Hypotheses disambiguation in retrospective for robust perception in ambiguous environments

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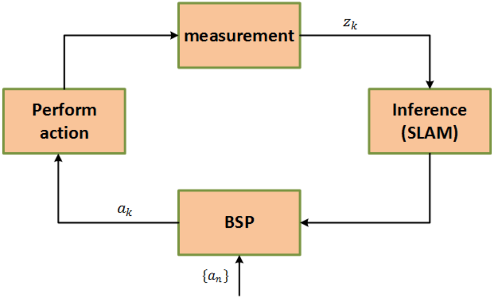
Introduction

Autonomous car

Xiaomi Roborock

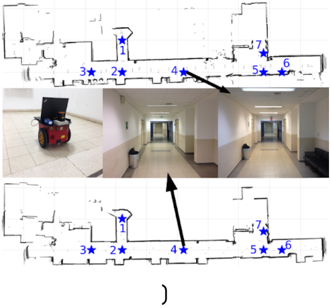
Quad Copter

- **Autonomous navigation high level framework.**



  - Estimation of the robot's location.
  - Choose and perform a course of action.
  - Receive a measurement, and re estimate the robot's location.

Motivation-Ambiguous environment



Angeli et al., TRO'08

Pathak, Thomas, Indelnan, IJRR 2018

Mu et al., IROS'16

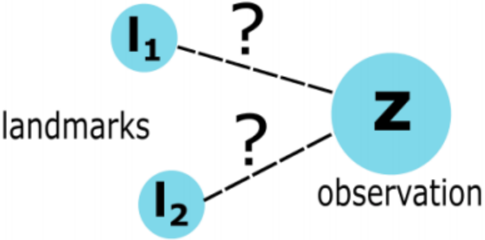
Motivation-Ambiguous environment

- Ambiguity can be due to (combination of):
  - Perceptually aliased scenes (two similar objects).
  - Limited/imperfect sensing (limited sensing range).

How can we deal with this ambiguity in inference and planning?

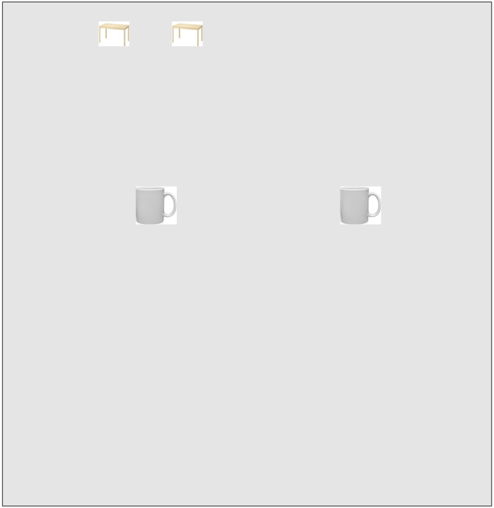
Motivation - Data Association (DA)

Given two identical landmarks, commonly it is assumed to be known which landmark generated the current measurement.

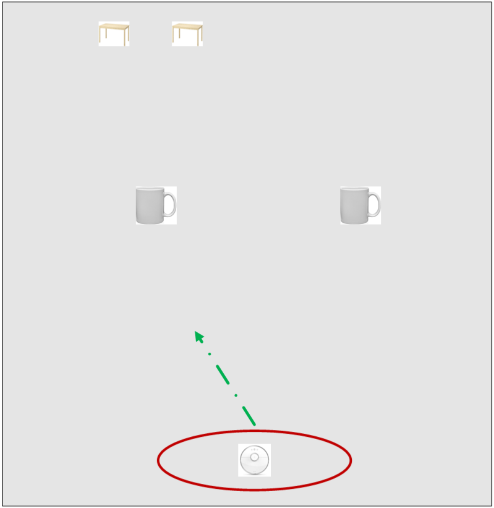


What happens in the case of making a **false data association**?.

- **A simplified static world illustration**



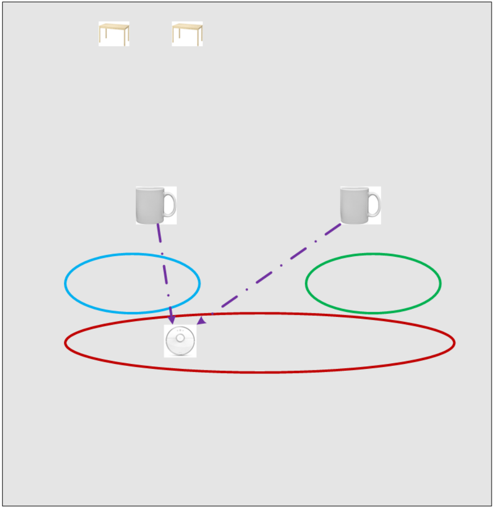
- **Robot's belief at time $k-2$**



The robot decides to take an action of **one step forward**.

