Multi-Robot Decentralized Belief Space Planning in Unknown Environments

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TECHNION
Israel Institute of Technology

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Introduction – Application

- Navigation and mapping in unknown environment with **multiple robots (MR)**

Navigation and mapping with MR:
Why is it interesting?

Autonomous cars

[google.com]

Space exploration

[nasa.gov]

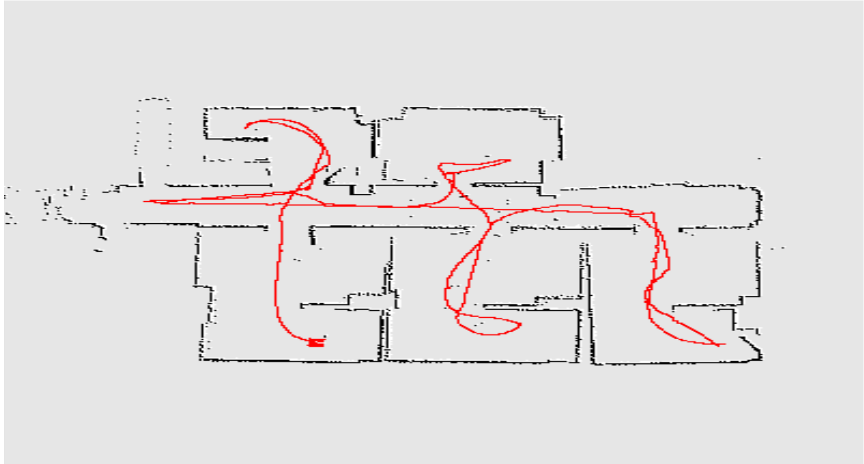
Search & rescue operations

[spectrum.ieee.org]

Introduction - SLAM

- Navigation and mapping in **unknown environment** without GPS
- Simultaneous localization and mapping (**SLAM**)

Estimation and Perception:
Where am I? What is the surrounding environment?



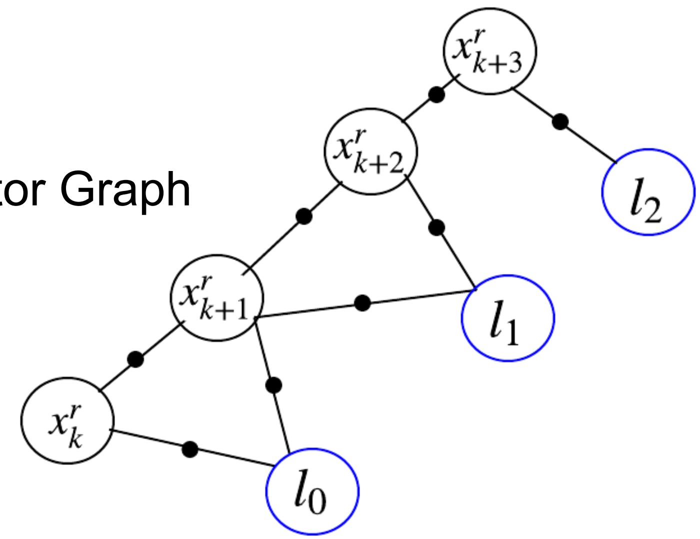
[societyofrobots.com]

Estimation ⇄ **Perception**

Introduction - SLAM

- Represent robot knowledge in a graph model
  - Vertices represent the variables. For example location of robot
  - Edges represent constrains between variables, also known as **factors**

Factor Graph

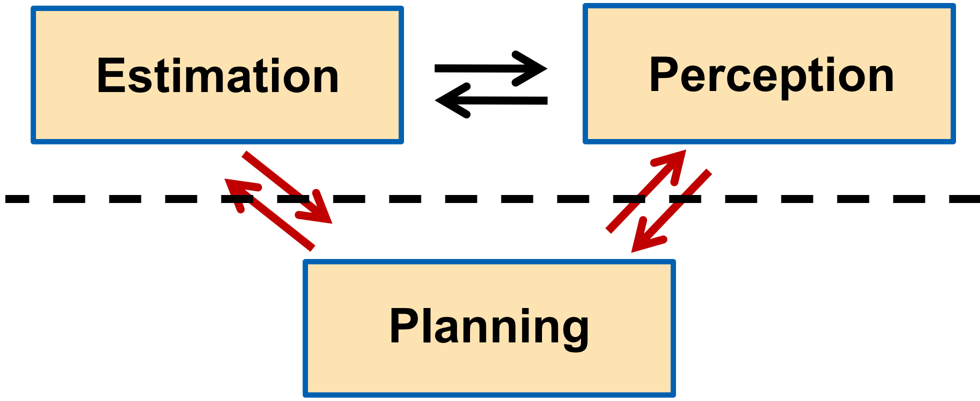


Estimation ⇄ **Perception**

- Allows computationally efficient probabilistic inference, given data
- For example, pose estimation given sensor data (e.g. images)

Introduction - Planning

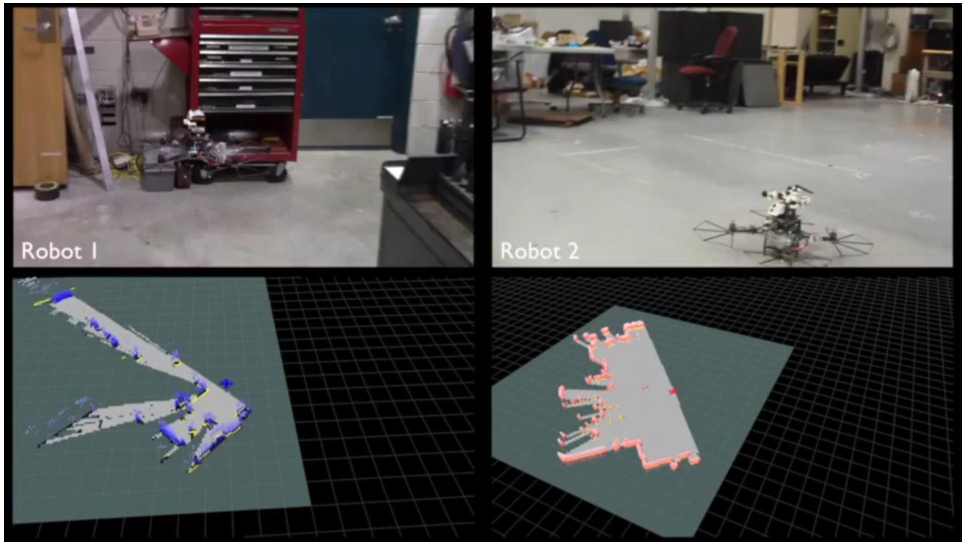
- Navigation and mapping in **unknown environment** without GPS
- Key components for autonomous operation include
  - Estimation and Perception:
    Where am I? What is the surrounding environment?
  - Planning: What to do next?



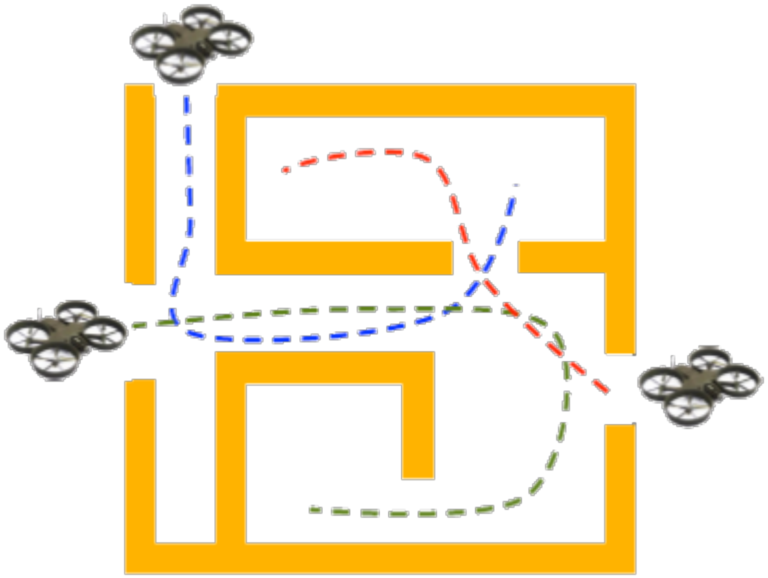
- Belief space planning (BSP) - fundamental problem in robotics

Introduction – Multi Robot SLAM

- Navigation and mapping in unknown environment with **multiple robots (MR)**
  - Robust and faster exploration/mapping
  - Higher accuracy in a multi robot collaborative framework



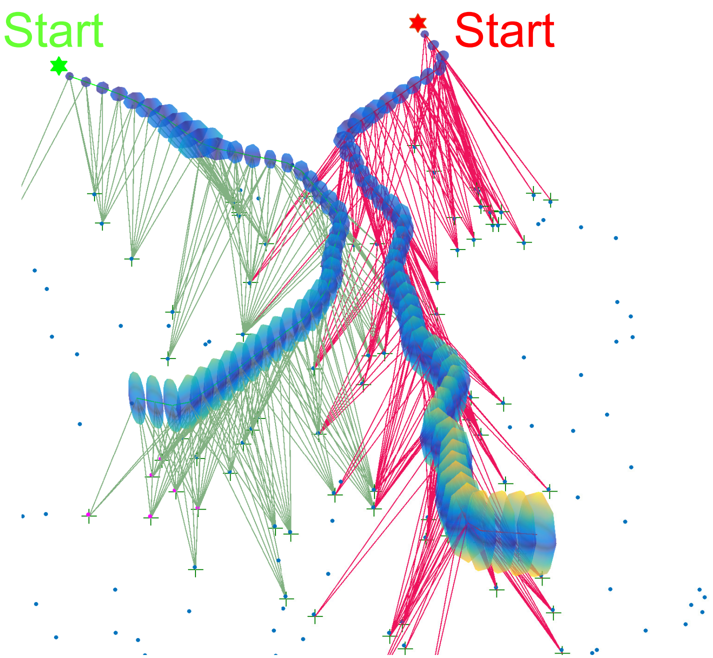
[J. Dong et al. '15 ICRA]



[Indelman '16 CSM, '14 ICRA]

Introduction - Multi Robot Belief Space Planning

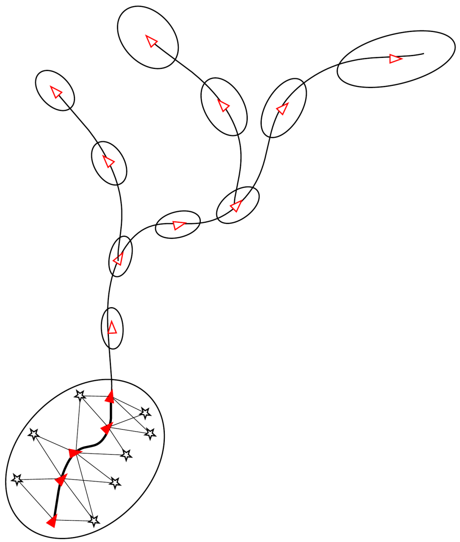
- Belief space planning in unknown environment with multiple robots (MR)
- Reason about uncertainty within planning and consider collaboration between robots



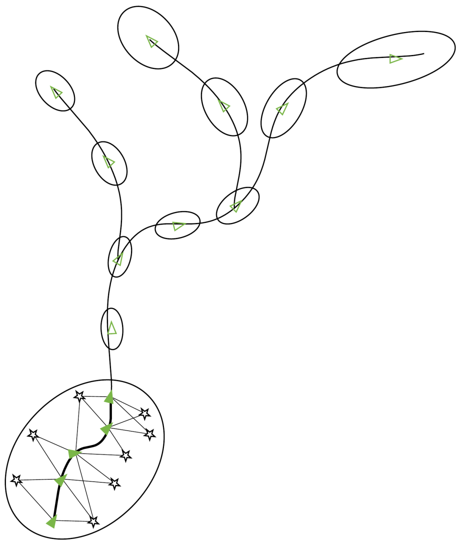
[Indelman '15 ISRR]

Related Work – Belief Space Planning (BSP)

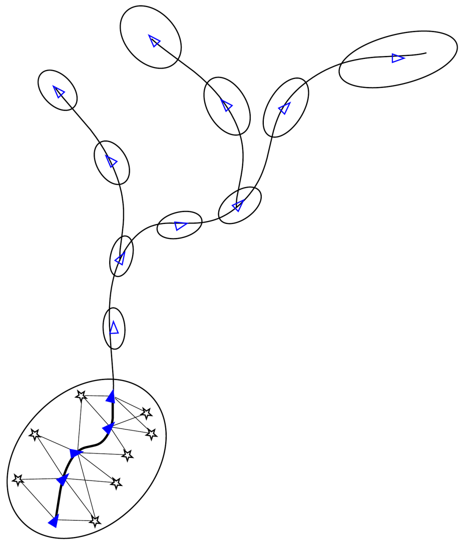
- Solving **multi-robot BSP** is in particular challenging
  - Involves considering **all combinations** of candidate paths of **all** robots
  - Existing approaches typically assume **environment/map is known**



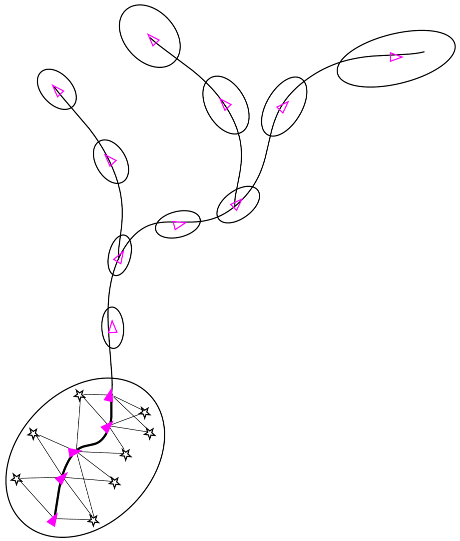
Red Robot



Green Robot



Blue Robot

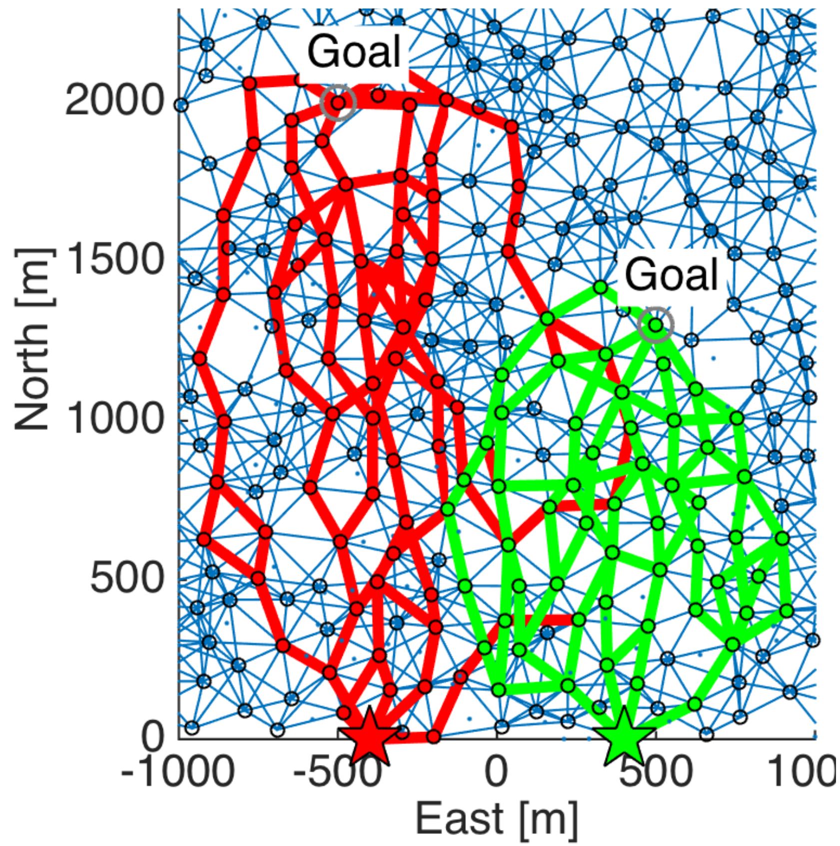


Purple Robot

Related Work – Sampling Methods

- Sampling Methods
  - Discretize the environment
  - Candidate trajectories
- Existing approaches
  - Rapidly exploring random trees (RRT)
  - Rapidly exploring random graph (RRG)
  - Probabilistic road map (PRM)

