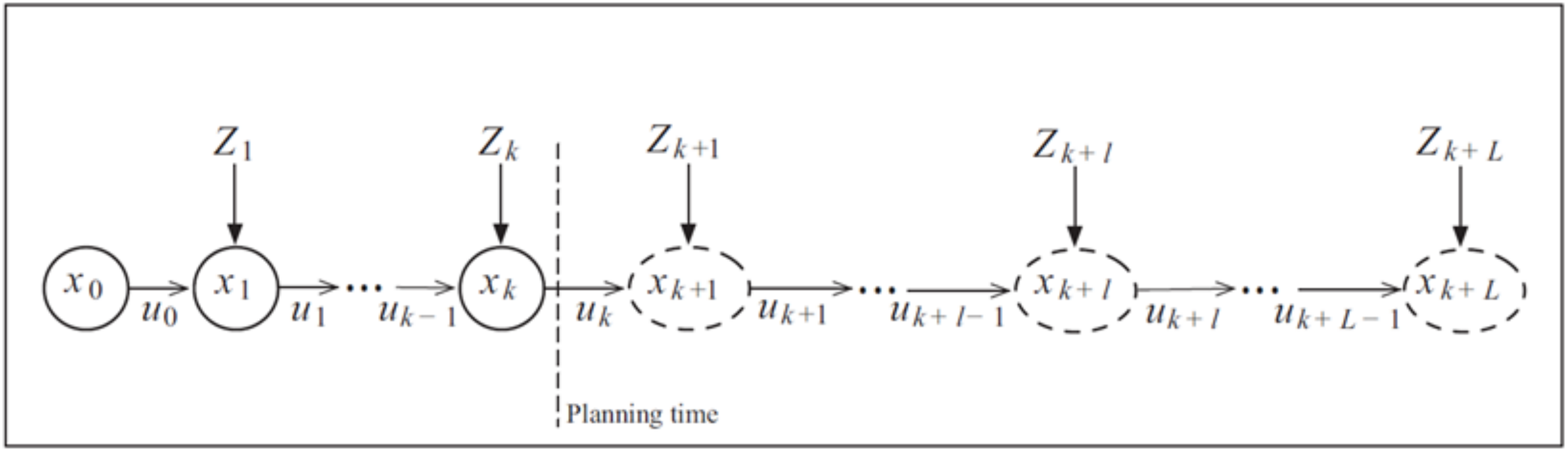
- Consider state vector $X_k \in \mathbb{R}^n$ at time t_k
  - e.g. history of robot poses, landmarks, etc.
  - n can be huge (> 10000), for example..

- Consider its belief $b[X_k] = \mathrm{N}\,(X_k^*, \Sigma_k)$
- Consider candidate actions $\mathcal{A} \doteq \{a_1, a_2, \ldots, a_N\}$
- Each candidate a_i provides different posterior belief $b[X_{k+L} \mid a_i]$
- The goal is to choose optimal action according to some objective:

$$a^* = \underset{a \in \mathrm{A}}{\operatorname{argmin}}\, J(a)$$

Problem Formulation

- Example from mobile robotics domain:
  - Given action $a = u_{k:k+L-1}$ and new observations $Z_{k+1:k+L}$, future belief is:



(Image is taken from Indelman15ijrr)

$$b\left[X_{k+L}\right] = p\left(X_{k+L}|Z_{0:k+L}, u_{0:k+L-1}\right) \propto p\left(X_k|Z_{0:k}, u_{0:k-1}\right) \prod_{l=k+1}^{k+L} \underbrace{p\left(x_l|x_{l-1}, u_{l-1}\right)}_{\text{motion model}} \underbrace{p\left(Z_l|X_l^o\right)}_{\text{measurement likelihood}}$$

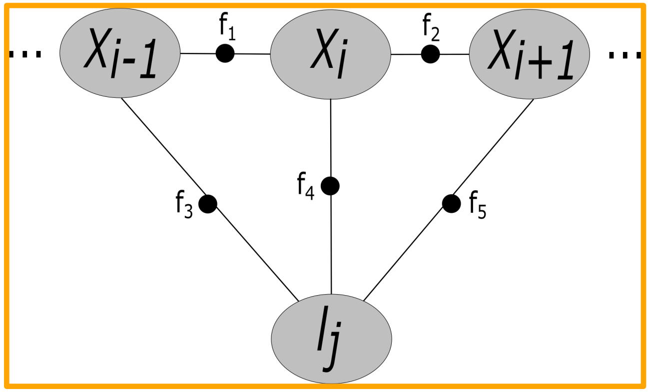
» L is planning horizon

Problem Formulation

- Consider state vector $X_k \in \mathbb{R}^n$ at time t_k

- Posterior at time t_k can be represented in general form via factor terms $F_i = \{f_i^1, \ldots, f_i^{n_i}\}$ for $0 \leq t_i \leq t_k$:

$$P(X_k \mid history) \propto \prod_{i=0}^{k} \prod_{j=1}^{n_i} f_i^j(X_i^j)$$



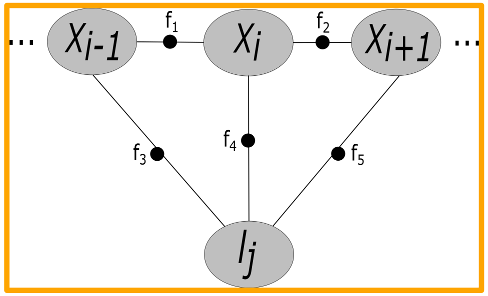
where each factor f_i^j:

  - have form $f_i^j(X_i^j) \propto \exp(-\frac{1}{2} \| h_i^j(X_i^j) - r_i^j \|^2_{\Sigma_i^j})$
  - with measurement model $r_i^j = h_i^j(X_i^j) + \upsilon_i^j, \quad \upsilon_i^j \sim N(0, \Sigma_i^j)$
  - and *involved* variables X_i^j

Problem Formulation

- State vector $X_k \in \mathbb{R}^n$ at time t_k
- Factors $F_i = \{f_i^1, \ldots, f_i^{n_i}\}$ for $0 \le t_i \le t_k$

$$P(X_k \mid history) \propto \prod_{i=0}^{k} \prod_{j=1}^{n_i} f_i^j (X_i^j)$$



- Maximum A Posteriori (MAP) inference:

$$\boxed{\underbrace{b[X_k]}_{\text{belief}} = P(X_k \mid history) = N\,(X_k^*, \Sigma_k) = N^{-1}(\eta_k^*, \Lambda_k)}$$

  – Usually information form is used.

Problem Formulation

- Consider candidate actions $\mathcal{A} \doteq \{a_1, a_2, \ldots, a_N\}$
- For each a_i we can model
  - Planning horizon L
  - New factors $F_{k+l} = \{f_{k+l}^{1}, \ldots, f_{k+l}^{n_{k+l}}\}$ for $1 \leq l \leq L$
  - New variables X_{new} (**<u>empty in not-augmented scenarios</u>**)
  - Noise-weighted Jacobian A of new factors with respect to state variables (more details later)

	NAug	**Aug**
U	X_{new}	X_{new}
F	empty	not empty

- Posterior belief (considering a_i) is then: $$b[X_{k+L}] \propto b[X_k] \prod_{l=k+1}^{k+L} \prod_{j=1}^{n_l} f_l^j(X_l^j)$$
- General objective function: $$J(a) = \mathop{\mathrm{E}}_{Z_{k+1:k+L}} \left\{ \sum_{l=0}^{L-1} c_l(b[X_{k+l}]) + c_L(b[X_{k+L}]) \right\}$$

Problem Formulation

- This work – information-theoretic objectives
  - (Differential) Entropy – measures uncertainty of estimation

$$\mathrm{H}\left(X\right)=-\int_{X} p(x)\cdot\log p(x)dx$$

  - BSP Information term (Unfocused):
    - (Differential) Entropy:

$$J_{\mathrm{H}}(a)=\mathrm{H}\left(b[X_{k+L}]\right)$$
$$a^{*}=\underset{a\in \mathrm{A}}{\operatorname{argmin}}\, J_{\mathrm{H}}(a)$$

    - Information Gain:

$$J_{IG}(a)=\mathrm{H}(b[X_{k}])-\mathrm{H}(b[X_{k+L}])$$
$$a^{*}=\underset{a\in \mathrm{A}}{\operatorname{argmax}}\, J_{IG}(a)$$

    - Mathematically identical
    - Each can be computationally preferable in different scenarios

Problem Formulation

- Assuming Gaussian Distributions
- Objectives for Not-Augmented Unfocused BSP:

	NAug	Aug
U	✓	
F		

$$J_{\mathrm{H}}(a) = dim.const - \frac{1}{2}\boxed{\ln\left|\Lambda_{k+L}\right|}\ , \quad J_{IG}(a) = \frac{1}{2}\ln\frac{\left|\Lambda_{k+L}\right|}{\left|\Lambda_{k}\right|}$$

- Objectives for Augmented Unfocused BSP:

	NAug	Aug
U		✓
F		

$$J_{\mathrm{H}}(a) = dim.const - \frac{1}{2}\boxed{\ln\left|\Lambda_{k+L}\right|}\ , \quad J_{IG}(a) = dim.const + \frac{1}{2}\ln\frac{\left|\Lambda_{k+L}\right|}{\left|\Lambda_{k}\right|}$$

