Formal Data Association aware & Robust Belief Space Planning

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Structure

- Preliminaries
  - Recalling MDP, POMDP & complexities
  - Belief-space planning (BSP)
- Data-association in BSP
  - State of the art
  - Considering it within BSP
- Formal approaches to planning
  - Temporal logic & Plan synthesis
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Constituents of autonomous navigation

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Question

What is autonomy? Any specific definition? Which context?

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What is autonomy? Any specific definition? Which context?

- **Inference** (estimation): Where am I?
- **Perception**: What is the environment perceived by sensors?
  e.g.: What am I looking at? Is that the same scene as before?
- **Planning**: What is the next best action(s) to realize a task?
  e.g.: where to look or navigate next?

Belief Space Planning - Why

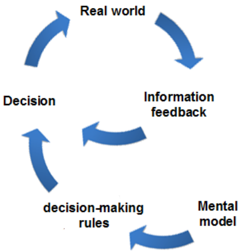
Recall *belief* MDP

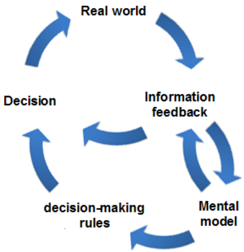
Intuitively, the aims of belief space planning are:

- solve the underlying POMDP
- reason directly over probability distribution over states
- harness some *structure* to get more efficient solution

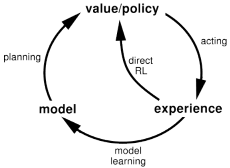
Planning - How

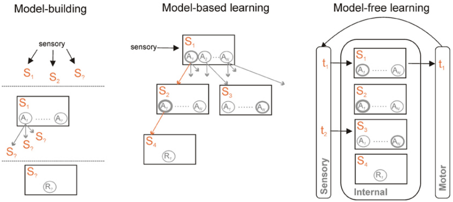
Sense-decide-act





Plan or learn? Model-based or model-free?





What is a model?

- AI: Any structure that provides some domain knowledge explicitly
- MDP: transition system $T(s, s') : \mathcal{S} \times \mathcal{S} \mapsto \mathbb{R}_{[0,1]}$
- POMDP:

$$b^{u,z}(x') = \frac{O(x', u, z)}{\mathbb{P}(z|u, b)} \cdot \sum_{x \in \mathcal{S}} T(x, u, x') b(x)$$

$$b^{u,z}(x') \propto O(x', u, z) \cdot \sum_{x \in \mathcal{S}} T(x, u, x') b(x)$$

The model $O(x', u, z)$ makes a crucial assumption whenever we say $z = h(x)$.

Belief Space Planning: formulation

- motion model: $\mathbb{P}(x_{i+1}|x_i, u_i)$, assume:
  $x_{i+1} = f(x_i, u_i) + w_i \ \wedge w_i \sim \mathcal{N}(0, \Sigma_w)$
- observation model: $\mathbb{P}(z_{i+1}|x_{i+1}, A_j)$, assume:
  $z_{i+1} = h(x_{i+1}, A_j) + v_i \ \wedge v_i \sim \mathcal{N}(0, \Sigma_v)$
- belief at current time 'k': $b[X_k] \triangleq \mathbb{P}(X_k|u_{0:k-1}, z_{0:k})$
- (myopic) objective function:
  $J(u_k) \triangleq \underset{z_{k+1}}{\mathbb{E}}\{c(\mathbb{P}(X_k|u_{0:k-1}, z_{0:k}))\} \equiv \underset{z_{k+1}}{\mathbb{E}}\{c(b[X_k])\}$
- optimal control: $u_k^* \triangleq \underset{u_k}{\arg\min} J(u_k)$

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Local linearization

Assumptions:

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Local linearization

Belief propagation:

$$b[X_{0:k}]p(x_{k+1}|x_k, u_k)p(\tilde{z}_{k+1}|x_{k+1}) \triangleq \mathcal{N}(\hat{X}^i_{0:k+1|k}, \Sigma^i_{0:k+1|k})\mathcal{N}(\tilde{h}_{k+1}, \tilde{\Sigma})$$

Maximum likelihood & MAP estimate:

$$X_k^* = \arg\min_{X_k} \left(\| X_k - \hat{X}_k \|^2_{\Lambda_0} + \| f(x_k, u_k) - x_{k+1} \|^2_{\Omega_w} + \| \tilde{h}(x_k) - \tilde{z}_{k+1} \|^2_{\Omega_\omega} \right)$$