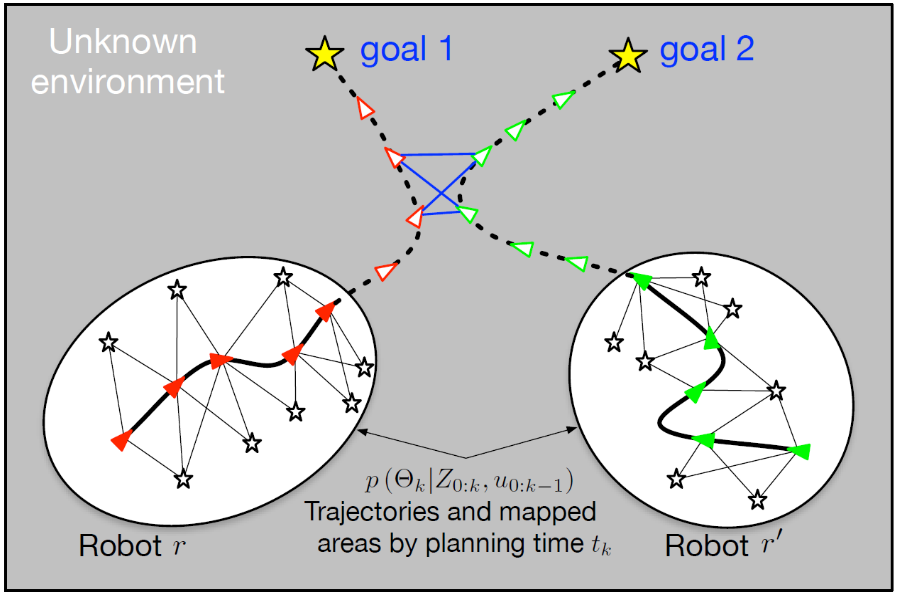
PRM



Related Work – Belief Space Planning (BSP)

– Existing approaches typically assume **environment/map is known**

- Recent works enable operation in unknown environments **[Kim et al. '14 IJRR] [Indelman et al. '15 IJRR] [Indelman '15 ISRR]**
- In particular, reason **about future observations of unknown environments within multi-robot belief space planning** **[Indelman '15 ISRR]**

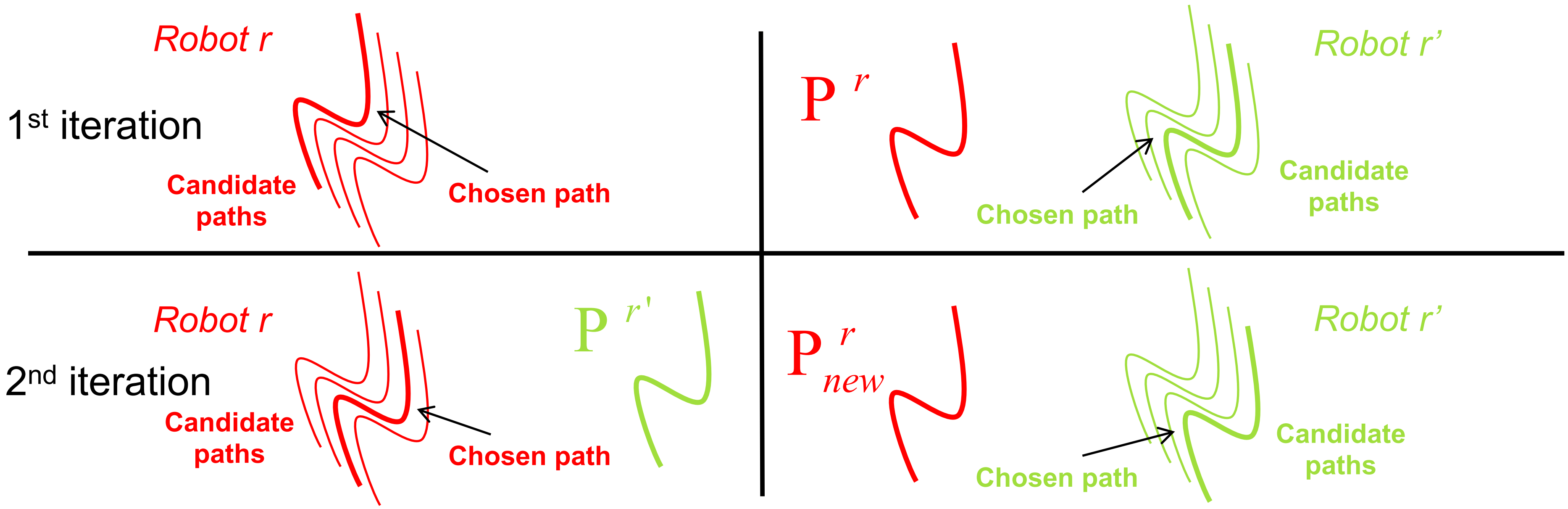


Related Work – Announced Path Approach

– Involves considering **all combinations** of candidate paths of **all** robots

- A common (sub-optimal) iterative approach to reduce computational effort: [Atanasov '14 TRO] [Levine '13 JAIS]

Calculations are performed from scratch at each iteration



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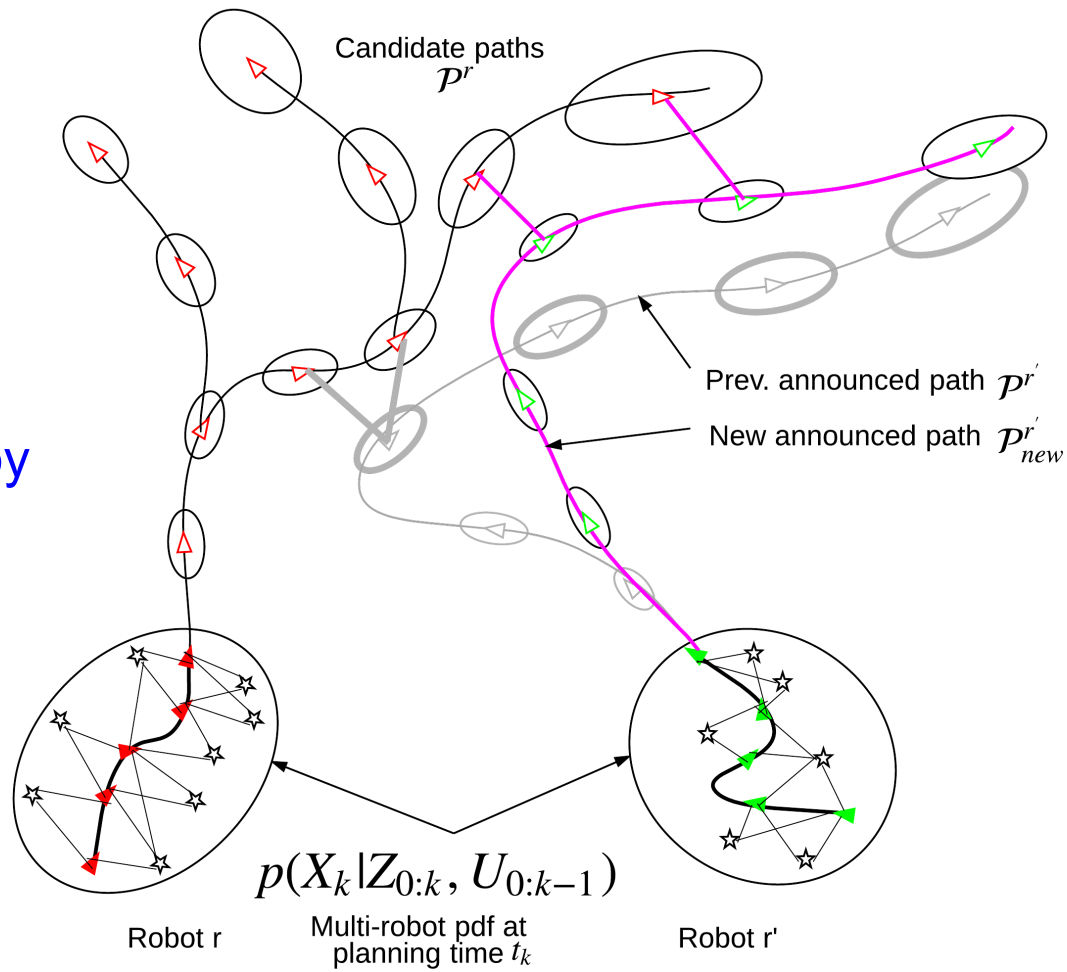
Contribution

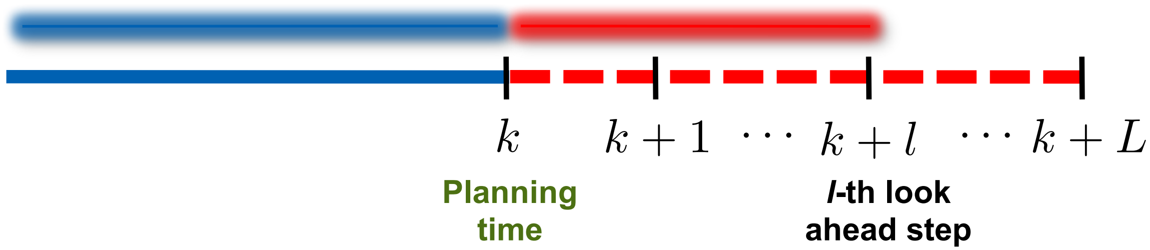
- Each time the announced path from some robot r' changes, each robot r has to re-evaluate its candidate paths

- **Key idea**
  - Not all paths are impacted due to change in the announced paths
  - Impacted paths can be efficiently re-evaluated by reusing calculations



Formulation - Multi-robot Belief Space Planning

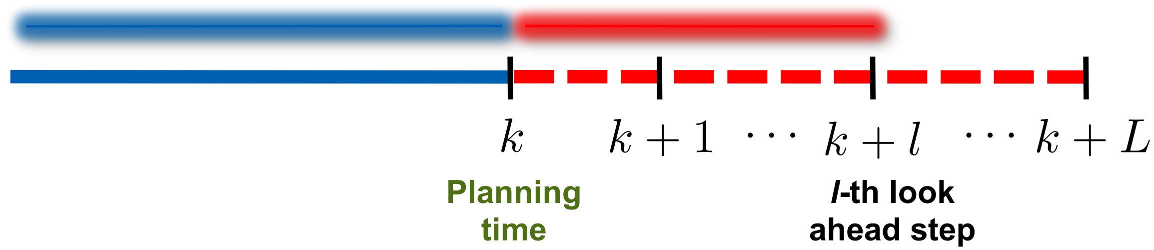
- Belief for robot r at a future time t_{k+l}: $b\left[X_{k+l}^{r}\right]$ B $p\left(X_{k+l}^{r} \mid Z_{0:k+l}^{r}, u_{0:k+l-1}^{r}\right)$



- X_{k+l}^{r} - Poses and landmarks estimated by robot r
- $Z_{0:k+l}^{r}$ - Observations available to robot r
- $u_{0:k+l-1}^{r}$ - Controls of robot r

Formulation - Multi-robot Belief Space Planning

- Belief for robot r at a future time t_{k+l}: $b\left[X^r_{k+l}\right] \triangleq p\left(X^r_{k+l} \mid Z^r_{0:k+l}, u^r_{0:k+l-1}\right)$



- Multi-robot objective function:

$$J(\mathrm{U}) = \mathrm{E}\left[\sum_{l=1}^{L}\sum_{r=1}^{R} c^r_l\left(b[X^r_{k+l}], u^r_{k+l}\right)\right]$$

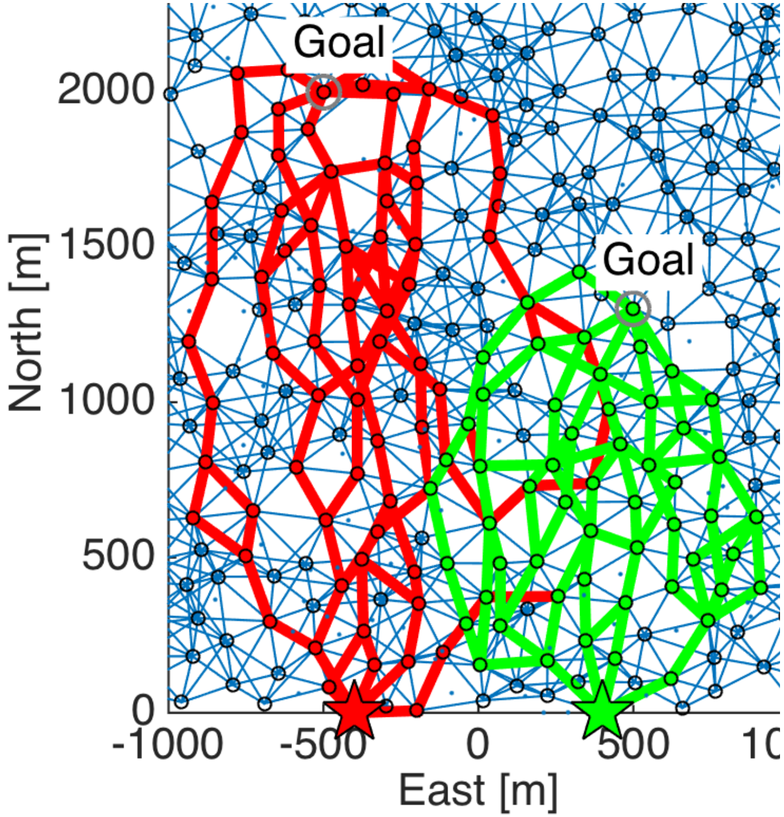
- Optimal controls for all R robots: $\mathrm{U}^{\star} = \underset{\mathrm{U}}{\operatorname{argmin}}\; J(\mathrm{U})$

Formulation - Multi-robot Belief Space Planning

- Probability distribution function (pdf) of multi robot at planning time t_k

$$b[\mathrm{P}] = p(X_k \mid Z_{0:k}, U_{0:k-1}) \prod_{r=1}^{R} \left[\prod_{l=1}^{L(\mathrm{P}^r)} p(x_{v_l}^r \mid x_{v_{l-1}}^r, u_{v_{l-1}}^r) \cdot p(Z_{v_l}^r \mid X_{k+l}^r) \prod_{\{i,j\}} p(z_{i,j}^{r,r'} \mid x_{v_i}^r, x_{v_j}^{r'}) \right]$$

- P - Some specific candidate paths for all robots $\mathrm{P} = \left[\mathrm{P}^r, \mathrm{P}^{r'}, \ldots \right]$



- Maximum a posteriori (MAP) inference:

$$b[\mathrm{P}] = N\left(\hat{X}(\mathrm{P}), \Lambda^{-1}(\mathrm{P}) \right)$$

- State transition and observation models:

$$x_{i+1}^r = f(x_i^r, u_i^r, w_i^r), \quad z_{i,j}^r = h(x_i^r, x_j^r, v_{i,j}^r)$$

Formulation - Multi-robot Belief Space Planning

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Prior

$$\Lambda(\mathrm{P}) = \Lambda_k + \sum_{r=1}^{R} \left[\sum_{l=1}^{L(\mathrm{P}^r)} \Lambda^{r,local}_l + \sum_{\{i,j\}} \Lambda^{r,r'}_{i,j} \right]$$

- Maximum a posteriori (MAP) inference:

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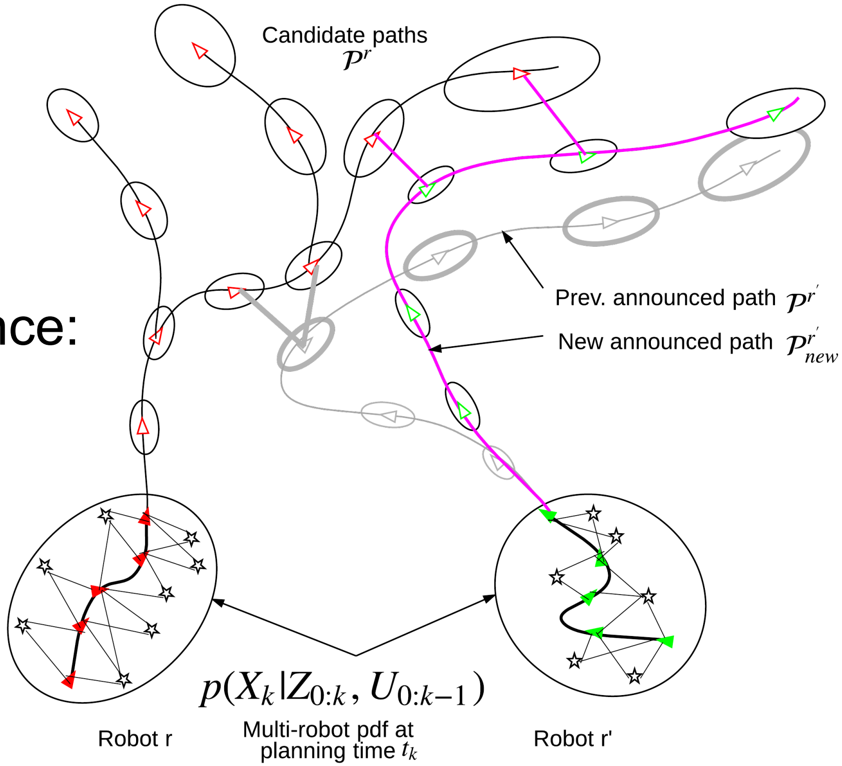

Formulation - Multi-robot Belief Space Planning

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Prior

Local information

$$\Lambda(\mathrm{P}) = \Lambda_k + \sum_{r=1}^{R} \left[\sum_{l=1}^{L(\mathrm{P}^r)} \Lambda_l^{r,local} + \sum_{\{i,j\}} \Lambda_{i,j}^{r,r'} \right]$$