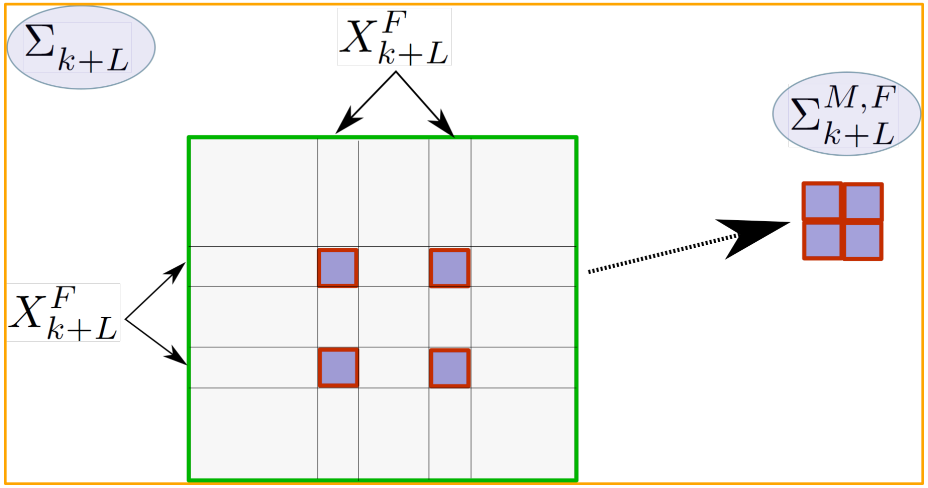
$O(N^3)$ complexity!!!

- Where
  - Λ_k is prior information matrix
  - Λ_{k+L} is posterior information matrix

Problem Formulation

- Focused setting:
  - Consider focused variables $X_{k+L}^{F} \subseteq X_{k+L}$
  - Its posterior marginal covariance:

$$(\Sigma_{k+L} = \Lambda_{k+L}^{-1})$$



  - Measure the posterior information (entropy, IG) for these variables:

	NAug	Aug
U		
F	✓	✓

$$J_{\mathrm{H}}^{F}(a) = \mathrm{H}(X_{k+L}^{F}) = dim.const + \frac{1}{2}\ln\left|\Sigma_{k+L}^{M,F}\right|$$

$$J_{IG}^{F}(a) = \mathrm{H}(X_{k}^{F}) - \mathrm{H}(X_{k+L}^{F}) = \frac{1}{2}\ln\frac{\left|\Sigma_{k}^{M,F}\right|}{\left|\Sigma_{k+L}^{M,F}\right|}$$

Standard Approaches

- Propagate posterior belief for each action
- Calculate determinants of large matrices
- Per-action complexity - $O(N^3)$, where N is posterior state dimension
- More information later..

BSP cases	Non-Augmented	Augmented
Unfocused	$J_{\mathrm{H}}(a) = dim.const - \frac{1}{2}\ln \boxed{\left\|\Lambda_{k+L}\right\|}$ $J_{IG}(a) = \frac{1}{2}\ln \boxed{\frac{\left\|\Lambda_{k+L}\right\|}{\left\|\Lambda_{k}\right\|}}$	$J_{\mathrm{H}}(a) = dim.const - \frac{1}{2}\ln \boxed{\left\|\Lambda_{k+L}\right\|}$ $J_{IG}(a) = dim.const + \frac{1}{2}\ln \boxed{\frac{\left\|\Lambda_{k+L}\right\|}{\left\|\Lambda_{k}\right\|}}$
Focused	$J_{\mathrm{H}}^{F}(a) = dim.const + \frac{1}{2}\ln \boxed{\left\|\Sigma_{k+L}^{M,F}\right\|}$ $J_{IG}^{F}(a) = \frac{1}{2}\ln \boxed{\frac{\left\|\Sigma_{k}^{M,F}\right\|}{\left\|\Sigma_{k+L}^{M,F}\right\|}}$	

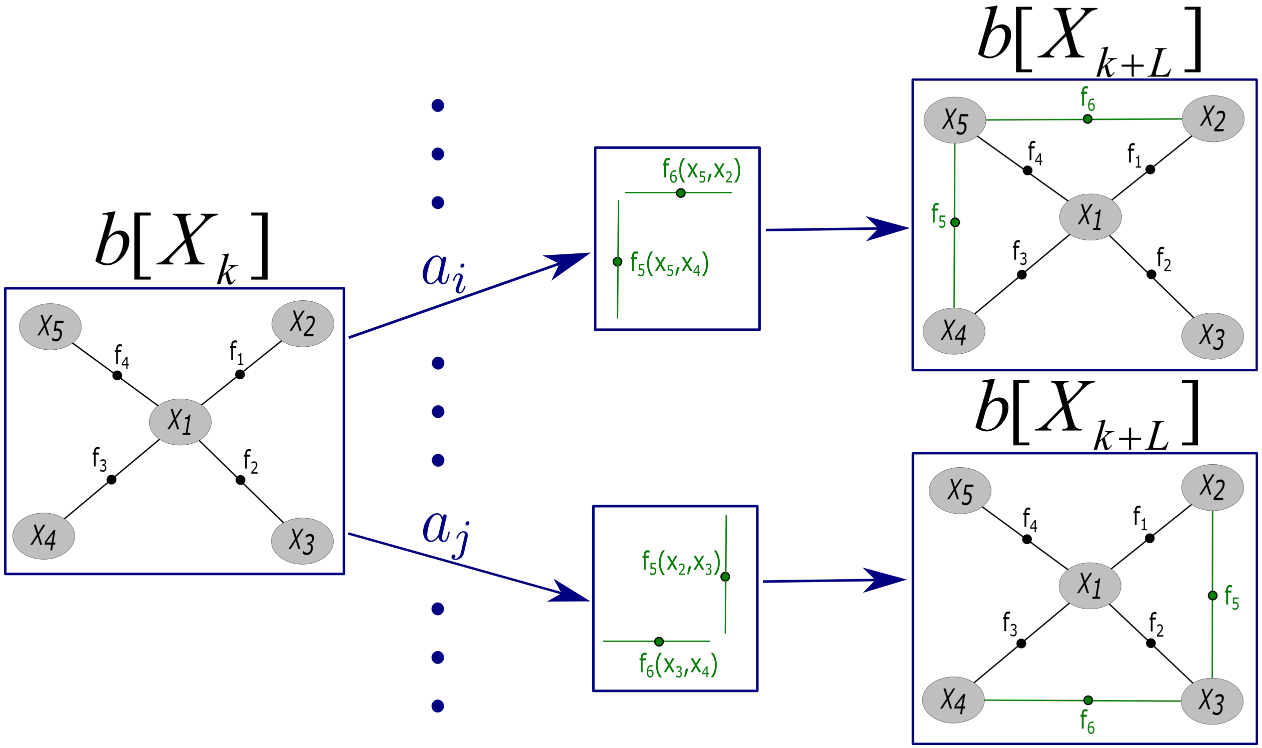
Our Approach *rAMDL*
(Re-use with Augmented Matrix Determinant Lemma)

- **Without** posterior propagation for each candidate action
- **Avoid** calculating determinants of large matrices through AMDL
- Calculation **Re-use**
- Per-action evaluation **does not depend on state** dimension
- Solve each of BSP problem types:

	NAug	Aug
U	✓	✓
F	✓	✓

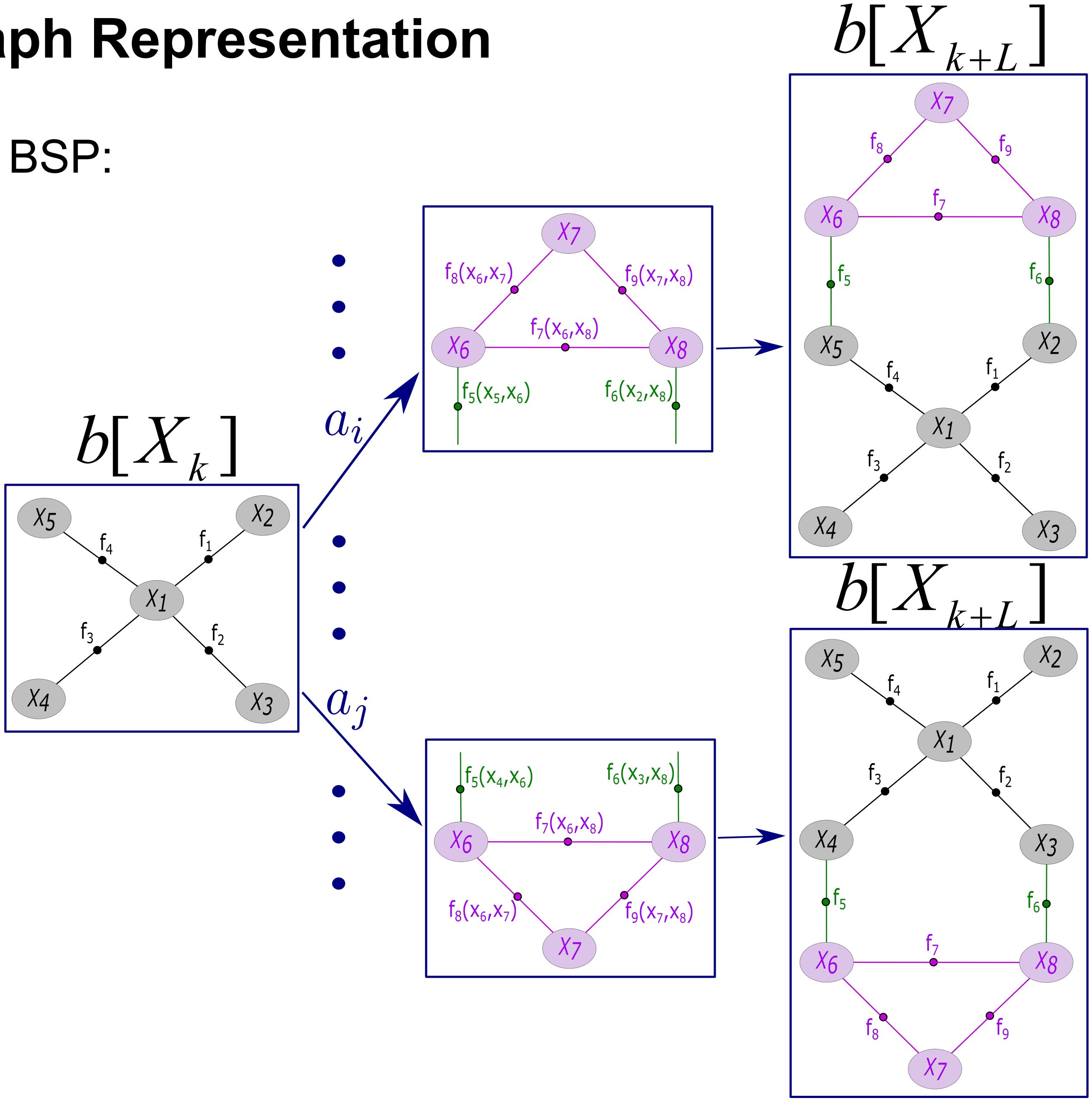
Factor Graph Representation

- Not-Augmented BSP:



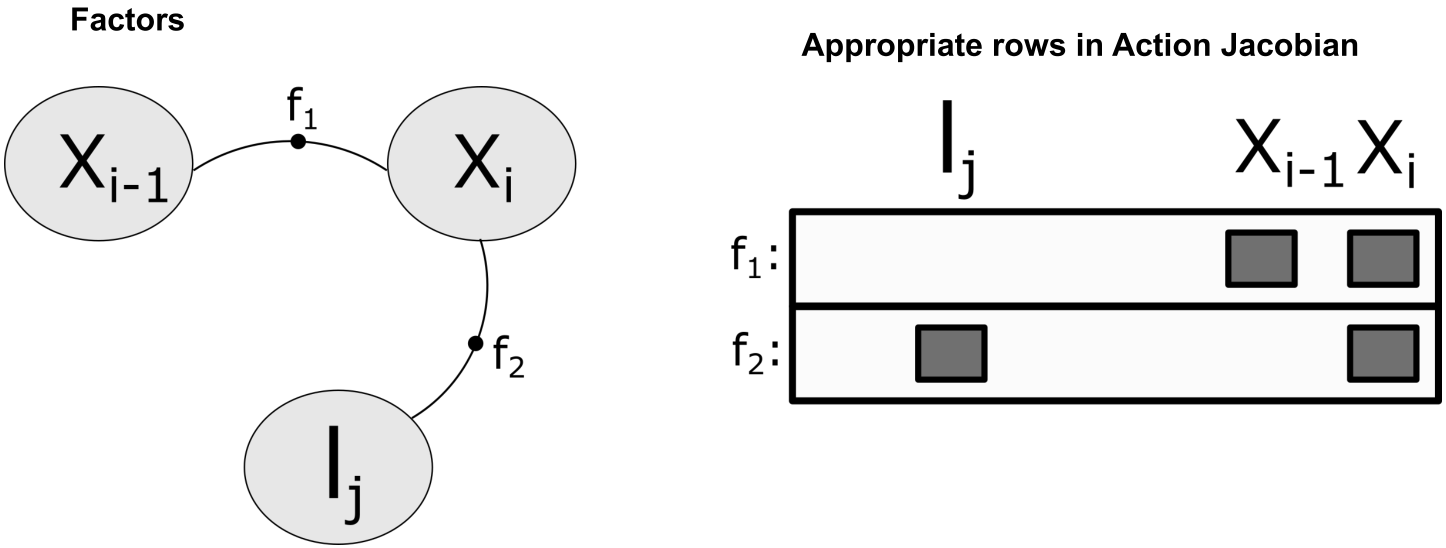
Factor Graph Representation

- Augmented BSP:



Jacobian Structure Sparsity

- Matrix A is Jacobian of **new** factors, with dimension $m \times N$
- Its rows represent **new** factors (measurements)
- Its columns represent state variables (old and new)
- Only variables *involved* in new factors will have non-zero columns in A
- Typically m and number of *involved* variables is very small



Not-Augmented BSP, Unfocused Setting

BSP cases	Non-Augmented	Augmented
Unfocused	✌	
Focused		

Posterior Information Matrix

	NAug	Aug
U	✓	
F	✓	

- Not-augmented case (no new variables were introduced by a_i):

Posterior belief:

$$b[X_{k+L}] \propto b[X_k] \prod_{l=k+1}^{k+L} \prod_{j=1}^{n_l} f_l^j(X_l^j)$$

Its information matrix:

$$\Lambda_{k+L} = \Lambda_k + A^T \cdot A$$

Matrix Determinant Lemma (MDL)

- We use MDL to reduce calculations:

$$\left|\Lambda_k + A^T \cdot A\right| = \left|\Lambda_k\right| \cdot \left|I_m + A \cdot \Sigma_k \cdot A^T\right|$$

where $\Sigma_k \equiv \Lambda_k^{-1} \in \mathbb{R}^{n\times n}$, $A \in \mathbb{R}^{m\times n}$

- Applying it to unfocused not-augmented BSP:

$$J_{IG}(a) = \frac{1}{2}\ln\frac{\left|\Lambda_{k+L}\right|}{\left|\Lambda_k\right|} = \frac{1}{2}\ln\frac{\left|\Lambda_k + A^T \cdot A\right|}{\left|\Lambda_k\right|}$$

$$= \frac{1}{2}\ln\left|I_m + A\cdot\Sigma_k\cdot A^T\right| = \frac{1}{2}\ln\left|I_m + {}^{I}\!A\cdot\Sigma_k^{M,{}^{I}\!X}\cdot({}^{I}\!A)^T\right|$$

where:

» ${}^{I}\!A$ is partition of A with all non-zero columns

» $\Sigma_k^{M,{}^{I}\!X}$ is prior marginal covariance of *involved* variables ${}^{I}\!X$

Not-Augmented BSP, Unfocused Setting

	NAug	Aug
U	✓	
F		

- Objective:

$$\boxed{J_{IG}(a) = \frac{1}{2} \ln \left| I_m + {}^{I}\!A \cdot \Sigma_k^{M,{}^{I}\!X} \cdot ({}^{I}\!A)^T \right|}$$

- Calculation complexity depends on m and $dim({}^{I}\!X)$
- Given $\Sigma_k^{M,{}^{I}\!X}$, does not depend on state dimension N
- Only **few entries** from the prior covariance are actually required!
- Very cheap
- For example, in measurement selection $m = 1$, $dim({}^{I}\!X) < 10$

Calculation Re-use

- **Key observations**: $J_{IG}(a) = \frac{1}{2} \ln \left| I_m + {}^{I}\!A \cdot \Sigma_k^{M,{}^{I}\!X} \cdot ({}^{I}\!A)^T \right|$
  - We can **avoid** posterior propagation and determinants of **large** matrices
  - Calculation of action impact **does not depend** on N
  - Still, we need $\Sigma_k^{M,{}^{I}\!X}$
  - Different candidate actions often **share** many *involved* variables ${}^{I}\!X$

- **We propose re-use of calculation**:
  - Combine variables *involved* in all candidate actions into set $X_{All} \subseteq X_k$
  - Perform one-time calculation of $\Sigma_k^{M,X_{All}}$ (depends on N)
  - Calculate $J_{IG}(a)$ for each action, using $\Sigma_k^{M,X_{All}}$

Not-Augmented BSP, Focused Setting

BSP cases	Non-Augmented	Augmented
Unfocused		
Focused	✌	

Not-Augmented BSP, Focused Setting

	NAug	Aug
U		
F	✓	

- Consider focused variables $X^F \subseteq X$ and unfocused $X^U = X / X^F$
- Partition $\Sigma_{k/k+L}$ and $\Lambda_{k/k+L}$ appropriately:

$$\Sigma = \begin{bmatrix} \Sigma^U & \Sigma^{U\ F} \\ (\Sigma^{U\ F})^T & \Sigma^F \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} \Lambda^U & \Lambda^{U,F} \\ (\Lambda^{U,F})^T & \Lambda^F \end{bmatrix}$$

- We have Λ_k and Λ_{k+L}, but for focused BSP we need $\left|\Sigma^F_{k+L}\right|$ (for entropy) or $\dfrac{\left|\Sigma^F_k\right|}{\left|\Sigma^F_{k+L}\right|}$ (for IG)
- How to calculate $\dfrac{\left|\Sigma^F_k\right|}{\left|\Sigma^F_{k+L}\right|}$ efficiently?