Linearization (Taylor's first-order approximation) around the nominal point $\hat{X}_k$

- $X_k = \hat{X} + \Delta X_k$
- $\| (f(\hat{x}_k, u_k) + \nabla_x f(\hat{x}_{k+1})\Delta x_k) - (\hat{x}_{k+1} + \Delta x_{k+1}) \|^2$
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Local linearization

An L_2-norm minimization of form:

$$\begin{pmatrix} {\Lambda_0}^{\frac{1}{2}} & 0 \\ \Omega_w^{\frac{1}{2}} \nabla_x f_{k+1} & -1 \\ 0 & \Omega_v^{\frac{1}{2}} \nabla_x \tilde{h}_{k+1} \end{pmatrix} \begin{pmatrix} \Delta X_k \\ \Delta x_{k+1} \end{pmatrix} - \begin{pmatrix} 0 \\ \Omega_w^{\frac{1}{2}} (f(\hat{x}_k, u_k) - \hat{x}_{k+1}) \\ \Omega_v^{\frac{1}{2}} (\tilde{h}(\hat{x}_{k+1}) - \tilde{z}_{k+1}) \end{pmatrix}$$

hence,

$\| \mathcal{A}\Delta X - b \|_2 \implies \Delta X = (\mathcal{A}^T \mathcal{A})^{-1} \mathcal{A}^T b$

also note (X is the nominal point)

the right most term becomes, $\begin{pmatrix} 0 \\ 0 \\ \Omega_v^{\frac{1}{2}} (\tilde{h}(\hat{x}_{k+1}) - \tilde{z}_{k+1}) \end{pmatrix}$

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Data association in belief space planning: Why

- What happens if the environment is ambiguous, perceptually aliased?
  - Identical objects or scenes
  - Objects or scenes that appear similar for some viewpoints
- Examples:
  - Two corridors that look alike
  - Similar in appearance buildings, windows, ?
- What if additionally, we have localization (or orientation) uncertainty?

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Therefore:

- DA is challenging.
- Wrong DA may lead to catastrophic failures.
- Can we incorporate DA within the planning framework?

Data association in belief space planning: How

Robust graph optimization

- **aim**: being resilient to incorrect data association
- **assumption**: considers passive case (i.e., data is given)
- **how**: reasoning on outliers overlooked by front-end
- examples: Sünderhauf et al. 12

Data association in belief space planning: How

Non-parametric representation of belief

- **aim**: Efficient inference over non-Gaussian belief
- **assumption**: the problem is sufficiently large to render parametric methods intractable
- **how**: combines factor-graph based inference with Gibbs sampling of GMM
- **similar to**: Gibbs sampling and data-appended inference methods, other non-parametric inference not utilizing factor-graphs
- examples: yet-to-be-published, Sudderth et al. 02 etc

Data association in belief space planning: How

Active hypothesis disambiguation

- **aim**: choose future view-points in order to disambiguate
- **assumption**: sensor is perfectly localized
- **how**: enumerates paths of varying disambiguation
- **similar to**: multi-hypothesis tracking (MHT)
- examples: Atanasov et al. '14, Agarwal et al. 16

DA-BSP

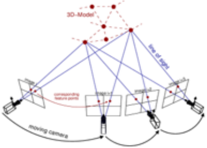
We develop **DA-BSP**: data association aware belief space planning.
We claim, it :

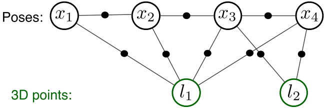
- relaxes the assumption of data association being **known** and **perfect**
- does not assume perfect localization
- reasons about the association explicitly and within the BSP
- is incorporated within overall plan-act-infer framework
- adapts the resulting hypotheses suitably to provide a scalable approach

DA-BSP

Context :

- robot operating in a partially known environment
- robot has sensors to take observations of different scenes or objects
- these observations are used for inference of variable of interest (e.g., pose)
- example: visual SLAM

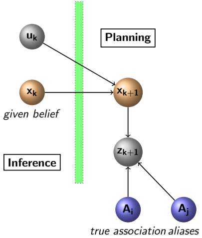


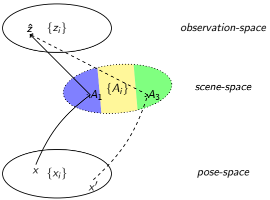


DA-BSP: formulation

Objective function:

$$J(u_k) = \int_{z_{k+1}} \overbrace{\mathbb{P}(z_{k+1}|H^-_{k+1})}^{(a)} \{c(\overbrace{\mathbb{P}(X_{k+1}|H^-_{k+1}, z_{k+1})}^{(b)})\}$$





Question

How to incorporate this within belief space planning?