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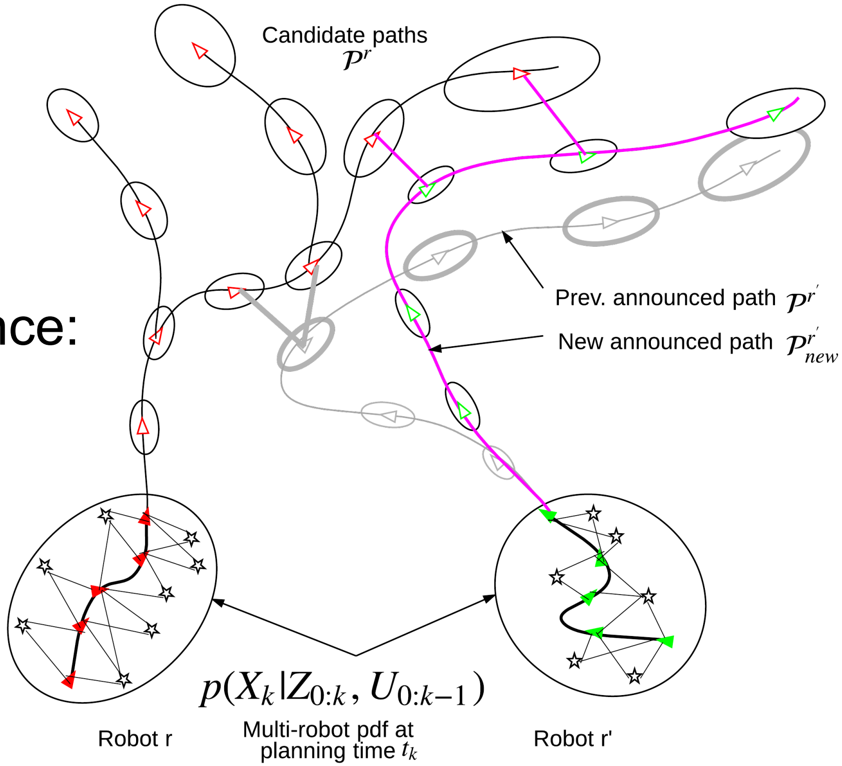

Formulation - Multi-robot Belief Space Planning

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Prior — Local information — Mutual observations

$$\Lambda(\mathrm{P}) = \Lambda_k + \sum_{r=1}^{R} \left[\sum_{l=1}^{L(\mathrm{P}^r)} \Lambda_l^{r,local} + {\color{red}\sum_{\{i,j\}} \Lambda^{r,r'}_{i,j}} \right]$$

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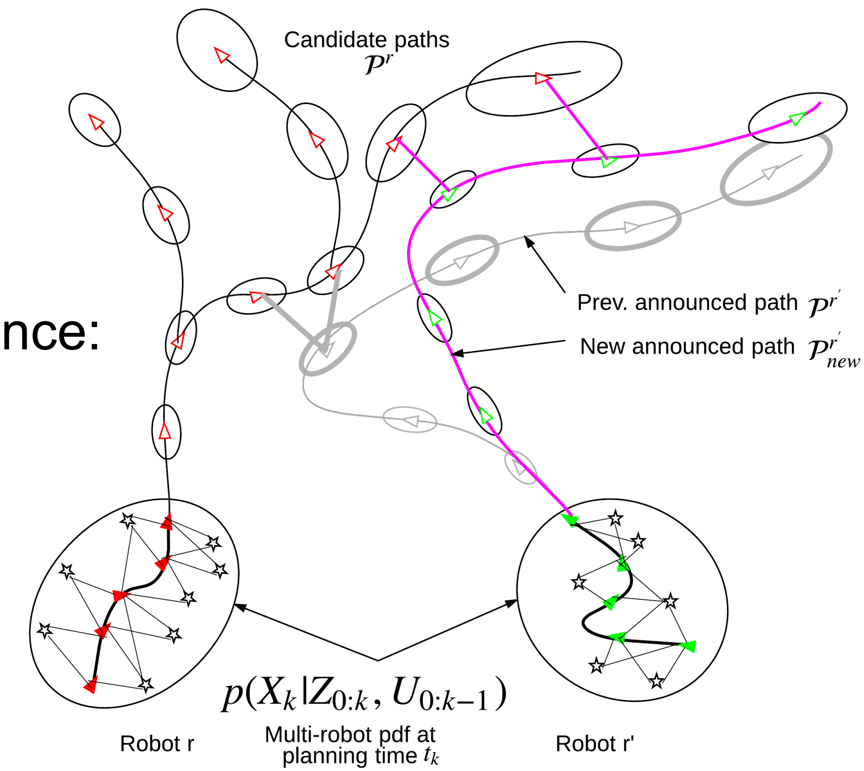

Formulation - Multi-robot Belief Space Planning

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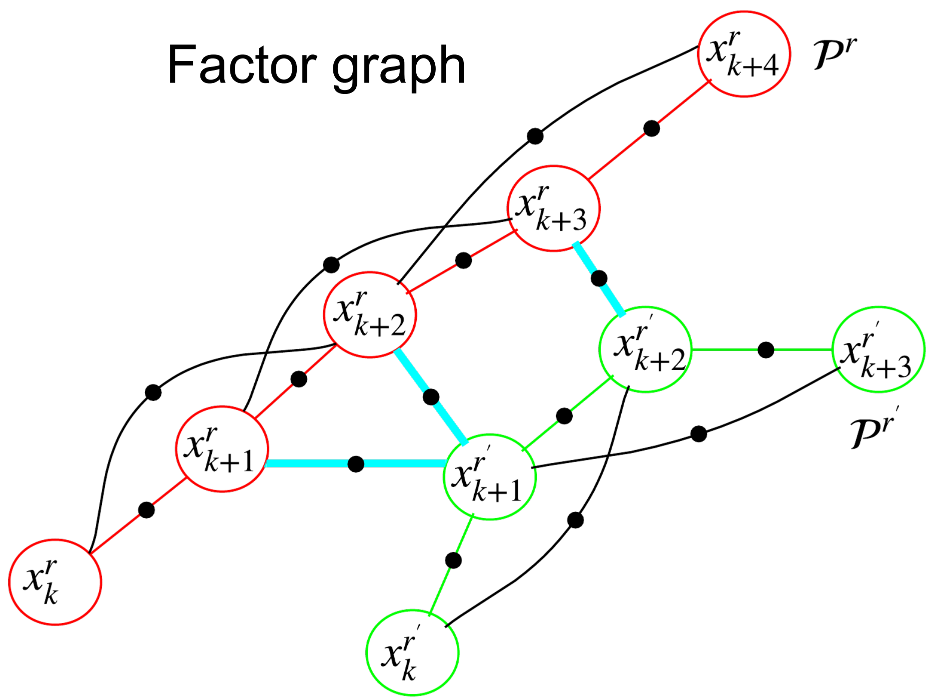
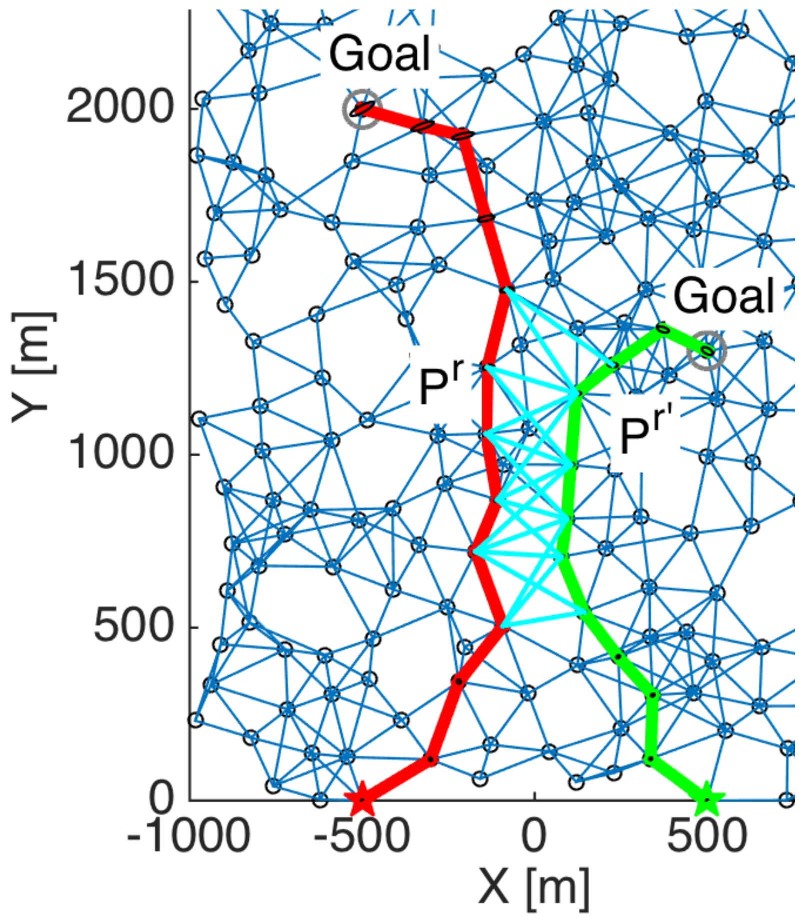


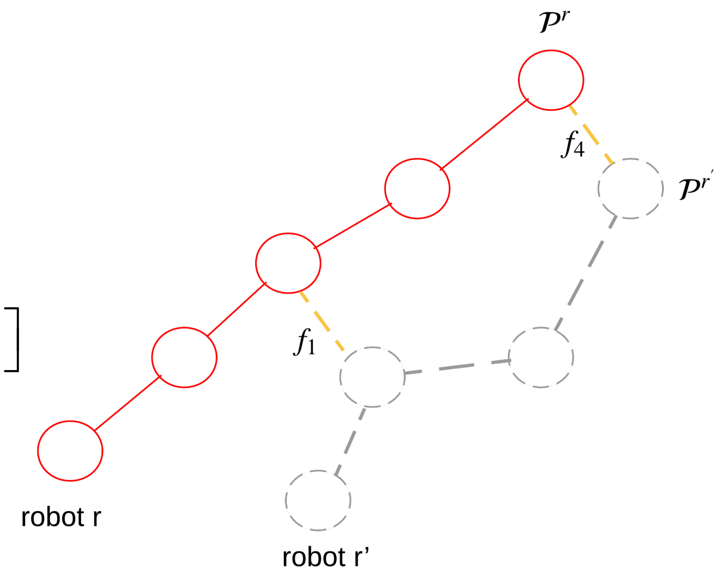


Factor Graph Inference

- Joint state $\mathrm{P}^{r}, \mathrm{P}^{r'}$:

$$b[\mathrm{P}^{r}, \mathrm{P}^{r'}] = p(X_k \mid \mathrm{Z}_{0:k}, \mathrm{U}_{0:k-1}) \prod_{l=1}^{L(\mathrm{P}^{r})} \left[p(x^{r}_{v_l} \mid x^{r}_{v_{l-1}}, u^{r}_{v_{l-1}}) \cdot p(Z^{r}_{v_l} \mid X^{r}_{k+l}) \right]$$

$$\prod_{l=1}^{L(\mathrm{P}^{r'})} \left[p(x^{r'}_{v_l} \mid x^{r'}_{v_{l-1}}, u^{r'}_{v_{l-1}}) \cdot p(Z^{r'}_{v_l} \mid X^{r'}_{k+l}) \right] \prod_{r=1}^{R} \left[\prod_{\{i,j\}} p(z^{r,r'}_{i,j} \mid x^{r}_{v_i}, x^{r'}_{v_j}) \right]$$



- Joint state $\mathrm{P}^{r}, \mathrm{P}^{r'}_{new}$:

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Factor Graph Inference

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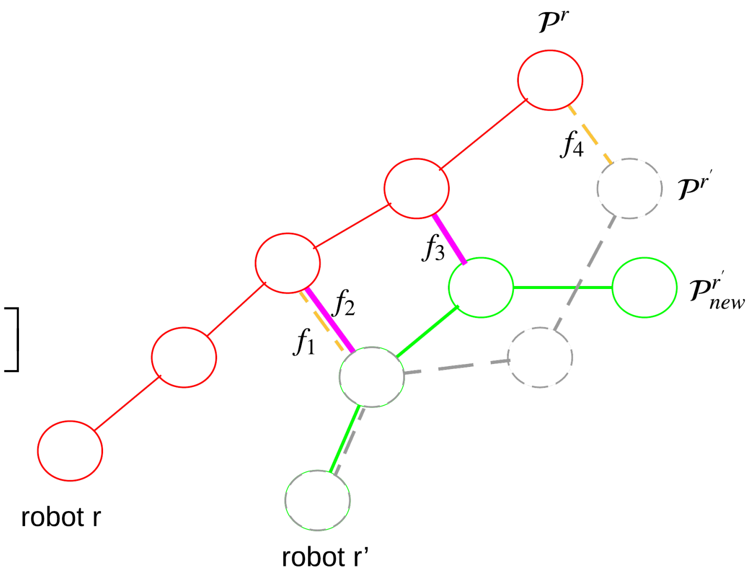

- Joint state $\mathrm{P}^{r}, \mathrm{P}^{r'}_{new}$:

Does not change

$$b[\mathrm{P}^{r}, \mathrm{P}^{r'}_{new}] = p(X_k \mid \mathrm{Z}_{0:k}, \mathrm{U}_{0:k-1}) \prod_{l=1}^{L(\mathrm{P}^{r})} \left[p(x^{r}_{v_l} \mid x^{r}_{v_{l-1}}, u^{r}_{v_{l-1}}) \cdot p(Z^{r}_{v_l} \mid X^{r}_{k+l}) \right]$$

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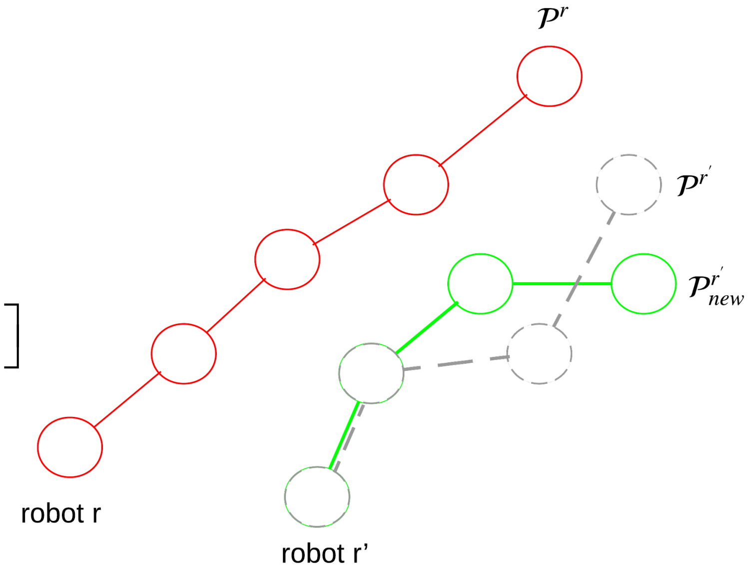

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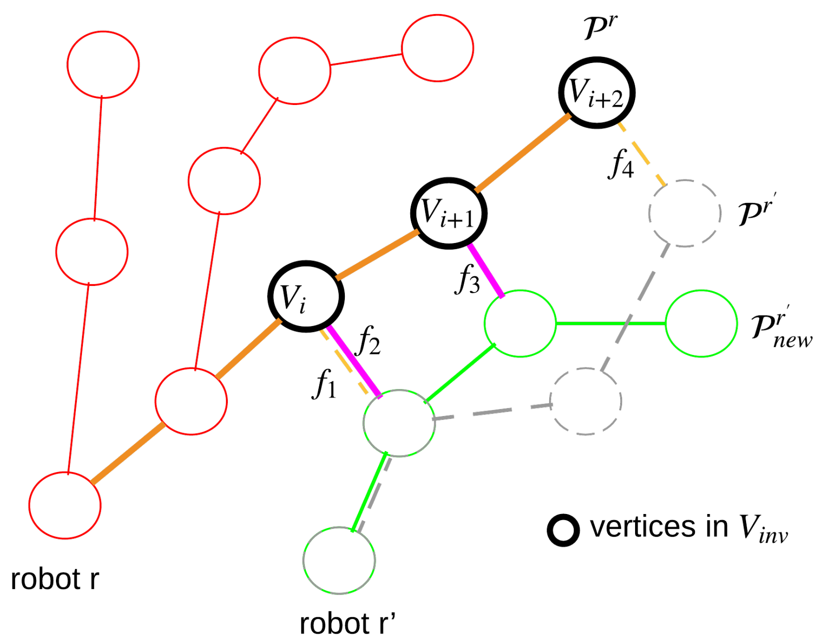
$$\prod_{l=1}^{L(\mathrm{P}^{r'})} \left[p(x^{r'}_{v_l} \mid x^{r'}_{v_{l-1}}, u^{r'}_{v_{l-1}}) \cdot p(Z^{r'}_{v_l} \mid X^{r'}_{k+l}) \right]$$