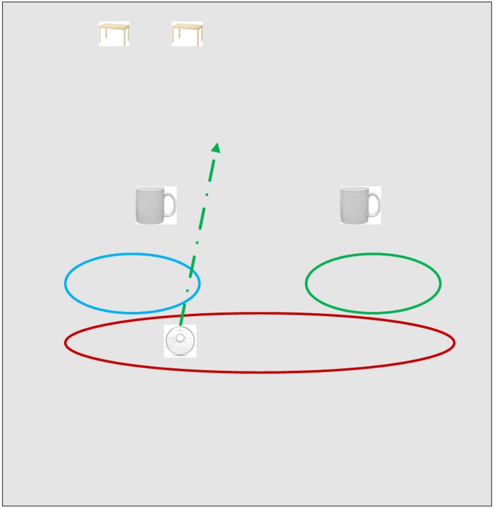
Motivation

- **Inference at time $k-1$**



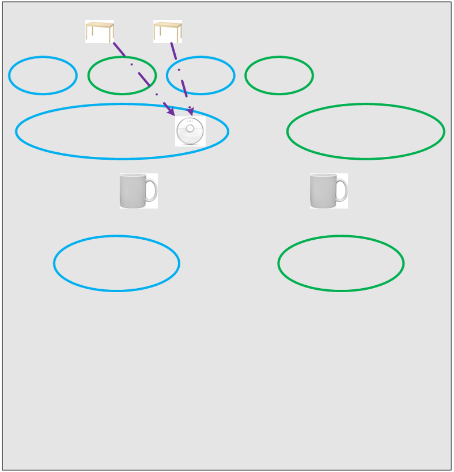
The robot receives a measurement of a single cup (but which one?).

- **Robot at time $k-1$**



The robot decides to take a **one step forward** action.

- **Robot's location at time k**



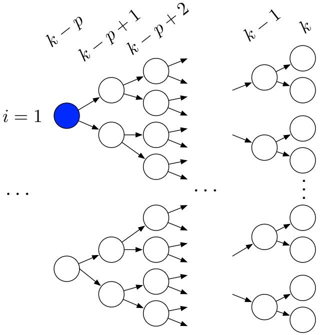
- The robot receives a measurement of a **single** table.
- **Can** information at time k help/impact inference estimation at time $k-1$?

Related work

- Indelman et al. CSM 2016: “Incremental Distributed Inference from Arbitrary poses and Unknown Data Association: Using Collaborating Robots to Establish a Common Reference”.
- N. Sünderhauf and P. Protzel ICRA 2013 : “Switchable Constraints vs. Max-Mixture Models vs. RRR - A Comparison of Three Approaches to Robust Pose Graph SLAM”.
- Fourie, Leonard and Kaess IROS 2016 : “A non-parametric belief solution to the Bayes tree- Incremental update of the GMM belief”
- S. Pathak and A. Thomas and V. Indelman IJRR 2018 : “A Unified Framework for Data Association Aware Robust Belief Space Planning and Perception”.
- Tchuiev, Feldman and Indelman IROS 2019 : “Data Association Aware Semantic Mapping and Localization via a Viewpoint-Dependent Classifier Model”.

Our Contribution

- Introduce the problem of hypotheses disambiguation in retrospective.



Our Contribution

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- Re-evaluation of a **strategic past event/point** using current information.
- Incremental calculation in order to reduce complexity.
- Re-evaluation of specific pas DA hypothsis using current information.
- Enhance hypotheses pruning also at *current* time.

- **Publications**:
  O. Shelly and V. Indelman. Hypotheses Disambiguation in Retrospective.
  - IEEE Robotics and Automation Letters (RA-L), 2022.
    IEEE Intl. Conf. on International Conference on Robotics and Automation (ICRA), 2022.