Not-Augmented BSP, Focused Setting

	NAug	Aug
U		
F	✓	

- Consider focused variables $X^F \subseteq X$ and unfocused $X^U = X / X^F$
- Partition $\Sigma_{k/k+L}$ and $\Lambda_{k/k+L}$ appropriately:

$$\Sigma = \begin{bmatrix} \Sigma^U & \Sigma^{U\,F} \\ (\Sigma^{U\,F})^T & \Sigma^F \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} \Lambda^U & \Lambda^{U,F} \\ (\Lambda^{U,F})^T & \Lambda^F \end{bmatrix}$$

- Connection through Schur complement:

$$\boxed{(\Sigma_k^F)^{-1} = \Lambda_k^{M,F} = \Lambda_k^F - (\Lambda_k^{U\,F})^T \cdot (\Lambda_k^U)^{-1} \cdot \Lambda_k^{U\,F}, \qquad \left|\Lambda_k\right| = \left|\Lambda_k^{M,F}\right| \cdot \left|\Lambda_k^U\right|}$$

- Can be shown that:

$$\boxed{\frac{\left|\Sigma_k^F\right|}{\left|\Sigma_{k+L}^F\right|} = \boxed{\frac{\left|\Lambda_{k+L}\right|}{\left|\Lambda_k\right|}} \cdot \boxed{\frac{\left|\Lambda_k^U\right|}{\left|\Lambda_{k+L}^U\right|}}}$$

Not-Augmented BSP, Focused Setting

	NAug	Aug
U		
F	✓	

- Solving: $$\boxed{\frac{\left|\Sigma_k^{M,F}\right|}{\left|\Sigma_{k+L}^{M,F}\right|} = \boxed{\frac{\left|\Lambda_{k+L}\right|}{\left|\Lambda_k\right|}} \cdot \boxed{\frac{\left|\Lambda_k^U\right|}{\left|\Lambda_{k+L}^U\right|}}}$$
- Term $\boxed{\dfrac{\left|\Lambda_{k+L}\right|}{\left|\Lambda_k\right|}}$ - through Determinant Lemma
- Note: $\Lambda_{k+L}^U = \Lambda_k^U + (A^U)^T \cdot A^U$ where A^U is partition of A

  with columns belonging to unfocused variables X^U
- Thus, term $\boxed{\dfrac{\left|\Lambda_k^U\right|}{\left|\Lambda_{k+L}^U\right|}}$ - also through Determinant Lemma
- Finally, IG of focused variables X^F can be calculated as:

$$\boxed{\begin{aligned} J_{IG}^F(a) &= \mathcal{H}(X_k^F) - \mathcal{H}(X_{k+L}^F) = \\ &\frac{1}{2}\ln\left|I_m + {}^I A \cdot \Sigma_k^{M,{}^I X} \cdot ({}^I A)^T\right| - \frac{1}{2}\ln\left|I_m + {}^I A^U \cdot \Sigma_k^{{}^I X^U|F} \cdot ({}^I A^U)^T\right| \end{aligned}}$$

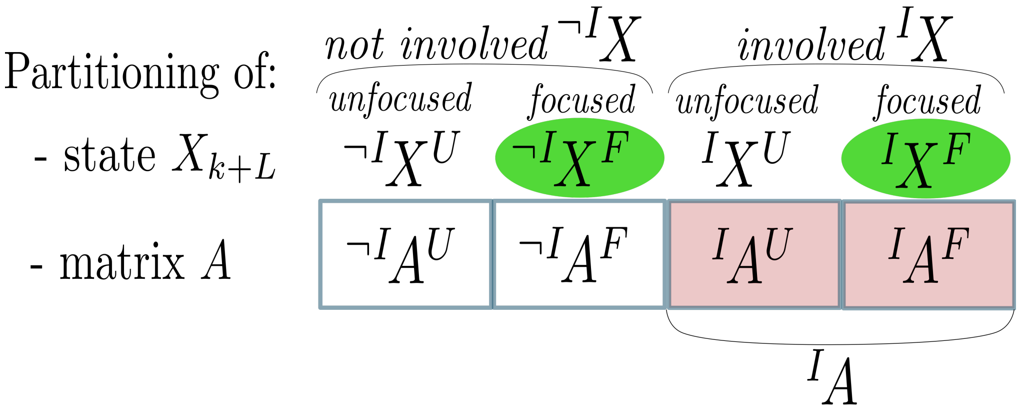
Not-Augmented BSP, Focused Setting

	NAug	Aug
U		
F	✓	

- Final solution:

$$J^F_{IG}(a) = \underline{\frac{1}{2}\ln\left|I_m + {}^IA\cdot\Sigma_k^{M,{}^IX}\cdot({}^IA)^T\right|} - \underline{\frac{1}{2}\ln\left|I_m + {}^IA^U\cdot\Sigma_k^{{}^IX^U|F}\cdot({}^IA^U)^T\right|}$$

where:



Partitioning of:

- state X_{k+L}: $\overbrace{\overset{unfocused}{{}^{\neg I}X^U}\quad\overset{focused}{{}^{\neg I}X^F}}^{not\ involved\ {}^{\neg I}X}\quad\overbrace{\overset{unfocused}{{}^IX^U}\quad\overset{focused}{{}^IX^F}}^{involved\ {}^IX}$

- matrix A: $\boxed{{}^{\neg I}A^U}\ \boxed{{}^{\neg I}A^F}\ \underbrace{\boxed{{}^IA^U}\ \boxed{{}^IA^F}}_{{}^IA}$

- Calculation complexity depends on m and $dim({}^IX)$
- Given $\Sigma_k^{M,{}^IX}$ and $\Sigma_k^{{}^IX^U|F}$, does not depend on state dimension N
- For example, in measurement selection $m = 1$, $dim({}^IX) < 10$

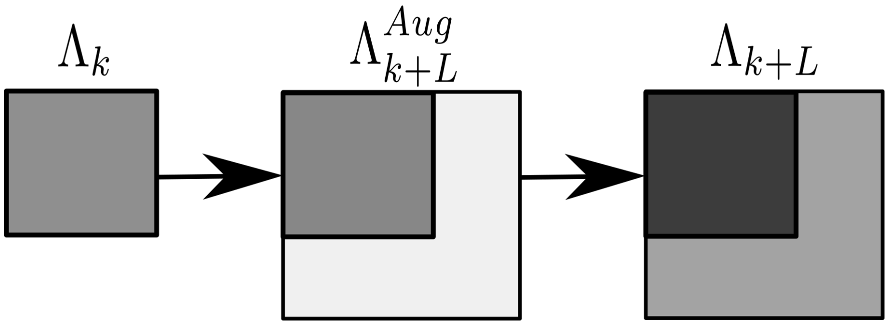
Augmented BSP, Unfocused Setting

BSP cases	Non-Augmented	Augmented
Unfocused		✌
Focused		

Posterior Information Matrix

	NAug	Aug
U		✓
F		✓

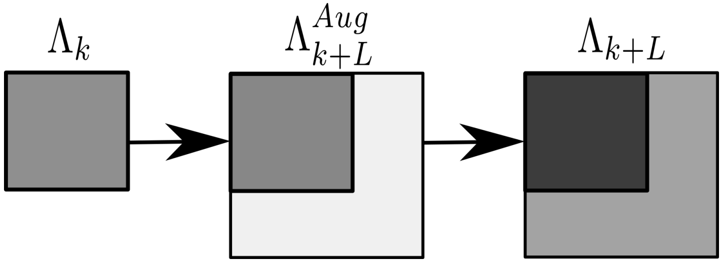
- Augmented case (new variables were introduced by a_i):



$$\Lambda_{k+L} = \Lambda_{k+L}^{Aug} + A^T \cdot A$$

  - Usual Matrix Determinant Lemma cannot be used

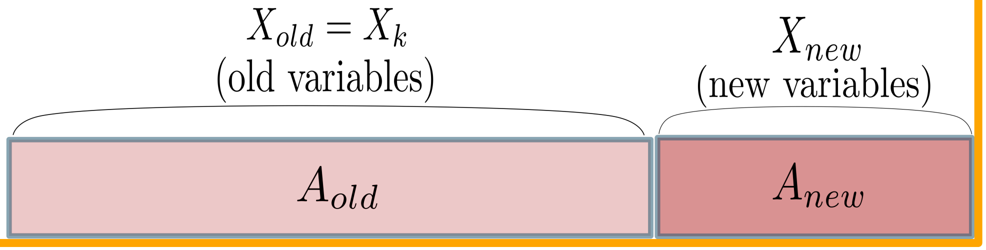
Augmented Matrix Determinant Lemma (AMDL)



$$\Lambda_{k+L} = \Lambda_{k+L}^{Aug} + A^T \cdot A$$

Partitioning of:

- state X_{k+L}
- matrix A



- **We developed Lemma**:

$$\frac{\left|\Lambda_{k+L}\right|}{\left|\Lambda_k\right|} = \frac{\left|\Lambda_{k+L}^{Aug} + A^T \cdot A\right|}{\left|\Lambda_k\right|} = \left|\Delta\right| \cdot \left|A_{new}^T \cdot \Delta^{-1} \cdot A_{new}\right|$$

$$\Delta \doteq I_m + A_{old} \cdot \Sigma_k \cdot A_{old}^T$$

Augmented BSP, <u>Unfocused</u> Setting

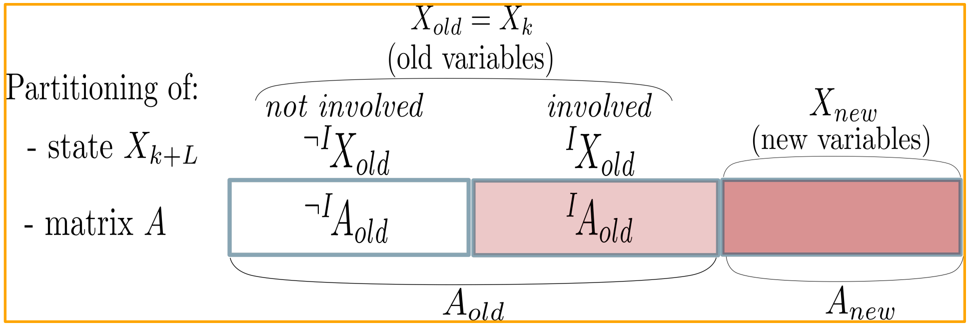
	NAug	Aug
U		✓
F		

- Objective:

$$J_{IG}(a) = dim.const + \frac{1}{2}\ln\left|C\right| + \frac{1}{2}\ln\left|A_{new}^{T}\cdot C^{-1}\cdot A_{new}\right|$$

$$C = I_m + {}^{I}A_{old}\cdot \Sigma_k^{M,{}^{I}X_{old}}\cdot ({}^{I}A_{old})^T$$

where



- Calculation complexity depends on m, $dim({}^{I}X_{old})$ and $dim(X_{new})$
- Given $\Sigma_k^{M,{}^{I}X_{old}}$, does not depend on state dimension N
- Only **few entries** from the prior covariance are actually required!
- Very cheap

Augmented BSP, Focused Setting

BSP cases	Non-Augmented	Augmented
Unfocused		
Focused		✌

Augmented BSP, Focused Setting

- Different cases:
  1. $X_{k+L}^{F} \subseteq X_{new}$, for example robot last pose
  2. $X_{k+L}^{F} \subseteq X_{old}$, for example mapped landmarks
  3. $X_{k+L}^{F} \subseteq \{X_{old} \cup X_{new}\}$, hard to find example

- We handle first 2 cases

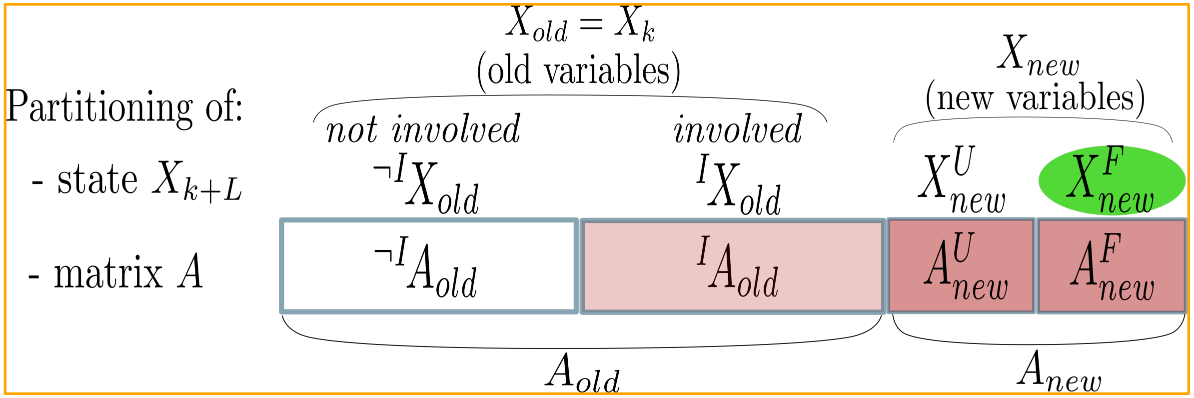
Augmented BSP, Focused Setting $\mathbf{X^F_{k+L} \subseteq X_{new}}$

	NAug	Aug
U		
F		✓

- Objective: $J^F_{\mathcal{H}}(a) = dim.const + \frac{1}{2}\ln\left|(A^U_{new})^T \cdot C^{-1} \cdot A^U_{new}\right| - \frac{1}{2}\ln\left|A^T_{new} \cdot C^{-1} \cdot A_{new}\right|$

$$C = I_m + {}^IA_{old} \cdot \Sigma_k^{M,{}^IX_{old}} \cdot ({}^IA_{old})^T$$

where



- Calculation complexity depends on m, $dim({}^IX_{old})$ and $dim(X_{new})$
- Given $\Sigma_k^{M,{}^IX_{old}}$, does not depend on state dimension N
- Only **few entries** from the prior covariance are actually required!
- Very cheap

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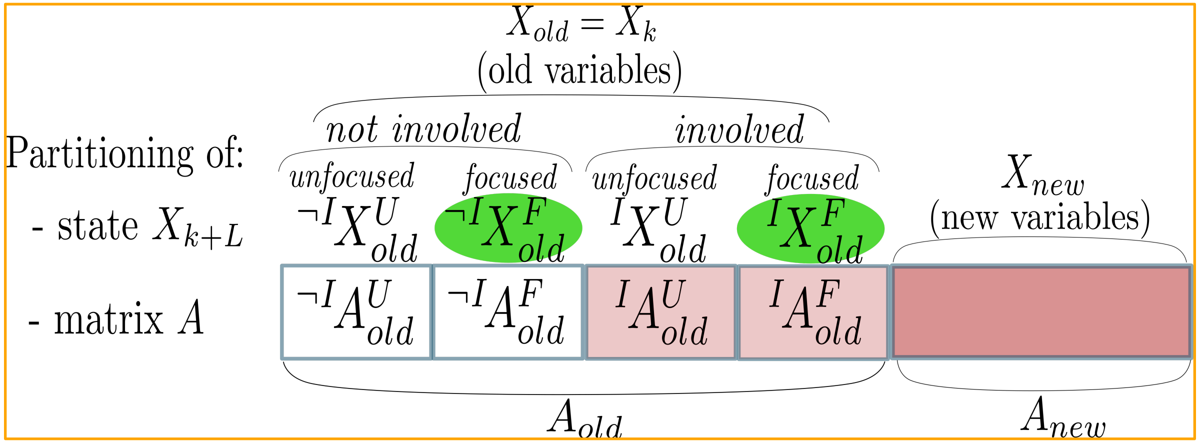
Augmented BSP, <u>Focused</u> Setting $\mathbf{X}^{\mathbf{F}}_{\mathbf{k+L}} \subseteq \mathbf{X}_{\mathbf{old}}$

	NAug	Aug
U		
F		✓

- Objective: $J^F_{IG}(a) = \frac{1}{2}\left(\ln\left|C\right| + \ln\left|A^T_{new} \cdot C^{-1} \cdot A_{new}\right| - \ln\left|S\right| - \ln\left|A^T_{new} \cdot S^{-1} \cdot A_{new}\right|\right)$

$$C = I_m + {}^I A_{old} \cdot \Sigma_k^{M,{}^I X_{old}} \cdot ({}^I A_{old})^T, \quad S \doteq I_m + {}^I A^U_{old} \cdot \Sigma_k^{{}^I X^U_{old}|F} \cdot ({}^I A^U_{old})^T$$

where



- Calculation complexity depends on m, $dim({}^I X_{old})$ and $dim(X_{new})$
- Given $\Sigma_k^{M,{}^I X_{old}}$ and $\Sigma_k^{{}^I X^U_{old}|F}$, does not depend on state dimension N

rAMDL Method - Summary

- We address all 4 BSP problem types:

BSP cases	Non-Augmented	Augmented
Unfocused	✓	✓
Focused	✓	✓

- **No need** for posterior belief propagation
- **Avoid** calculating determinants of large matrices
- Calculation **Re-use**
- Per-action evaluation **does not depend on state** dimension
- **Exact** and **general** solution

Standard Approaches

- *From-Scratch:*
  1. For each candidate a_i :
     - 1.1. Propagate belief $\Lambda_{k+L} = \Lambda_k + A^T \cdot A$
     - 1.2. Unfocused case - compute $\left|\Lambda_{k+L}\right|$
     - 1.3. Focused case – compute Schur Complement of X_{k+L}^F and $\left|\Sigma_{k+L}^{M,F}\right|$
  2. Select action with minimal posterior entropy

  – Per-action complexity - $O(N^3)$ for each candidate, N is posterior state dimension (can be **huge**)