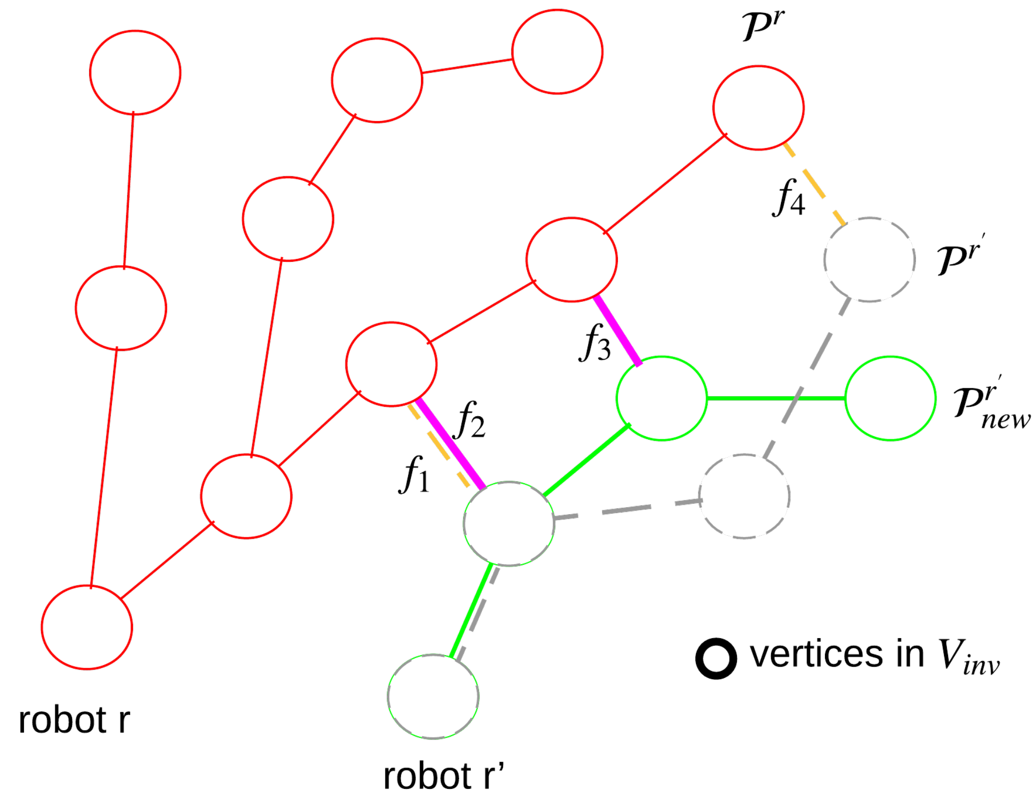
Algorithm Overview – High Level

- Key Idea
  - Not all paths are impacted due to change in the announced paths
  - Impacted paths can be efficiently re-evaluated by reusing calculations
- Iterate **over vertices** in previous and new announced paths
- Identify **involved** vertices in multi-robot factors – collect into set V_{inv}
- Identify and **mark** involved paths from all candidate paths



Algorithm Overview – Details

- Key Idea
  - Iterate vertices $v^{r'}$ that belong to $\mathcal{P}^{r'}$ or $\mathcal{P}^{r'}_{new}$
    - Find all nearby vertices $\{v^r\} \subseteq V^r$ to $v^{r'}$
    - Add v_i to V_{inv}
  - Identify multi-robot factors to be added or removed
  - Mark all candidate paths that go through vertex



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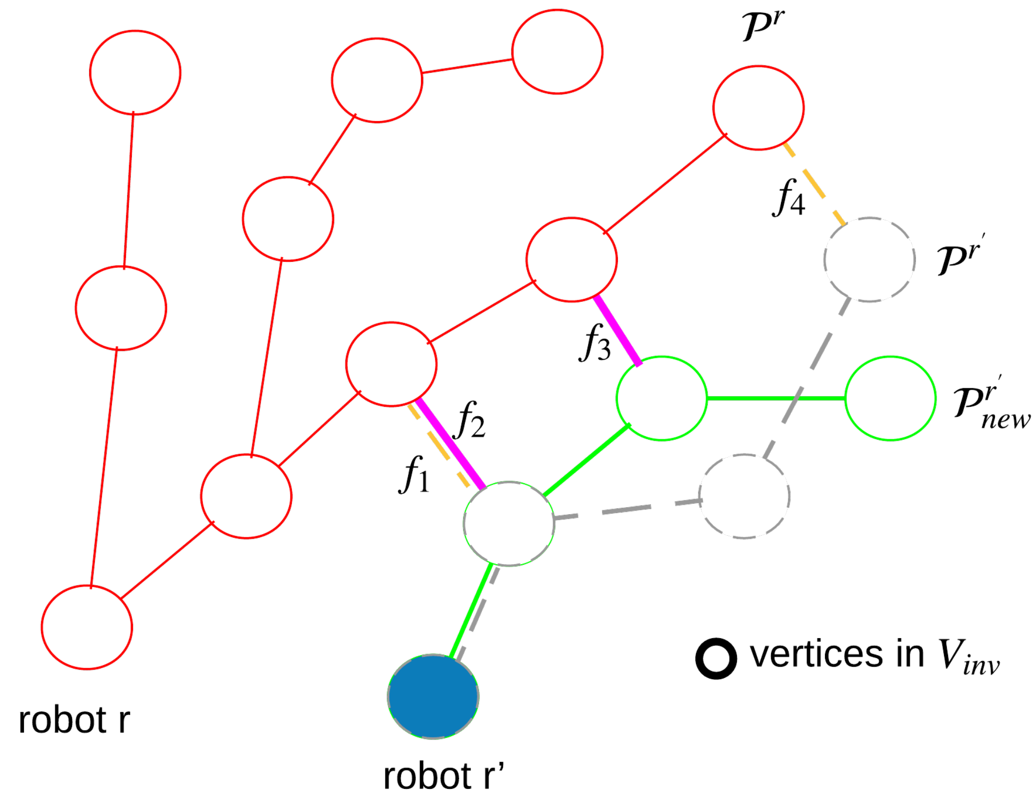

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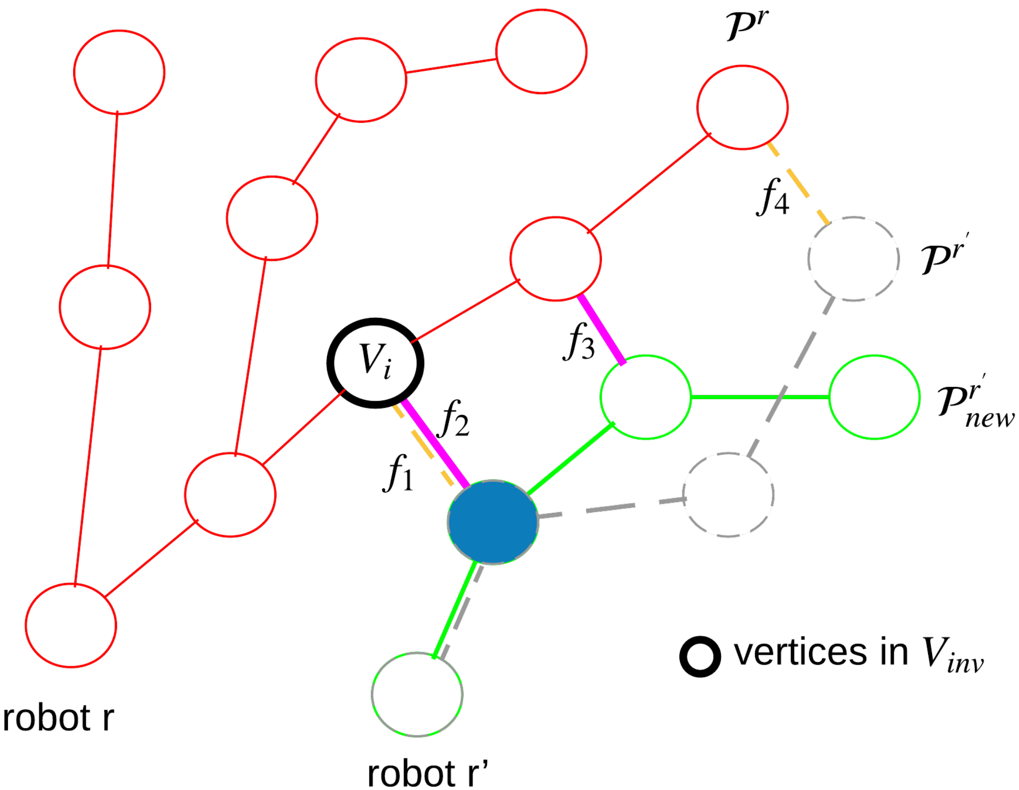

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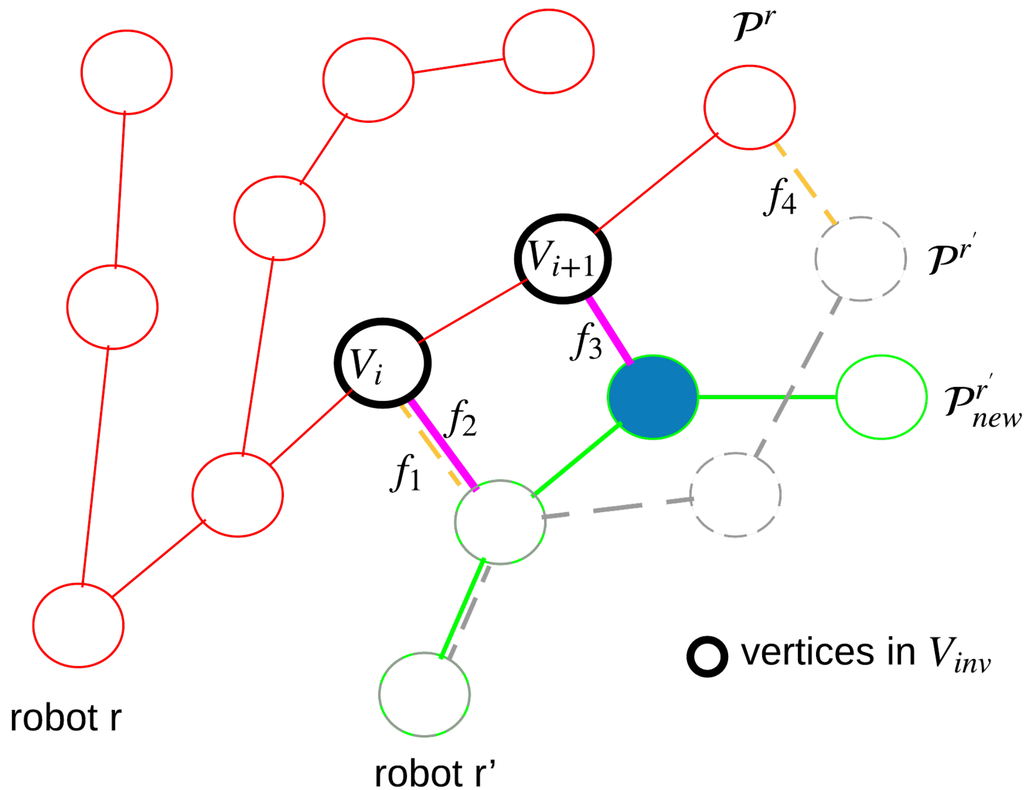

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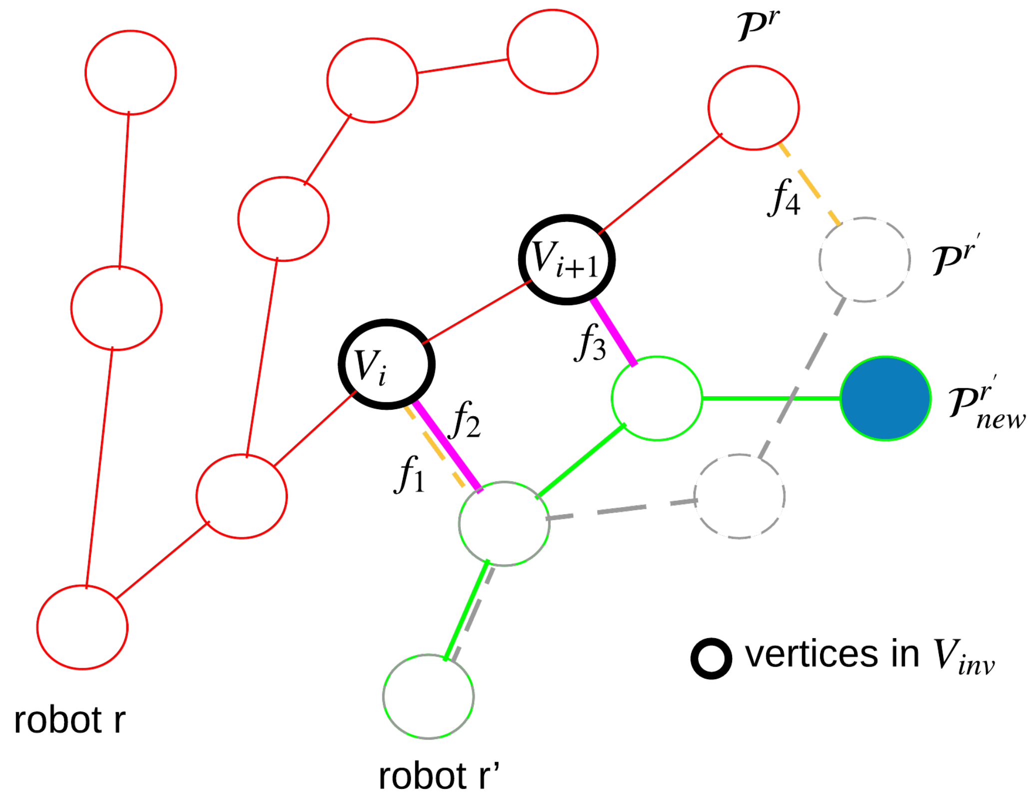

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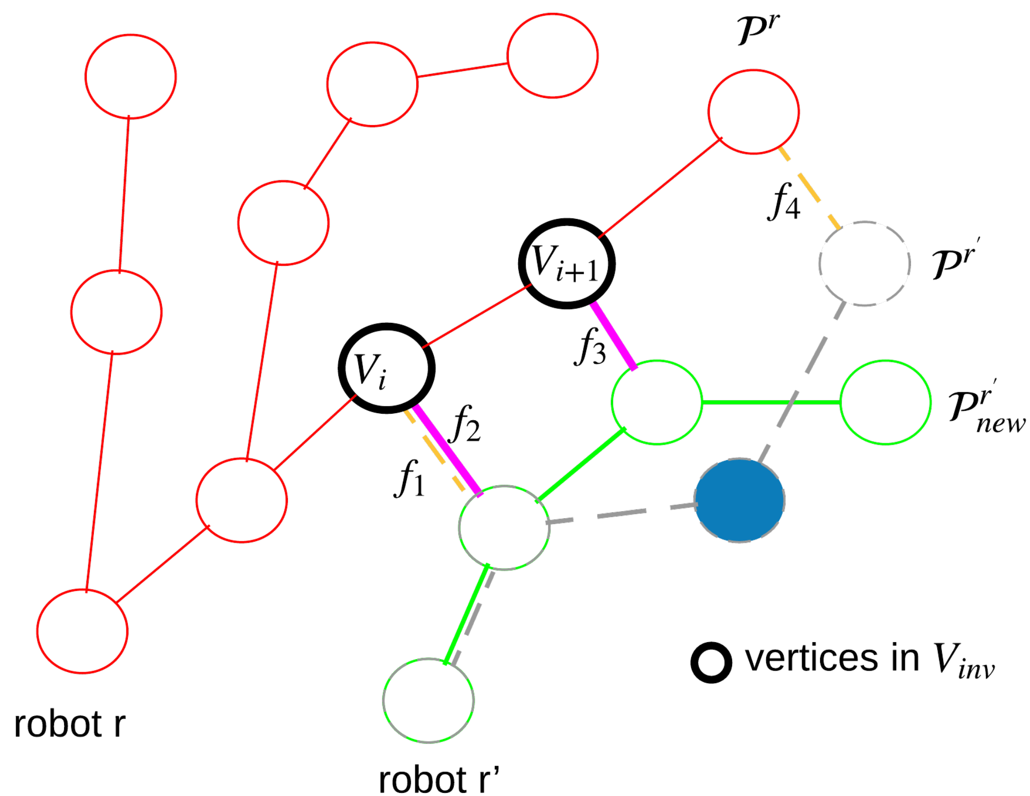
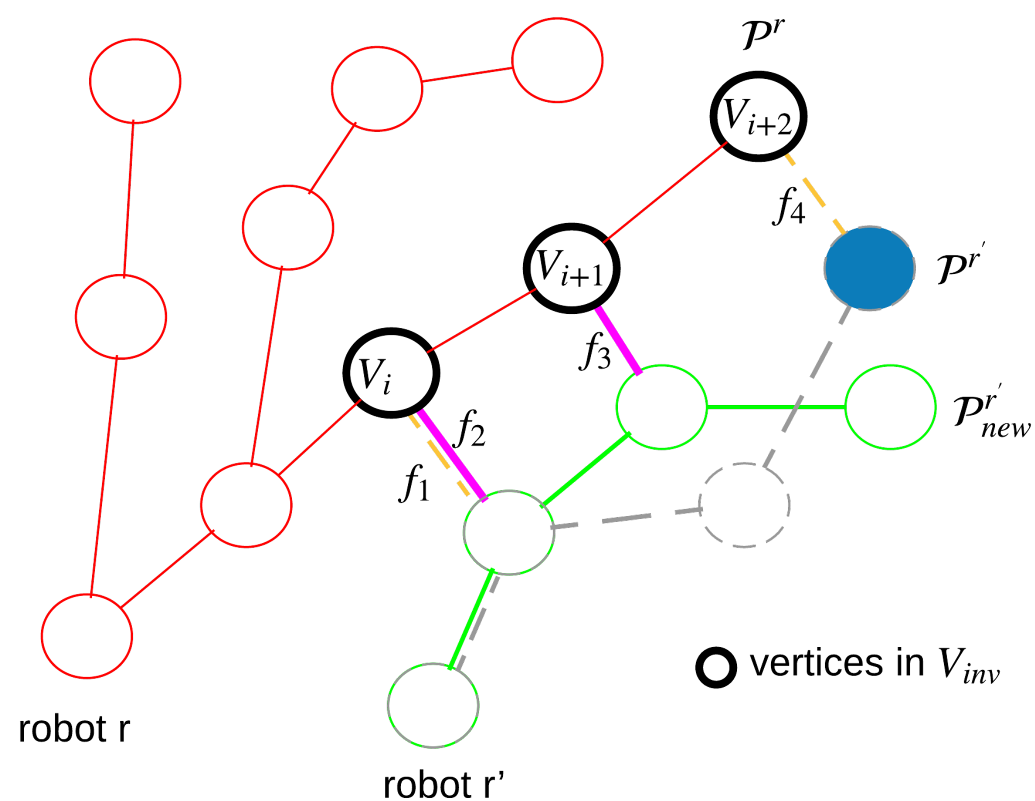