DA-BSP

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Key idea

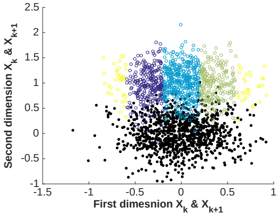
To reason explicitly over all *future* associations of future observations

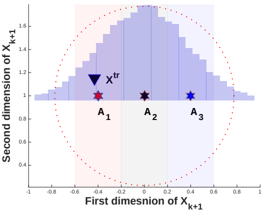
Re-interpreting the terms in previous equation:

- (a): observation likelihood i.e. likelihood that specific z_{k+1} is captured
- (b): cost associated with the posterior *conditioned* on this observation

Abstract example

$$f(x,u) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot x + d \begin{cases} [0,1]^T & \text{if } u = up \\ [1,0]^T & \text{if } u = right \end{cases}, \qquad (1)$$
$$h(x, A_i) = h_i(x) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot (x - x_i) + s_i.$$





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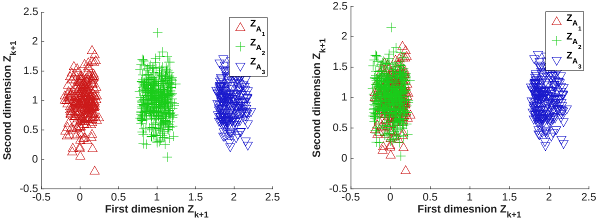

Figure Pose and observation space. (a) black-colored samples $\{x_k\}$ are drawn from $b[X_k] \doteq \mathcal{N}([0,0]^T, \Sigma_k)$, from which, given control u_k, samples $\{x_{x+1}\}$ are computed, colored according to different scenes A_i being observed, and used to generate observations $\{z_{k+1}\}$. (b) Stripes represent locations from which each scene A_i is observable, histogram represents distribution of $\{x_{x+1}\}$, which corresponds to $b[X^-_{k+1}]$. (c)-(d) distributions of $\{z_{k+1}\}$ without aliasing and when $\{A_1, A_3\}_{\textbf{aliased}}$.

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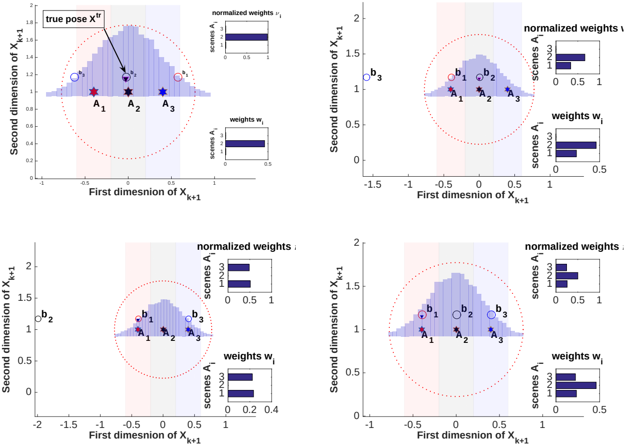
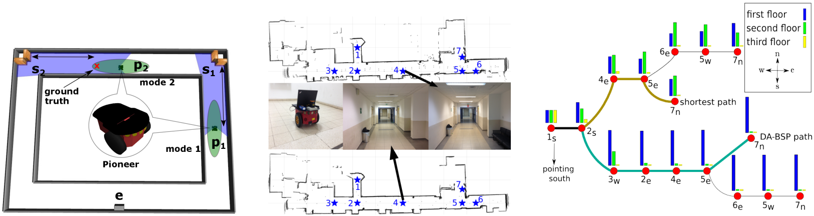


Realistic example

Gazebo simulation & real-world experiment



FigureUsing Pioneer robot in simulation and real-world. (a) a counter-example for hypothesis reduction in absence of pose-uncertainty in prior (b) two (of three) severely-aliased floors, and belief space planning for it (c) DA-BSP can plan for fully disambiguating path (otherwise sub-optimal) while usual BSP with *maximum likelihood* assumption can not

DA-BSP: conclusion

benefits

- considers data-association within the belief space planning framework
- relaxes the assumption that the data association is known or given
- incorporates both *perceptual aliasing* and *localization uncertainty*
- being general enough, has numerous applications

challenges

- the belief is no longer a Gaussian, hence compactness is sacrificed BSP-linearization
- in some pathological cases, number of beliefs can grow exponentially
- efficient pruning heuristics would be required in such cases (esp. in non-myopic planning)

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Preliminaries: Linear Temporal Logic

Syntax:

- $p \in AP \triangleq p$ is LTL formula.
- if Ψ and Φ are LTL formula $\neg\Psi, \Phi \vee \Psi, \mathcal{X}\Psi$ and $\Psi\mathcal{U}\Phi$ are LTL formulae.
- Boolean operators are $\neg, \vee, \wedge, \top, \bot$.