Standard Approaches

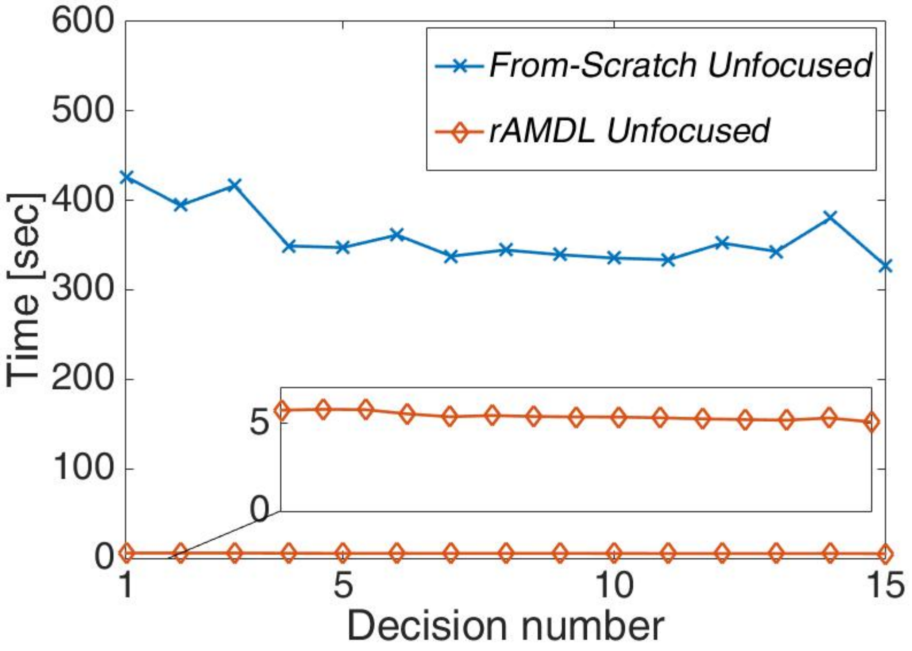
- *Incremental Smoothing And Mapping (iSAM):*
  - Uses iSAM2 incremental inference solver [Kaess et al. 2012] to propagate belief
  - Belief is represented by square-root information matrix R_k
  - Uses incremental factorization techniques (Givens Rotations) for inference
  - Complexity – hard to analyze, but faster than *From-Scratch*
  - Still, per-candidate calculation depends on N

Simulation Results

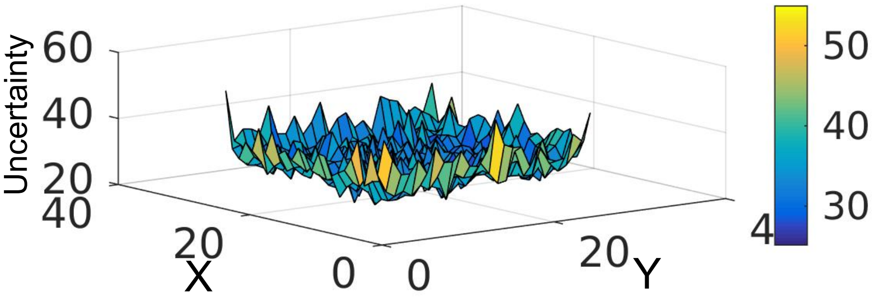
- Not-Augmented BSP
  - Sensor Deployment
  - Measurement Selection in SLAM
- Augmented BSP
  - Autonomous Navigation in Unknown Environment

Application to Sensor Deployment Problems

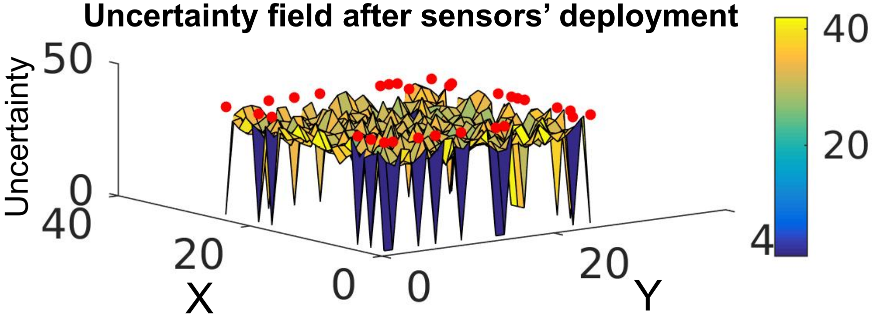
- Significant time reduction in *Unfocused* case



Uncertainty field (dense prior information matrix)



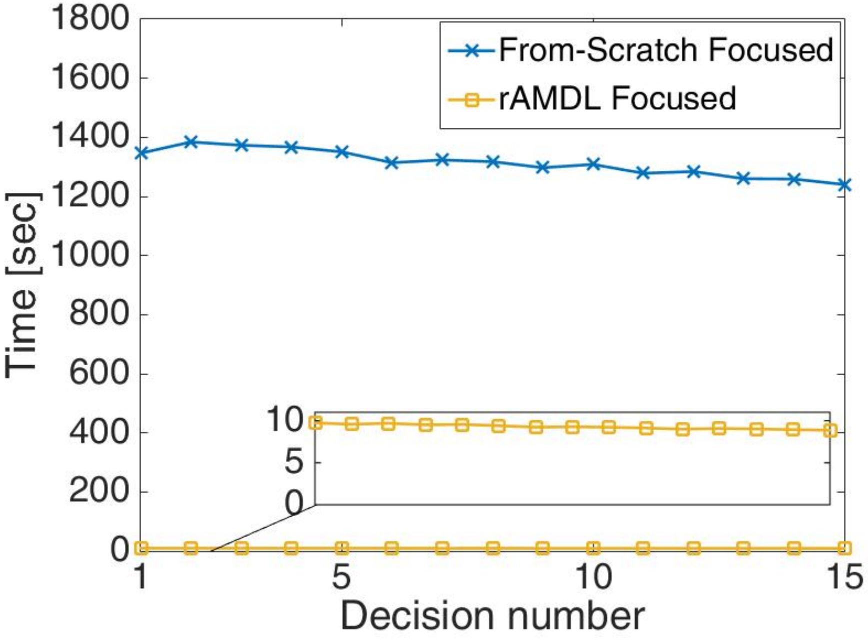
Uncertainty field after sensors' deployment

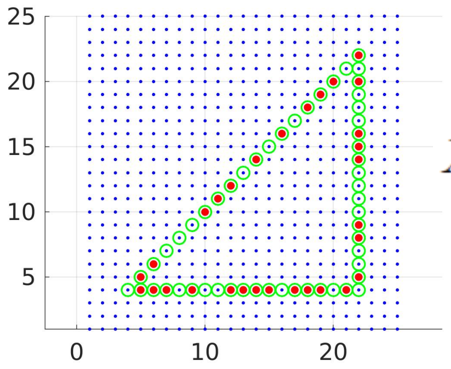


D. Kopitkov, Efficient Belief Space Planning in High-dimensional State Spaces by Exploiting Sparsity and Calculation Re-use, Graduate Seminar, March 2017

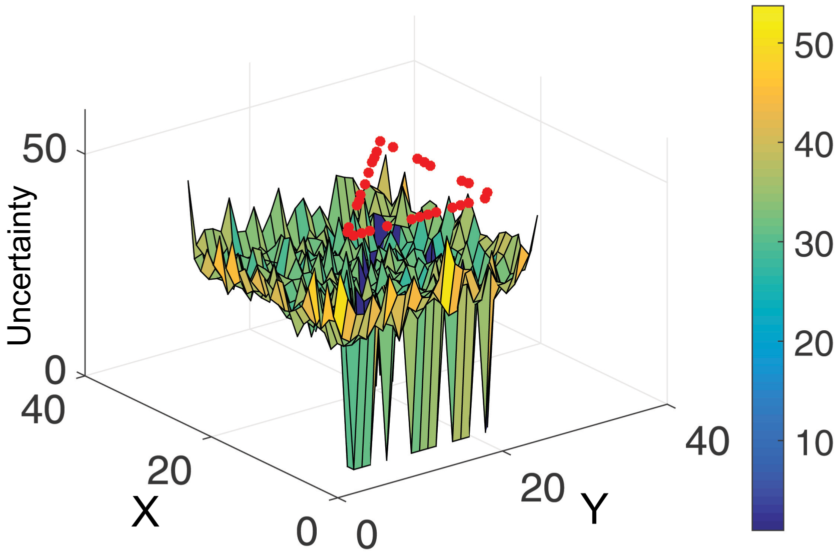
Application to Sensor Deployment Problems