Algorithm Overview – Details

- Key Idea
  - Iterate vertices $v^{r'}$ that belong to $\mathcal{P}^{r'}$ or $\mathcal{P}^{r'}_{new}$
    - Find all nearby vertices $\{v^r\} \subseteq V^r$ to $v^{r'}$
    - Add v_i to V_{inv}
  - Identify multi-robot factors to be added or removed
  - Mark all candidate paths that go through vertex



Algorithm Overview – Details

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  - Iterate vertices $v^{r'}$ that belong to $\mathcal{P}^{r'}$ or $\mathcal{P}^{r'}_{new}$
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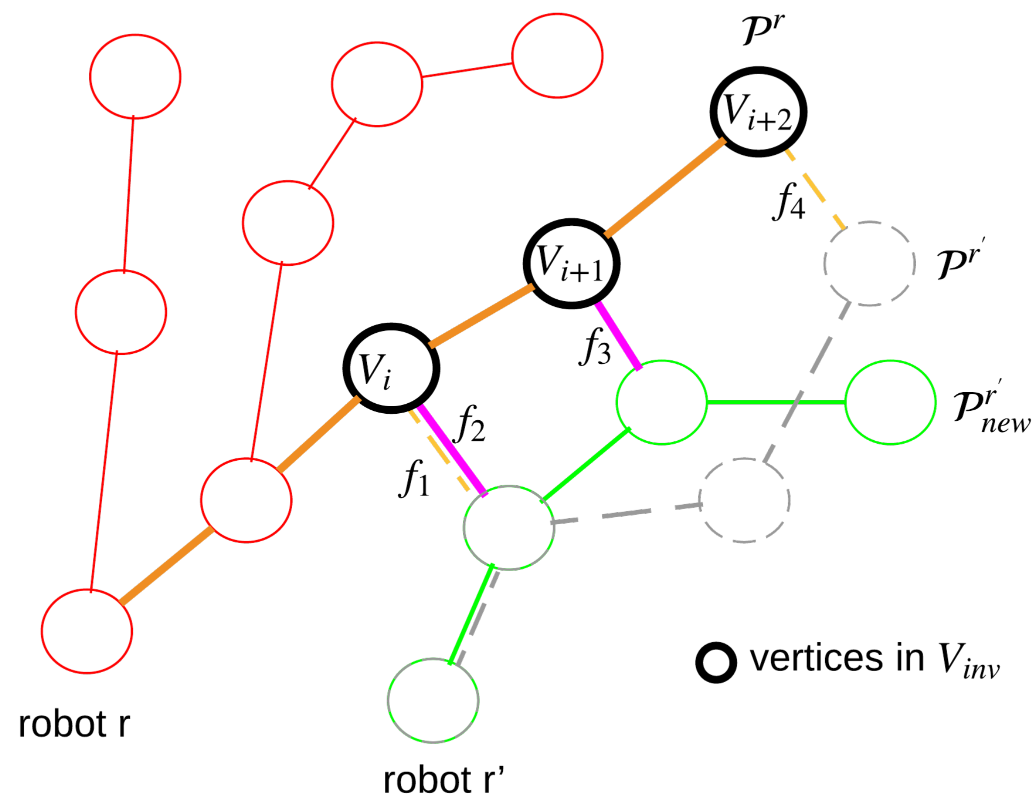
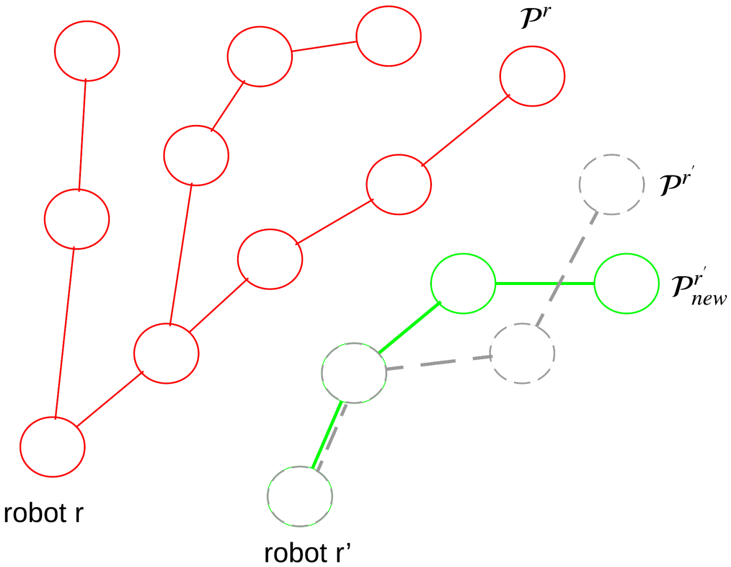


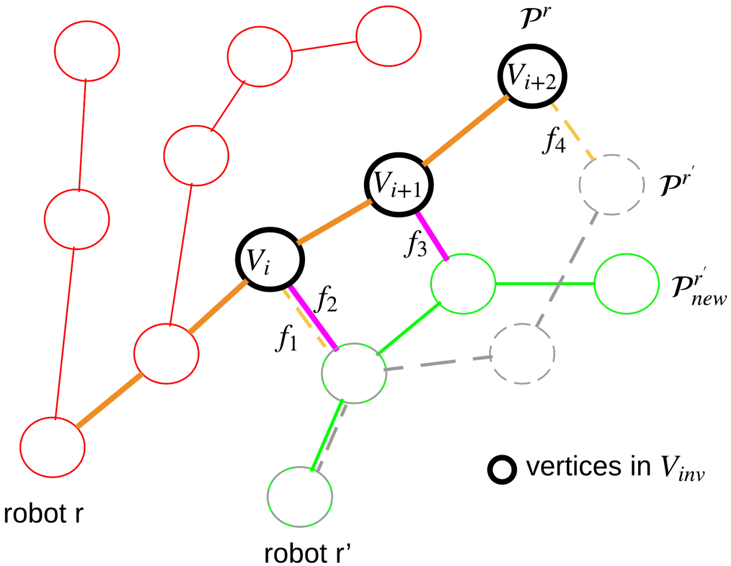
Algorithm Overview

- **Non-marked** paths
  - Calculate once the change $\Delta J^{r'}$ B $\sum_{l=1}^{L} \Delta c_l^{r'}$
  - Add $\Delta J^{r'}$ to old objective function



Algorithm Overview

- **Non-marked** paths
  - Calculate once the change $\Delta J^{r'}$ B $\sum_{l=1}^{L} \Delta c_l^{r'}$
  - Add $\Delta J^{r'}$ to old objective function



- **Marked** paths
  - Iterate over vertices in V_{inv}, add or remove multi-robot factors
  - Evaluate objective function from new Information matrix

Our method

$$\Lambda'_{k+l} = \Lambda_{k+l} - \sum_{\substack{f \in FG \\ f \notin FG' \\ f.t \le t_{k+l}}} \Lambda(f) + \sum_{\substack{f \in FG' \\ f \notin FG \\ f.t \le t_{k+l}}} \Lambda(f)$$

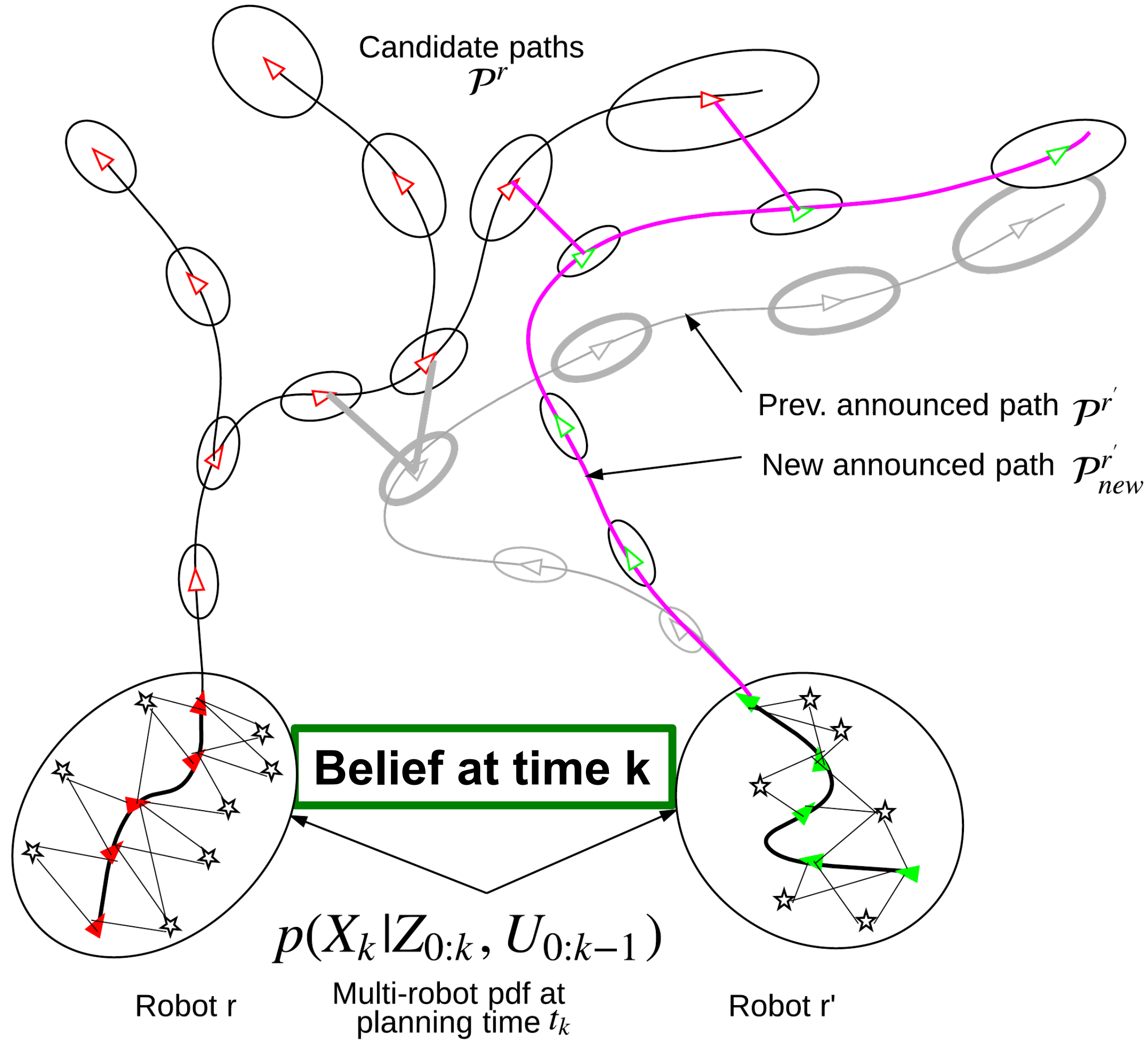
Standard method

$$\Lambda(\mathrm{P}) = \Lambda_k + \sum_{r=1}^{R} \left[\sum_{l=1}^{L(\mathrm{P}^r)} \Lambda_l^{r,local} + \sum_{\{i,j\}} \Lambda_{i,j}^{r,r'} \right]$$

From previous step!

Algorithm Overview – Prior Correlation

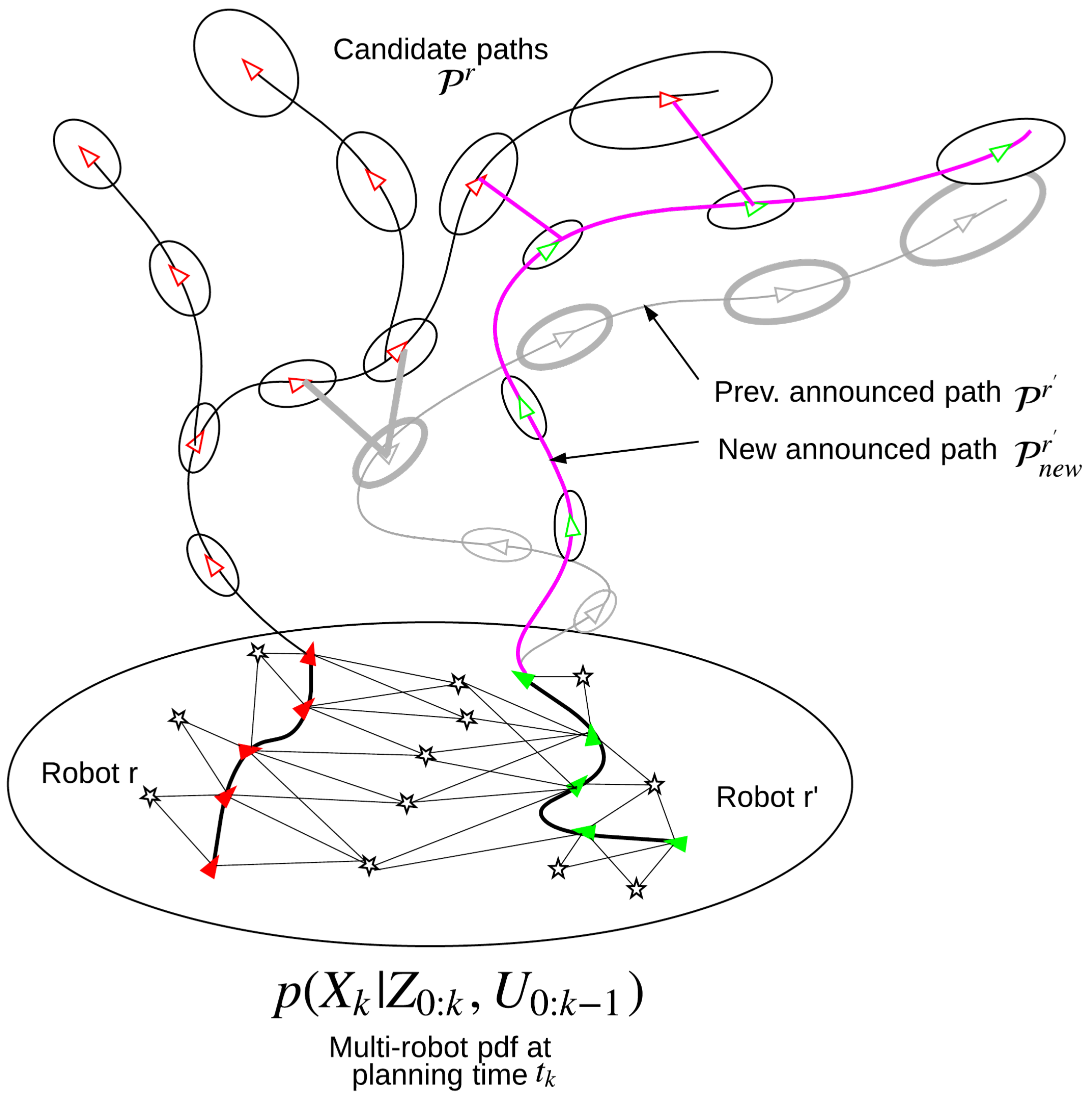
- No prior correlation at time k



Algorithm Overview – Prior Correlation

- With prior correlation at time k

Belief at time k

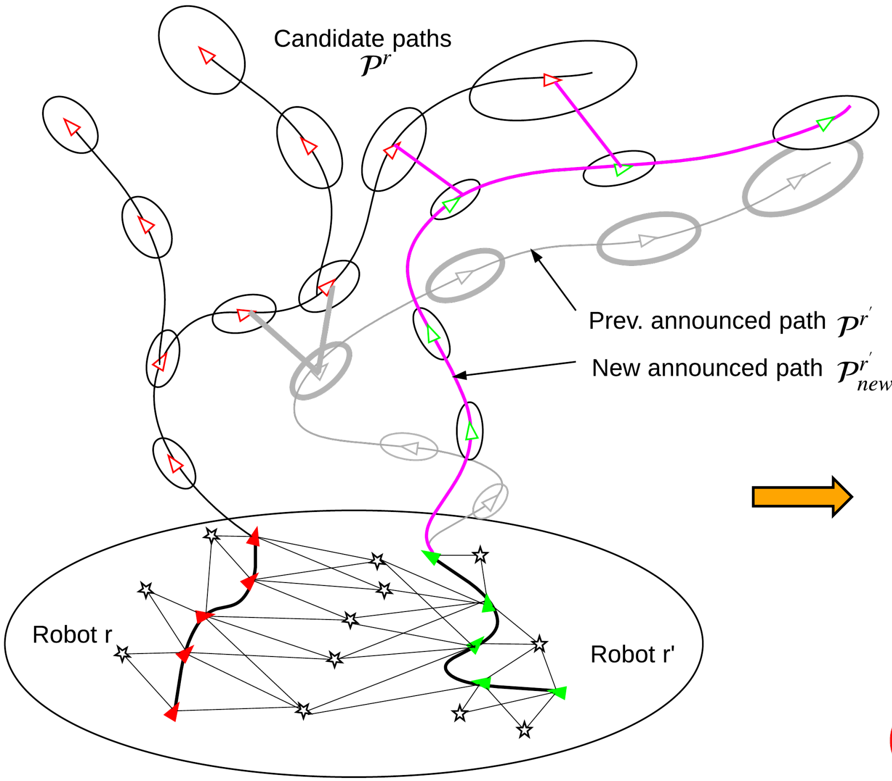


Algorithm Overview – Prior Correlation

- So far – robots' beliefs at time k were assumed to be not correlated
- In practice, correlation may exist, e.g.:
  - Robots have observed a mutual scene (or landmarks)
  - Robots made a direct observation of each other
- Our approach handles these cases as well (next slides)