Semantics :

- $w \models p$ if $p \in a_0$
- $w \models \neg\Psi$ if $w \not\models \Psi$
- $w \models \Phi \vee \Psi$ if $w \models \Phi \vee w \models \Psi$
- $w \models \mathcal{X}\Psi$ if $w_1 \models \Psi$
- $w \models \Phi\mathcal{U}\Psi,\ \exists i,\ i \geq 0$ s.t. $w_i \models \Psi \wedge \forall k,\ 0 \leq k < i,\ w_k \models \Phi$

Synthesing a plan

- Using high-level LTL specification (e.g., Hadas et al.)
- Using signal temporal logic (e.g., Belta et al.)
- Harnessing *co-safe* LTL. Lahijanian et al. AAAI-15.
- Optimal temporal planning in probabilistic semantic maps. Fu et al. ICRA-16.

Formal Data association aware BSP

Incorporating collisions:
$\phi = \Box\left(\|x - x_{obs,x}\|_d > \delta\right)$ where $\delta \in \mathbb{R}$ is the safe distance from the closest (in the sense of metric d) obstacle to x denoted by $x_{obs,x}$

Formal Data association aware BSP

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Uncertainty budget:

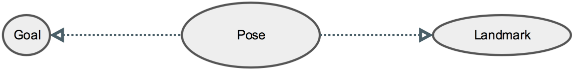

Figure Schematic of an infeasible planning problem. The landmark uncertainty is high enough to deny the robot sufficient localization after loop-closure, while moving directly towards the goal fails to contain the uncertainty within a maximum allowed limit.

Formal Data association aware BSP

Overall objective function:

$$\begin{aligned} \underset{u}{\text{minimize}} \quad & \mathrm{J}(b_u^L, \Psi_1) \\ \text{subject to} \quad & b_u^k \models \Psi_2,\ k \in \{1, 2, \ldots, L\} \end{aligned} \tag{2}$$

Schematic example:

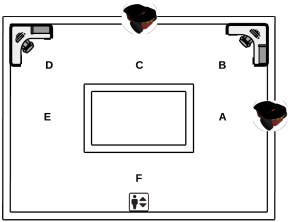

Figure Schematics of a lost janitor robot (figure not to scale). The prior belief is a multimodal Gaussian, with 4 modes, two each floor. Note that there is significant aliasing between the floors.

Formal DA-BSP vs BSP

Table Examples of formal BSP and DA-BSP

Planning	Property	LTL-formula	Comment	Example
BSP	Reaching target	$\diamond p_{goal}$	eventually the goal is reached	$p_{goal} = \|\hat{x} - x_G\|_d < \sigma_g$
	Avoiding obstacle	$\Box p_{safe}$	obstacles are avoided at each step	$p_{safe} = \min_i \|\hat{x} - x^i_{Ob}\|_d > \sigma_{safe}$
	Bounded uncertainty	$\Box p_{unc}$	pose uncertainty within a bound	$p_{unc} = tr(\Sigma_x) < \sigma_\Sigma$
DA-BSP	Reaching target	$\diamond p_{goal}$	goal is reached	$p_{goal} = \min_j \|\hat{x}_j, x_G\|_d < \sigma_g$
	Avoiding obstacle	$\Box p_{safe}$	obstacles are avoided at each step	$p_{safe} = \min_{i,j} \|\hat{x}_j - x^i_{Ob}\|_d > \sigma_{safe}$
	Active disambiguation	$\diamond p_{disamg}$	eventually, disambiguation	$p_{disamg} = \|\{A_{\mathbb{N}}\}\| = 1$
	Efficient propagation	$\Box p_{pruned}$	parsimonious data association	$p_{pruned} = \|\{A_{\mathbb{N}}\}\| < \sigma_N$

Formal DA-BSP vs BSP

H.264 avi

Thanks

Thanks to audience and my colleagues [1]!
Questions or comments, ☺

[1] *Sadegh, Vadim & Alessandro*