- Significant time reduction in *Focused* case

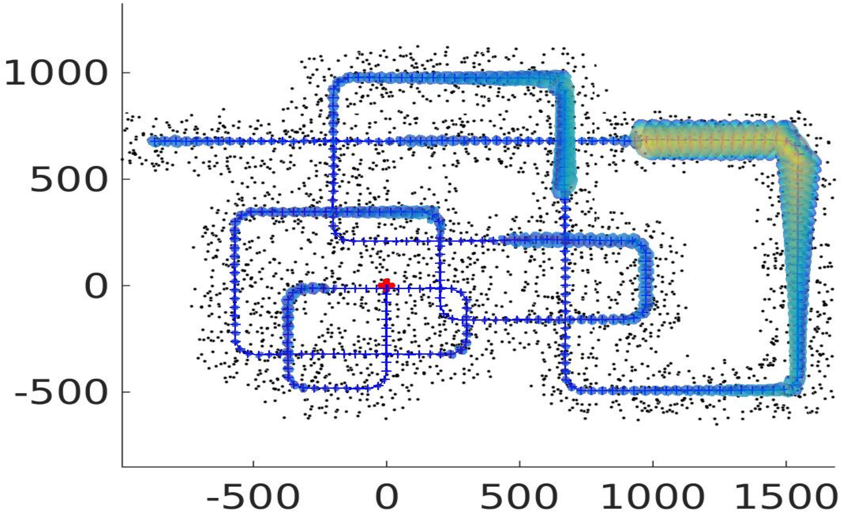


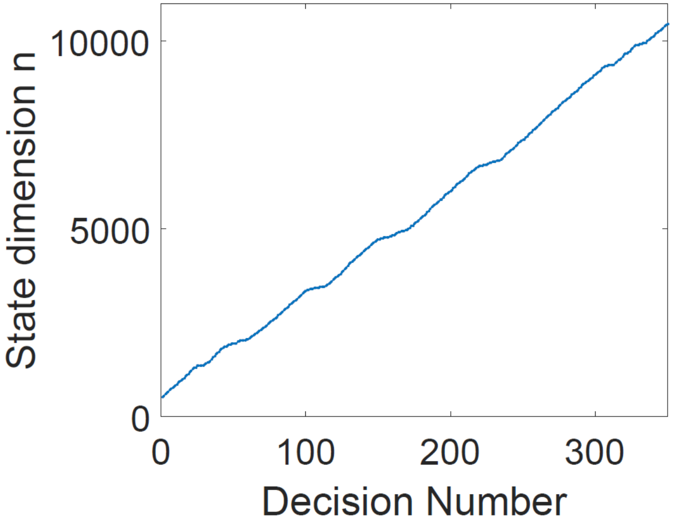


$$X^F_{k+L} \subseteq X_{k+L}$$

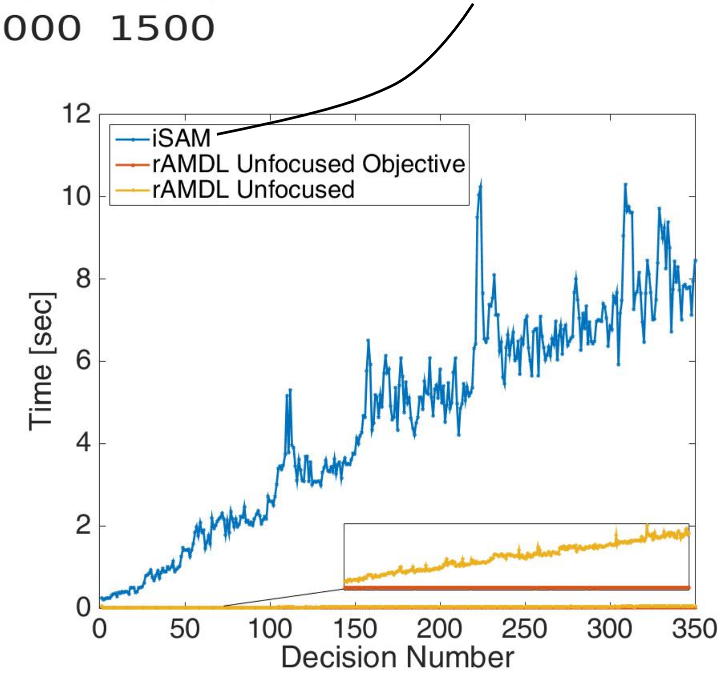


Application to Measurement Selection (in SLAM Context)



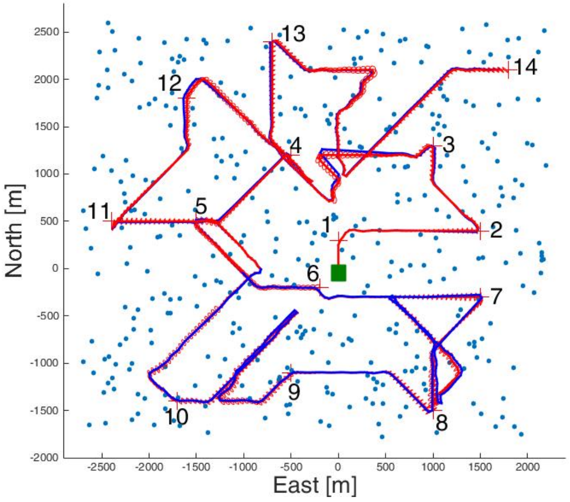


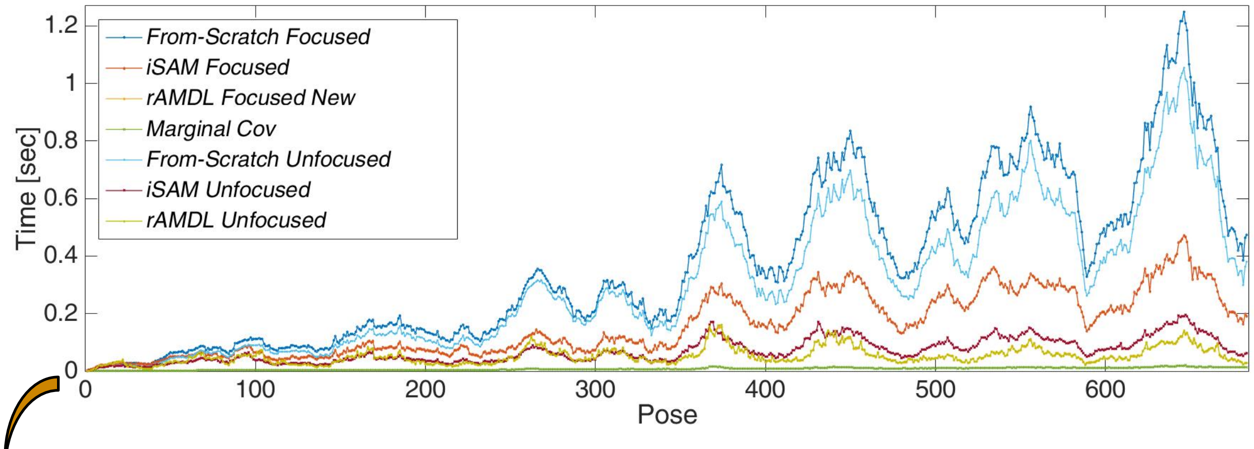
‘iSAM’: for each action, calculate posterior sqrt information matrix via **iSAM2**, then its determinant



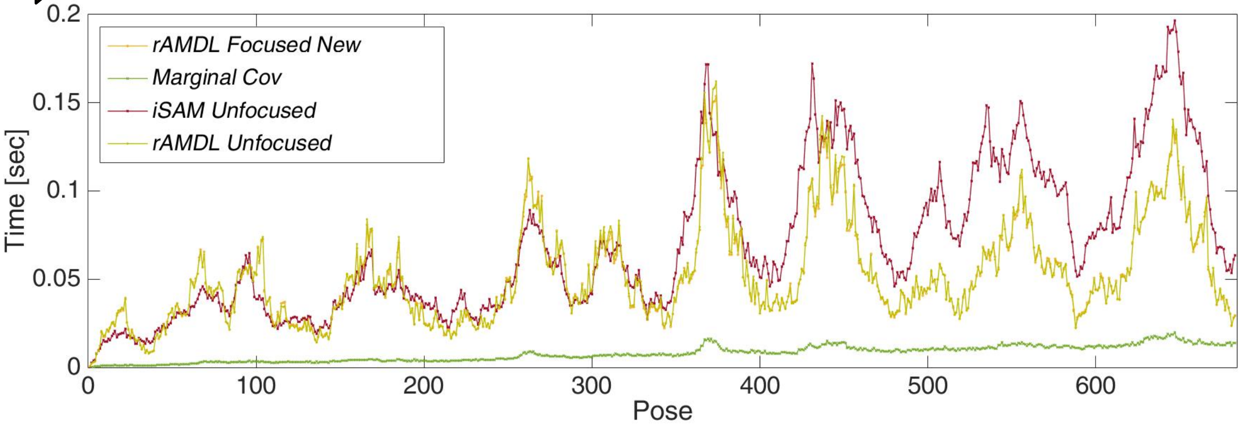
Application to Autonomous Navigation in Unknown Environment

- Significant time reduction in *Focused* case – focus on robot's last pose x_{k+L}