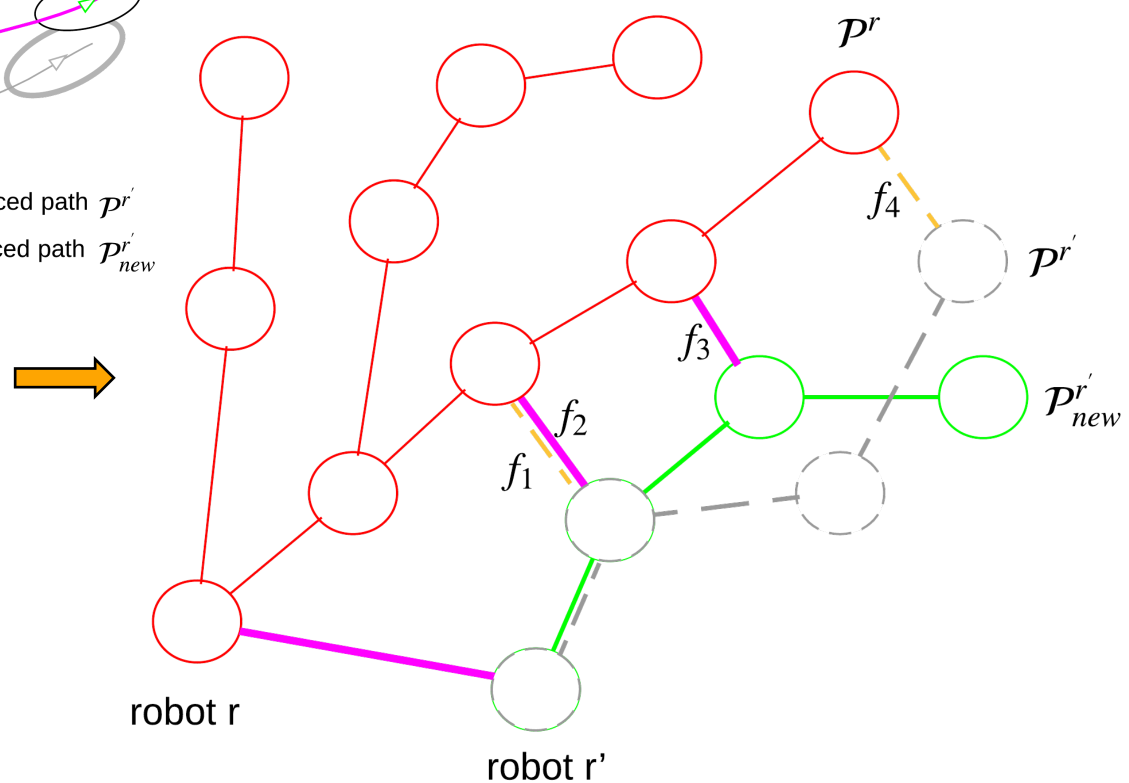
Algorithm Overview – Prior Correlation

- Two possible cases
  - **With multi-robot factors**
  - Without multi-robot factors



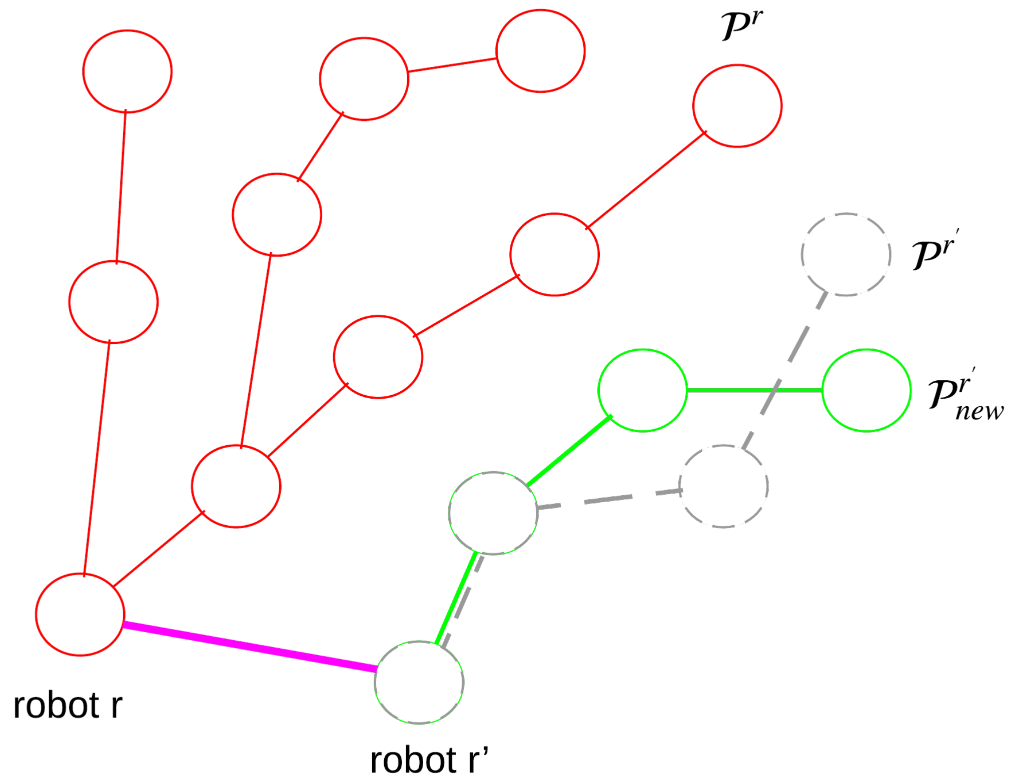
$p(X_k | Z_{0:k}, U_{0:k-1})$

Multi-robot pdf at planning time t_k



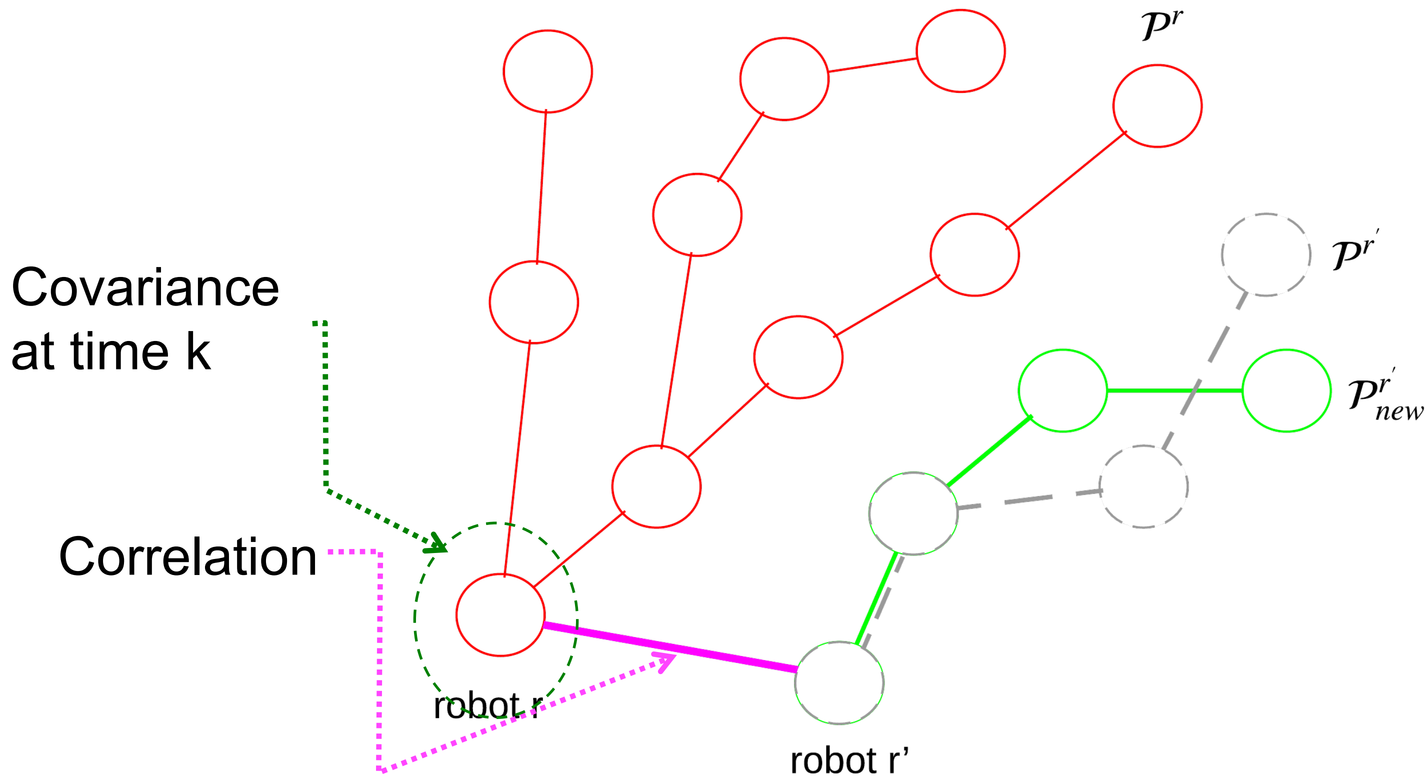
Algorithm Overview – Prior Correlation

- Two possible cases
  - With multi-robot factors
  - **Without multi-robot factors**



Algorithm Overview – Prior Correlation

- Two possible cases
  - With multi-robot factors
  - **Without multi-robot factors**
- If covariance changes significantly, mark all paths
- Otherwise, do not mark

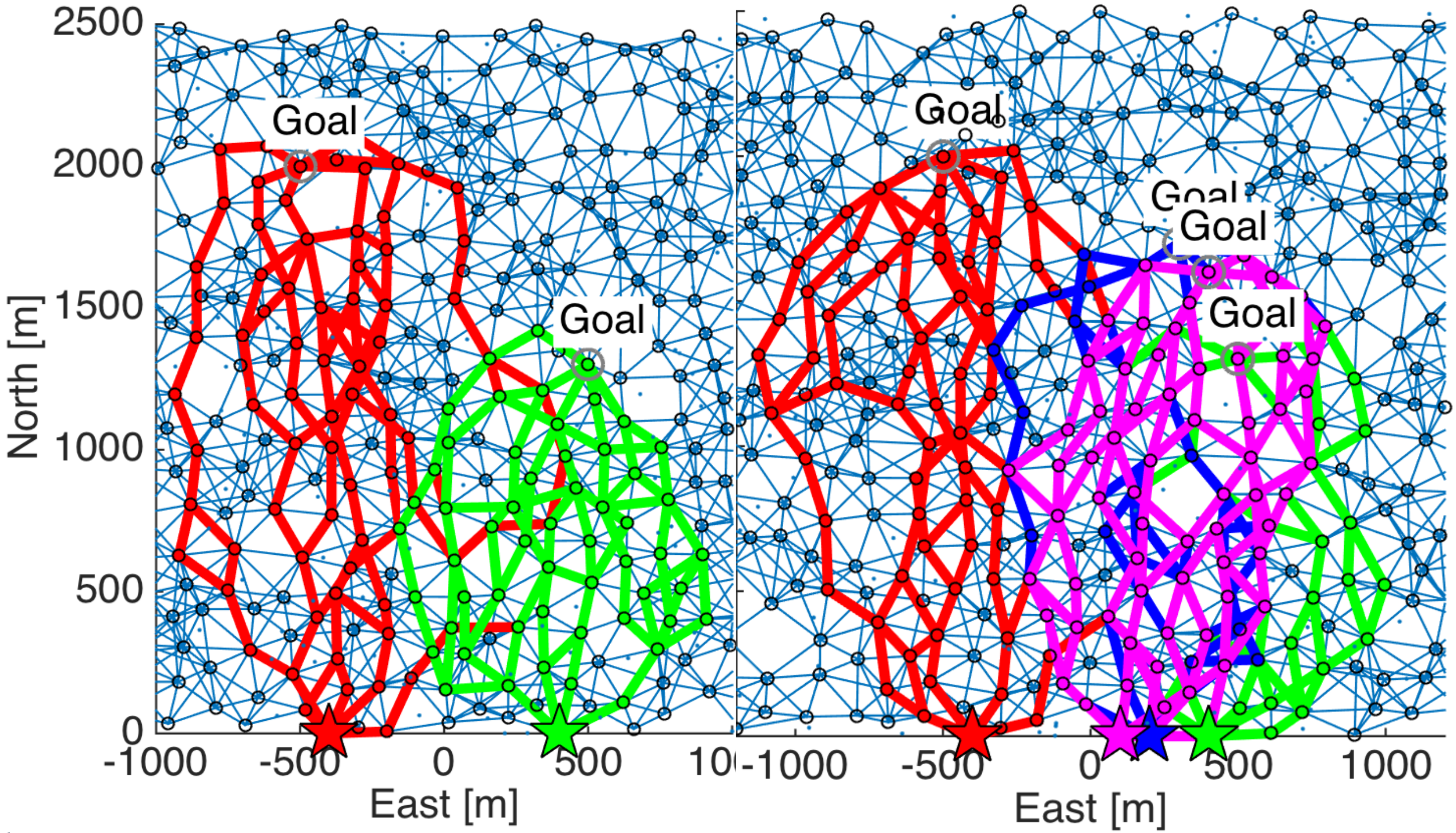


Results

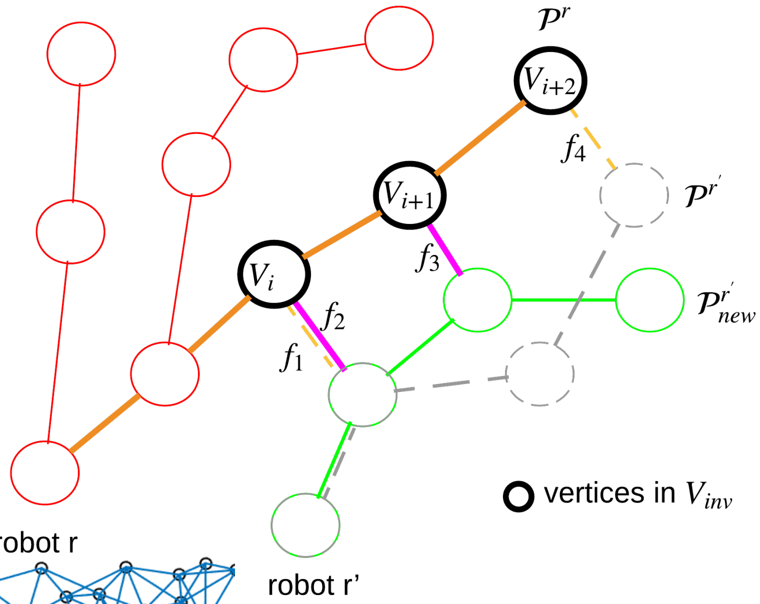
- Scenario: multiple robots autonomously navigating to goals in unknown environment
- Simulation results
  - 25 candidate paths per robot
  - PRM
  - No GPS
- Next slides:
  - Basic scenario: In-depth study for single goal
  - Larger scale scenario: multiple goals and planning sessions, SLAM in between