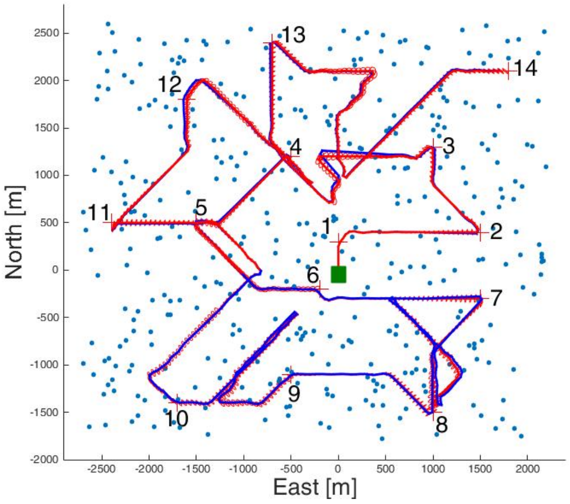


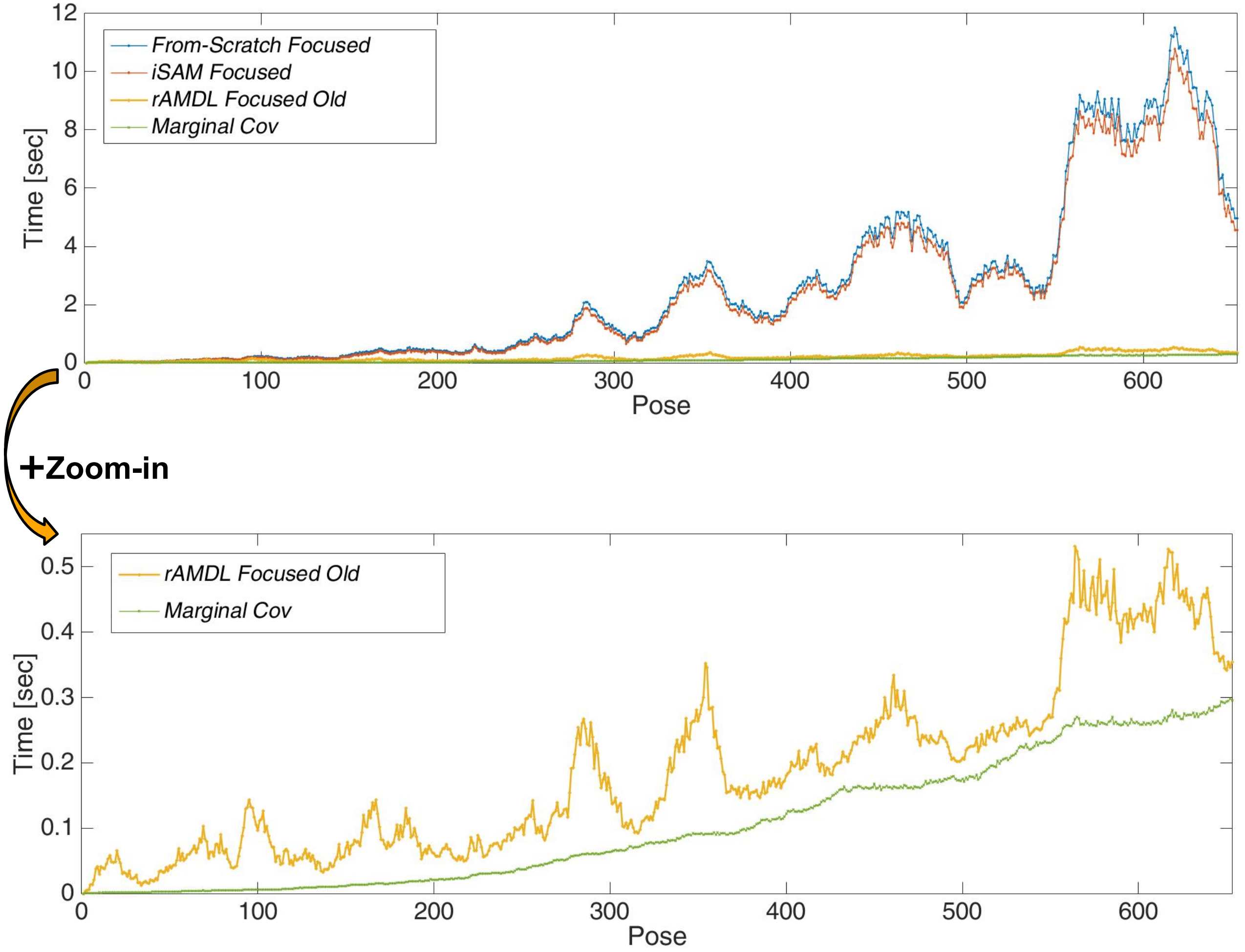
+Zoom-in



Application to Autonomous Navigation in Unknown Environment

- Significant time reduction in *Focused* case – focus on mapped landmarks





Conclusions

rAMDL (**R**e-use with **A**ugmented **M**atrix **D**eterminant **L**emma):

- Exact (identical to original objectives)
- General (any measurement model)
- Per-candidate complexity does not depend on state dimension
- Unfocused and Focused problem formulations
- **Not-Augmented** and **Augmented** cases
- Applicable to Sensor Deployment, Measurement Selection, Graph Sparsification, Active SLAM and many more..

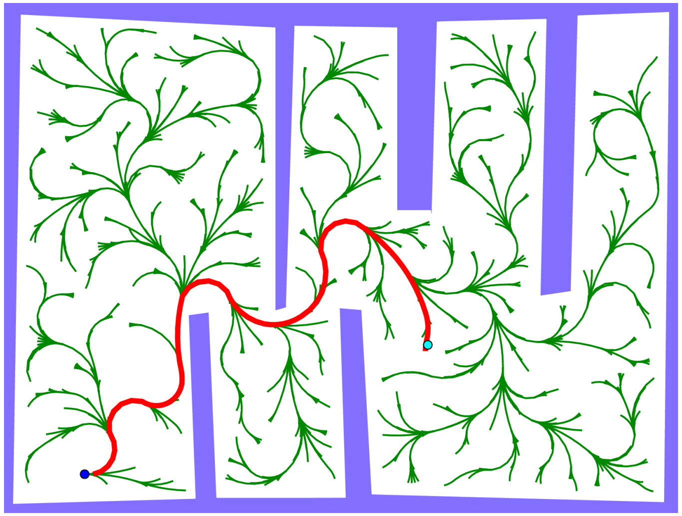
Thanks For Listening

Questions?

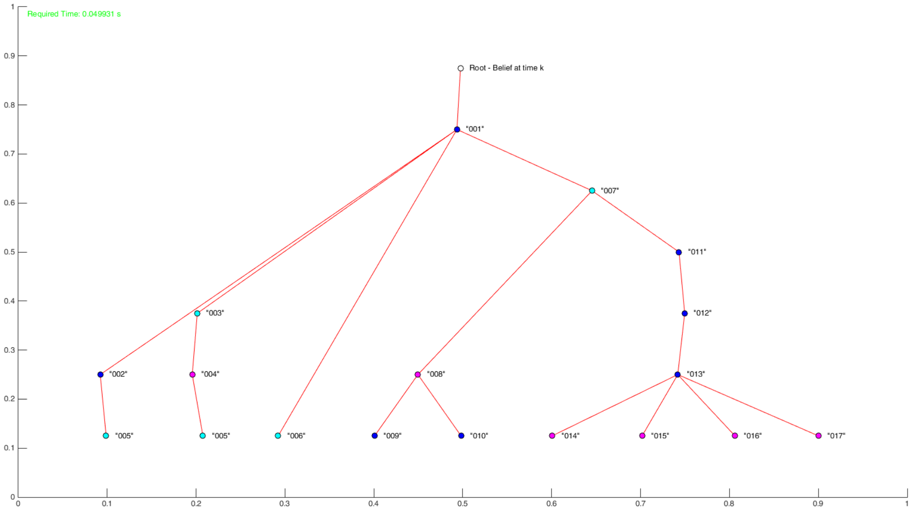
Future Research

Tree of Actions

- Consider tree of candidates



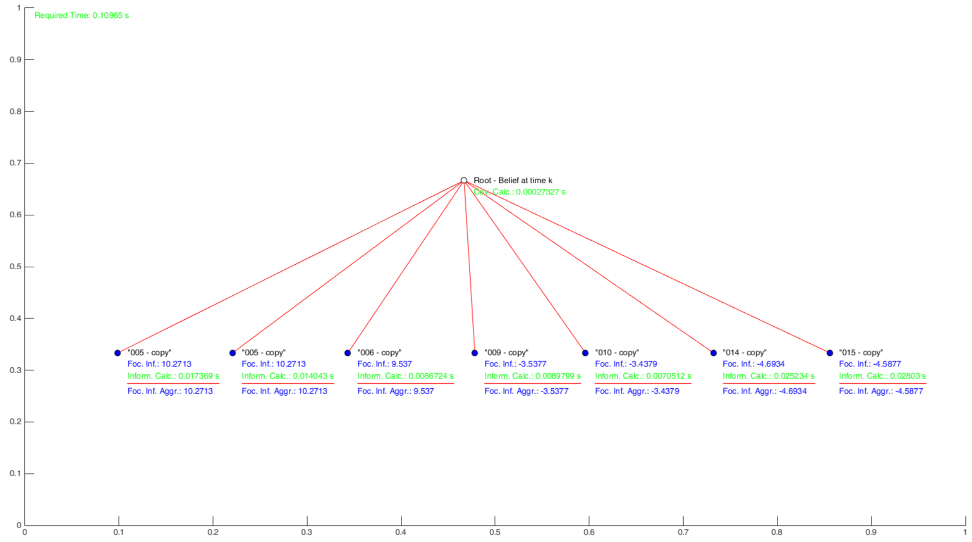
(Image is taken from "http://mrs.felk.cvut.cz/research/motion-planning")



- Some parts of actions are shared
- Can calculation be re-used?

Tree of Actions

- Yes, it can
- Propagate covariance of only required entries
- Calculate information objective through rAMDL



- Preliminary results – very fast solution