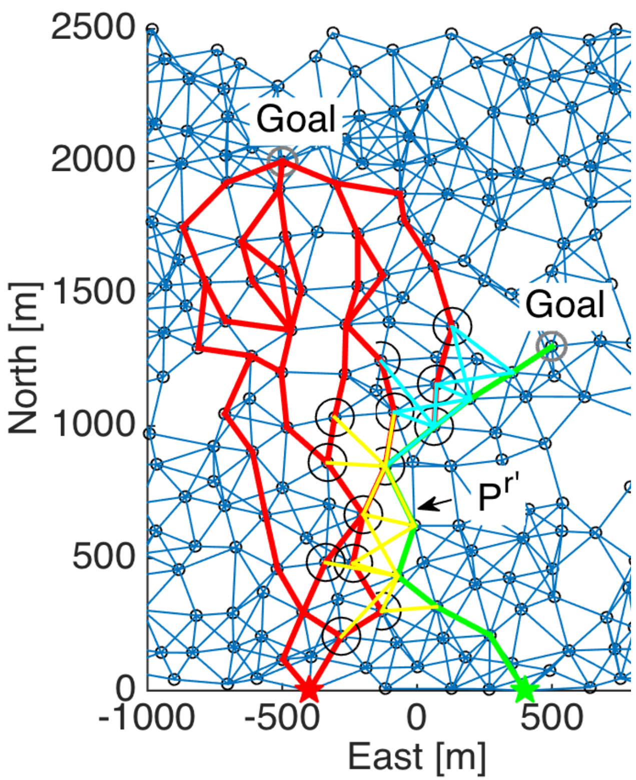
Results - Basic scenario

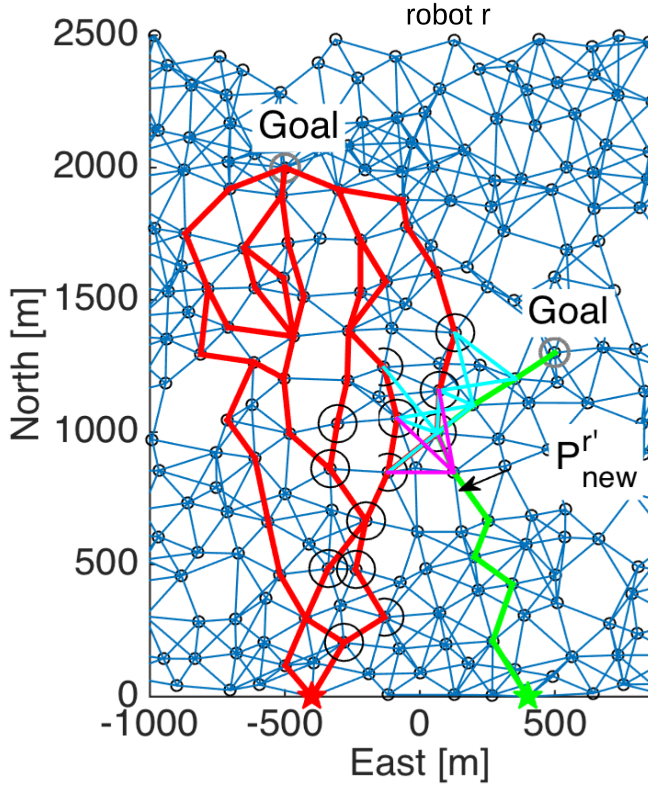
2 robots: 25 candidate paths **4 robots: 25 candidate paths**



Results - Basic scenario



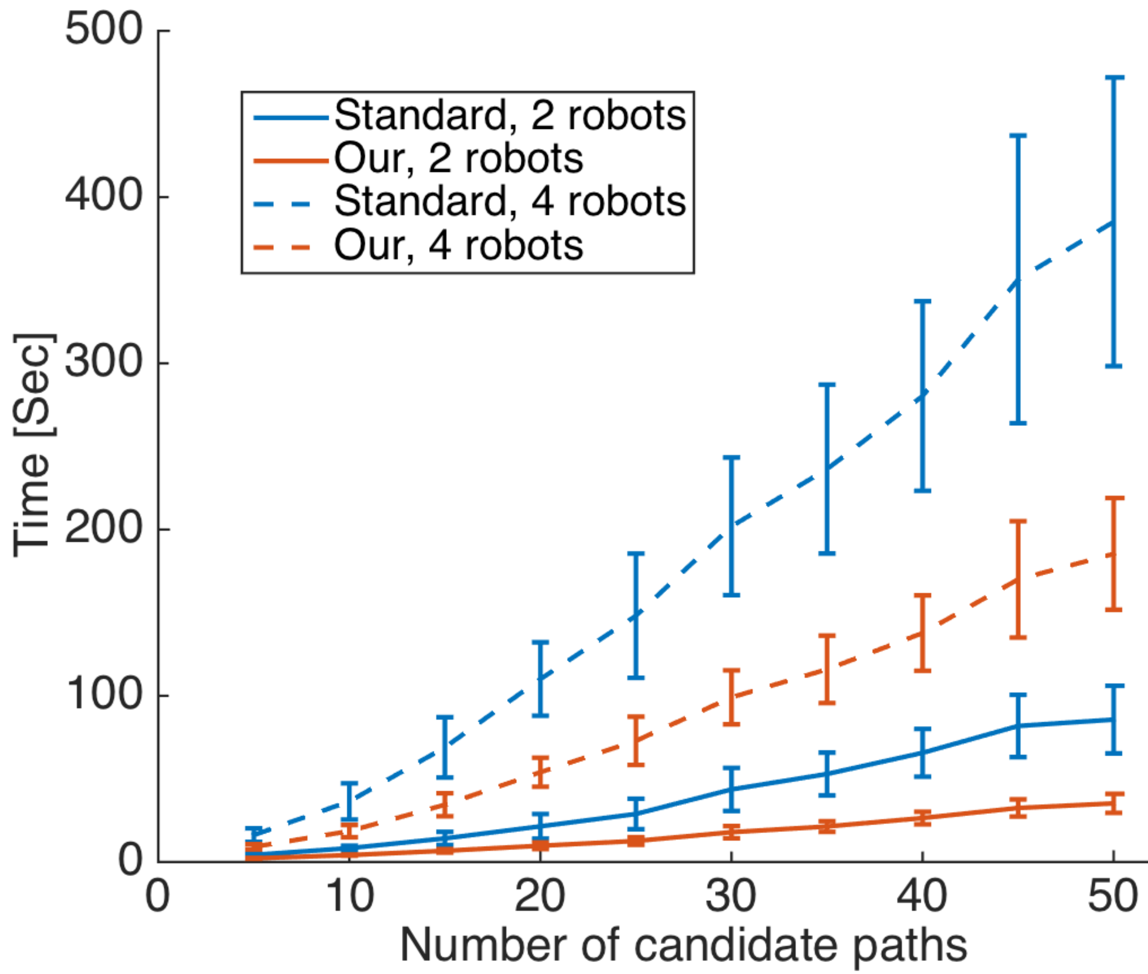




Results - Basic scenario

- **Statistical study of 50 runs (2 and 4 robots):**

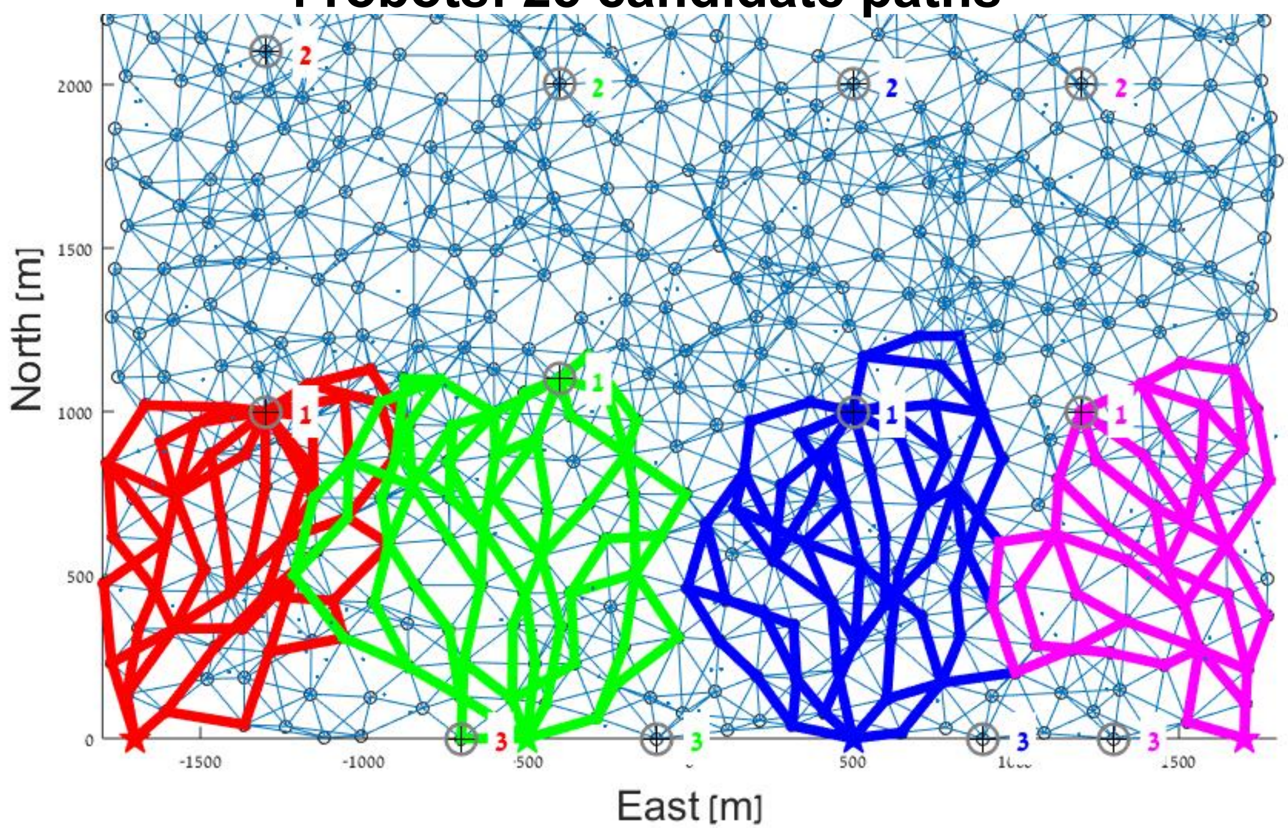
- **More than twice efficient**
- **Same results as Standard approach**



Results – Larger Scale Scenario

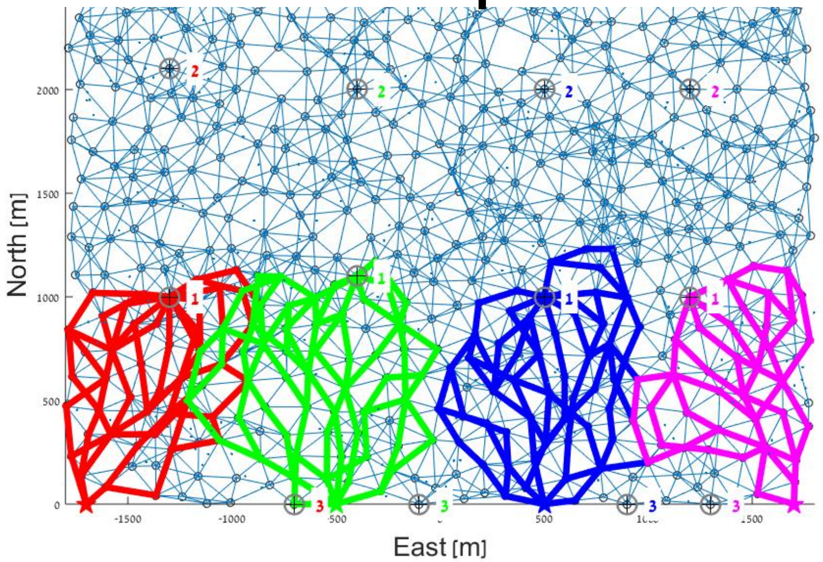
- Each robot has multiple goals
- Multiple planning sessions
- SLAM session given calculated robot paths (actions)
- Two first robots start with high position uncertainty

4 robots: 25 candidate paths

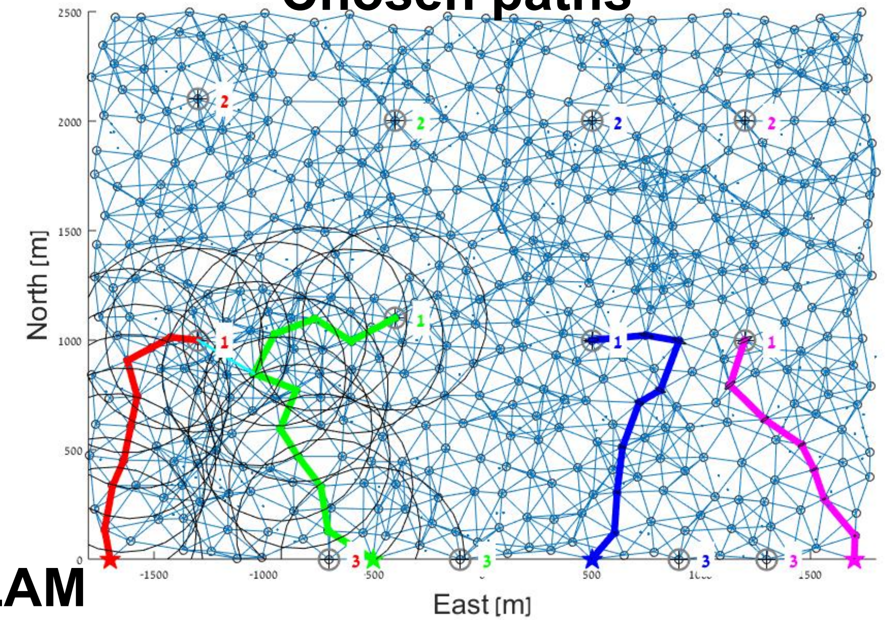


Results – Larger Scale Scenario – Planning 1

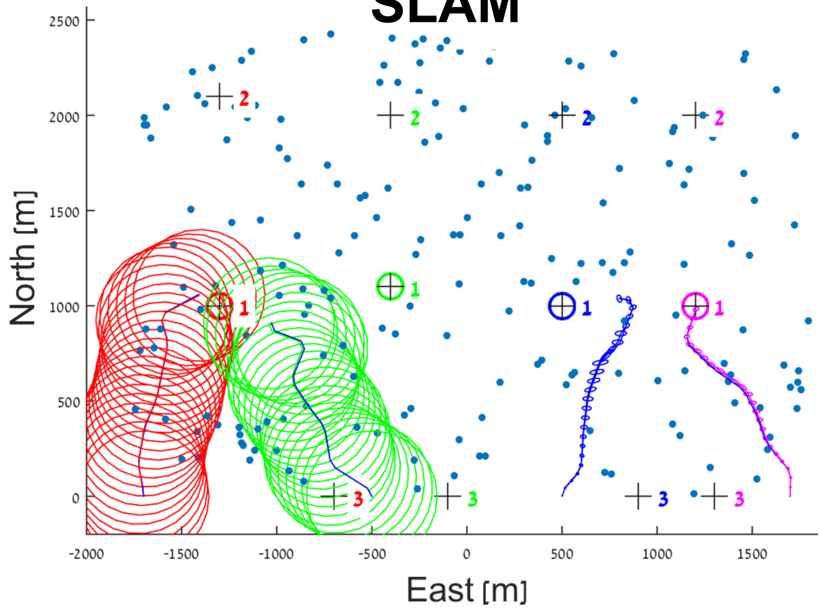
Candidate paths



Chosen paths

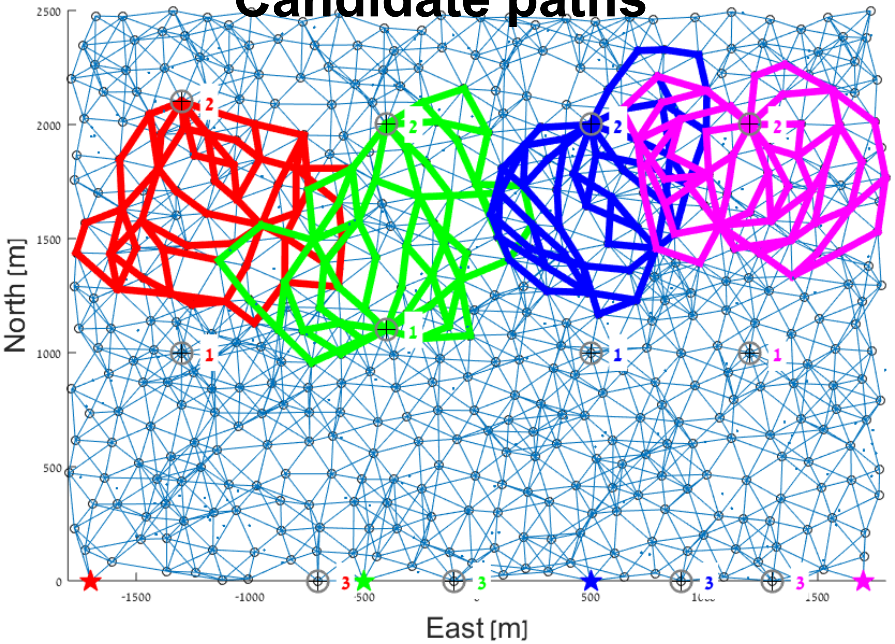


SLAM

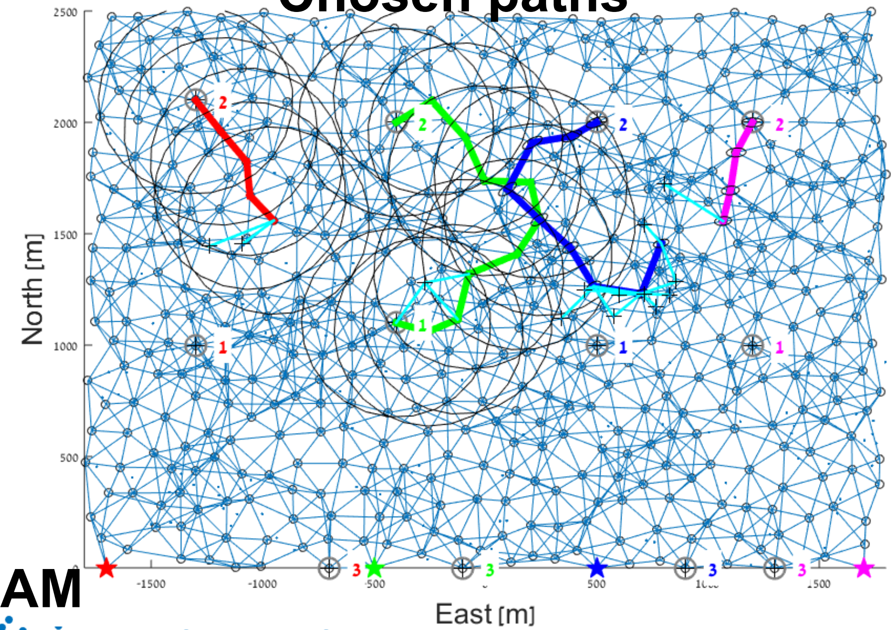


Results – Larger Scale Scenario – Planning 2

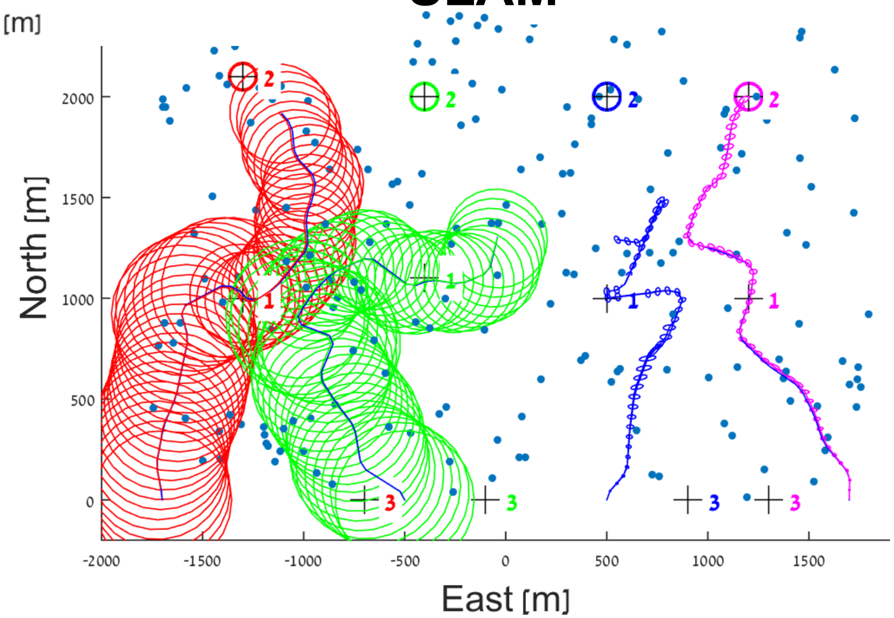
Candidate paths



Chosen paths

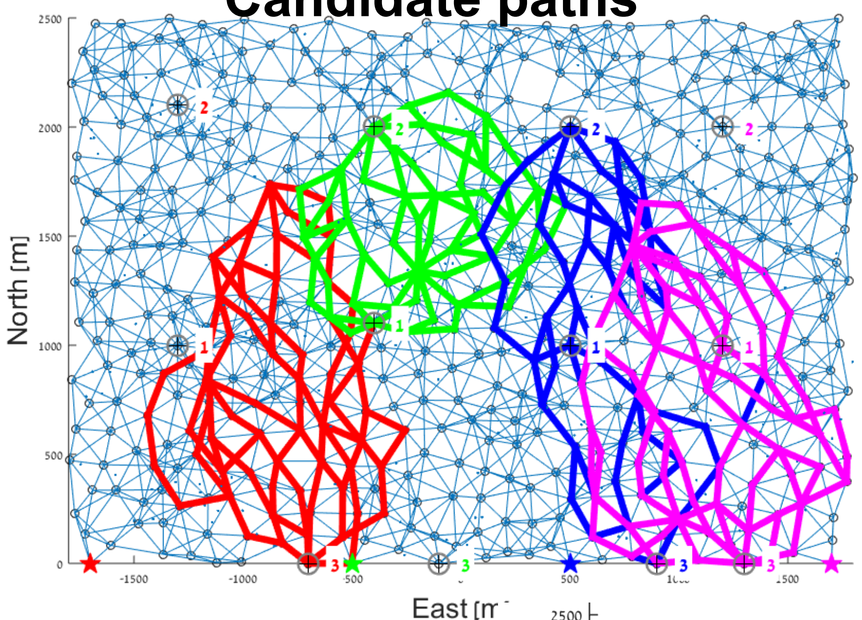


SLAM

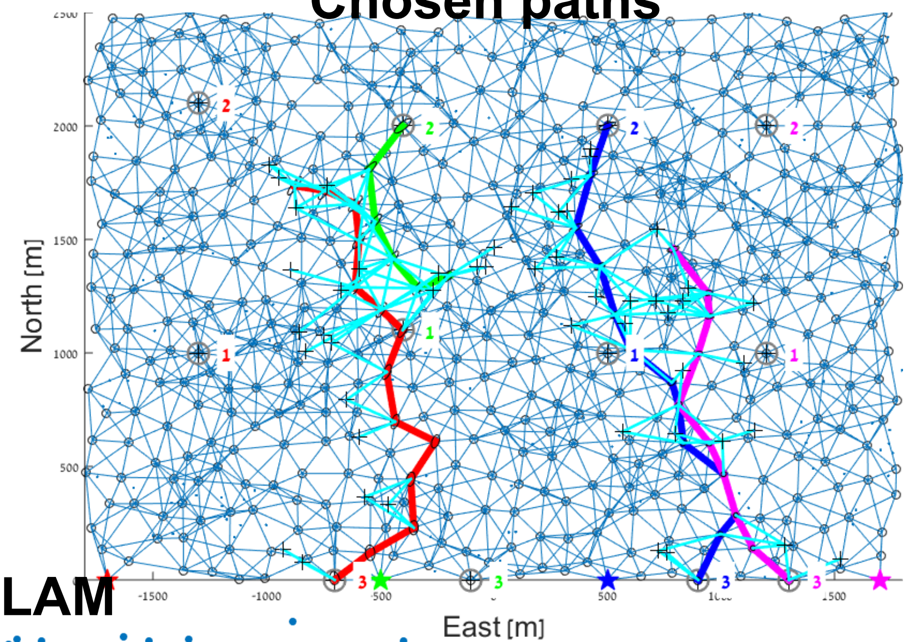


Results – Larger Scale Scenario – Planning 3

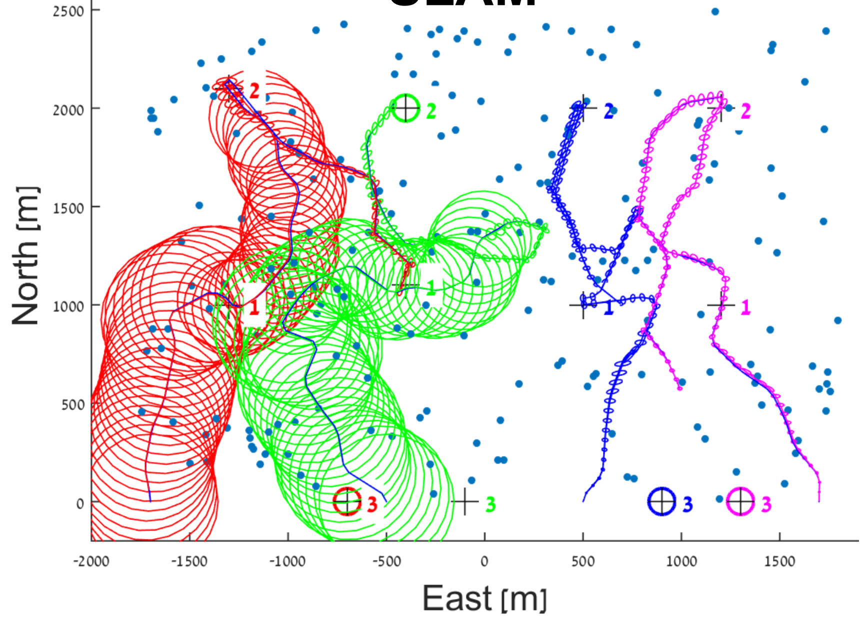
Candidate paths



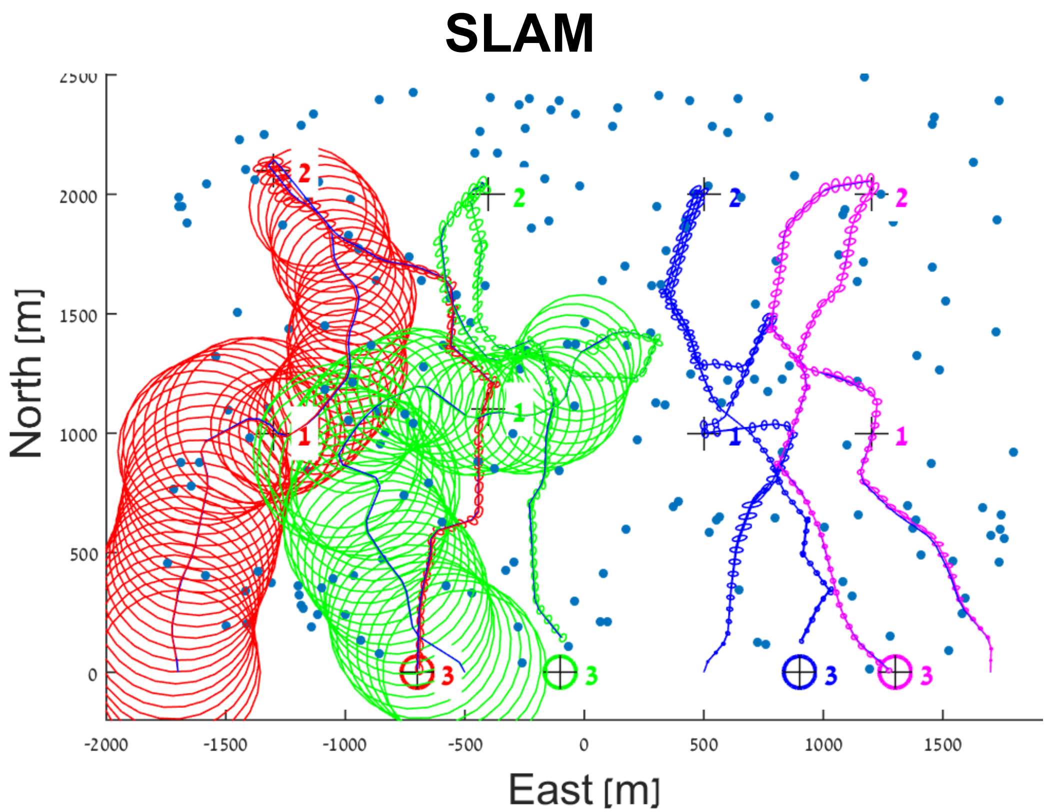
Chosen paths



SLAM



Results – Larger Scale Scenario – Final Result

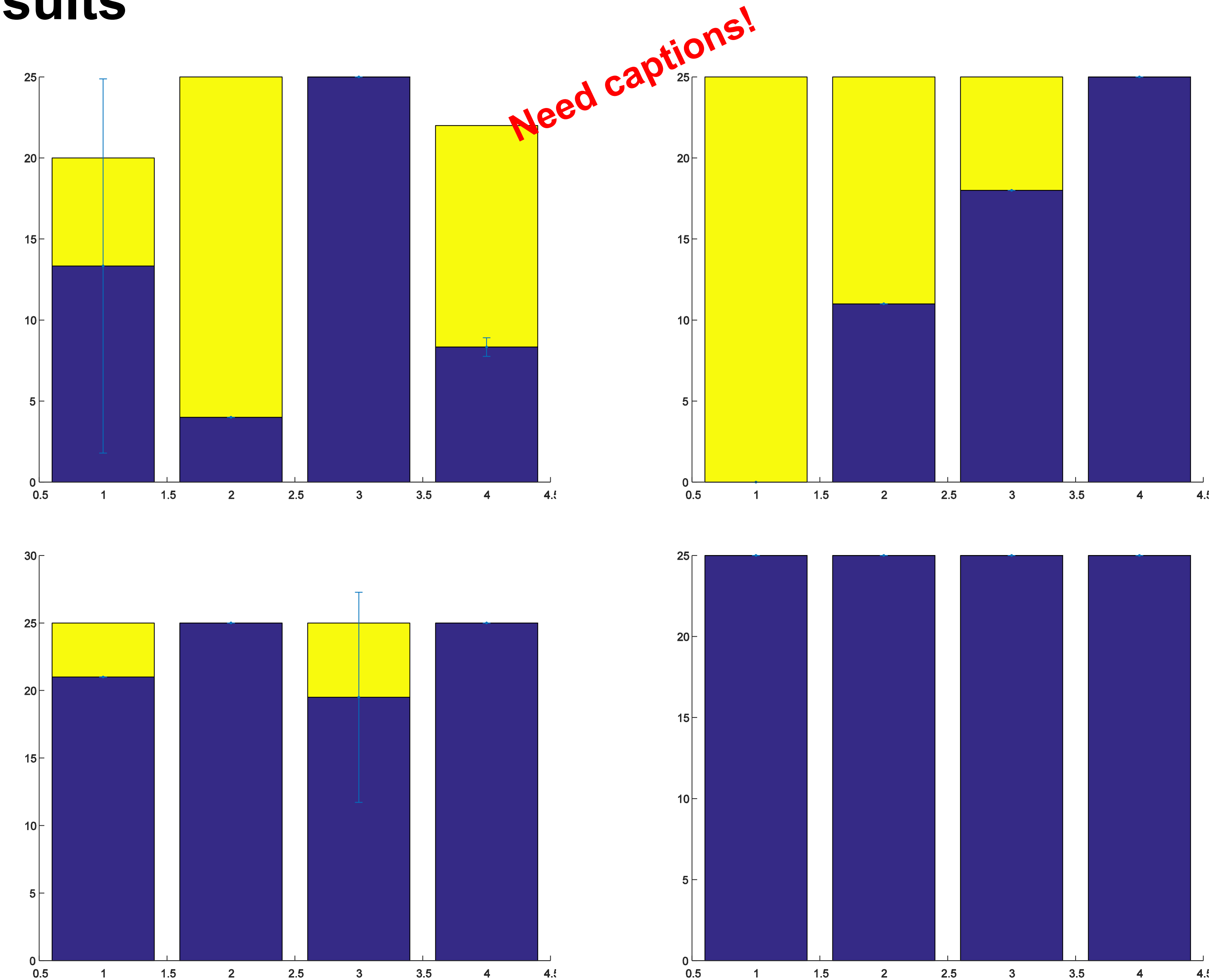


Conclusions and Future Work

- Collaborative multi-robot belief space planning in unknown environments
- **Contribution**:
  - Identify impacted paths due to change in announced paths
  - Efficiently re-evaluate belief only for impacted paths
  - One-time re-calculation for all non-impacted paths
  - Performance study in simulation

- Future work includes:
  - Concept may be generalized to other BSP approaches
  - Implement method in an incremental setting (e.g. RRG, RRT)
  - Extend approach to active cooperative localization and target tracking

Results



Results

Timing results

Introduction – Localization

- Navigation in **known environment** or with GPS.

  Localization: Where am I?

What happens when map is unknown and without GPS?