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- **Notations**
  - $b[X_k] = P(X_k \mid H_k)$ - The belief at time k.
    - $X_k = \{x_0, ...x_k\}$ - Robot pose vector.
    - $H_k = \{\underbrace{U_{k-1}}_{\text{actions}}, \underbrace{Z_k}_{\text{measurements}}\}$ - History vector, where $U_{k-1} = u_{0,\ldots,k-1}$ and $Z_{K-1} = z_{1,\ldots,k}$.
  - $b^i[X_k] = P(X_k \mid \gamma_k = i, H_k)$ - The i hypothesis at time k.
    - $\gamma_k = \beta_{1,\ldots,k}$ - A vector of DA.
    - β_k - A specific DA of z_k, where β_k is equal to given landmark, $l_j, j \in [1..L]$.
  - $b^{i-}[X_k] = P(X_k \mid H_k^-)$ Propagated belief, where $H_k^- = \{U_{k-1}, Z_{k-1}\}$.

- **Assumptions**
  - **Map is given*** - represented by static landmarks.
  - Observation and motion model with Gaussian noise.

$$
\begin{aligned}
x_k &= f(x_{k-1}, u_{k-1}) + w \\
z_k &= h(x_k, l) + v
\end{aligned}
$$

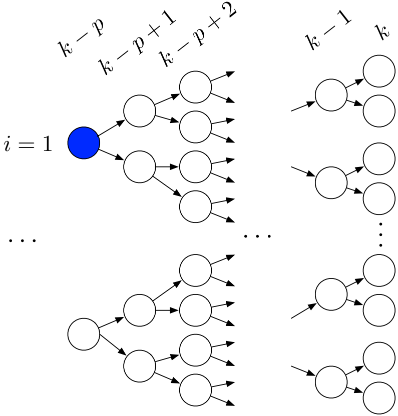
Where $w \sim \mathcal{N}(\mu_w, \Sigma_w)$ and $v \sim \mathcal{N}(\mu_v, \Sigma_v)$.

- **GMM Belief**
  When removing DA assumption we marginalize over all possible associations.

$$b[X_k] = \sum_{i=1}^{M_k} \underbrace{\mathbb{P}(\gamma_k = i \mid H_k)}_{w_k^i} \cdot \underbrace{\mathbb{P}(X_k \mid \gamma_k = i, H_k)}_{b^i[X_k]}$$

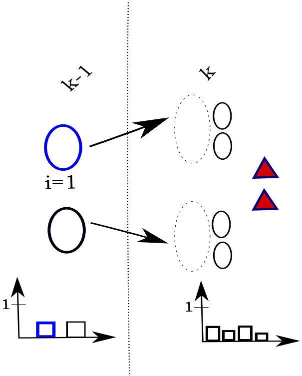
  - w_k^i - Probability of the i'th hypothesis, via its weight.
  - $b^i[X_k]$ - Belief over X_k of the i'th hypothesis.

- **Problem formulation**
  - Given the belief b^i_{k-p}, we wish to update the weight $w^i_{k-p|k-p}$, with gathered information up to time k, via $w^i_{k-p|k} = \mathbb{P}(\gamma_{k-p} = i \mid H_k)$.



- **Single step update $p = 1$**
  - By performing Bayes rule we receive,

$$w^i_{k-1|k} = \underbrace{\frac{\mathbb{P}(z_k \mid \gamma_{k-1} = i, H^-_k)}{\mathbb{P}(z_k \mid H^-_k)}}_{\text{update factor}} \cdot w^i_{k-1}$$



- **Update factor calculation**

$$\mathbb{P}(z_k|\gamma_{k-1}=i,H_k^-)=\sum_{g=1}^{|L|}\int_{x_k}\mathbb{P}(z_k,\overbrace{\beta_k}^{*}=g,x_k|\gamma_{k-1}=i,H_k^-)dx_k=$$

  - β_k - Random variable on all landmark index's associations.
  - We marginalize over all possible landmarks and current state.

$$\mathbb{P}(z_k|\gamma_{k-1}=i,H_k^-)=\sum_{g=1}^{|L|}\int_{x_k}\underbrace{\mathbb{P}(z_k\mid\beta_k=g,x_k)}_{a}\cdot\underbrace{\mathbb{P}(\beta_k=g\mid x_k,l_g)}_{b}\cdot\underbrace{b^{i-}[x_k]}_{c}\,dx_k$$

  - a - Observation model.
  - b - Sets integral finite borders per l_g, via Ω_{l_g}.
  - c - Propagated belief at time k.

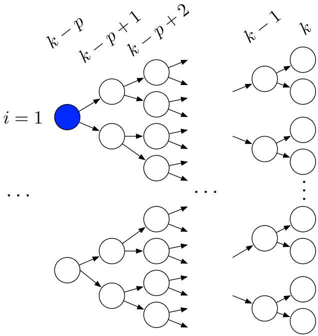
- **Update factor calculation**
  - Since the integral has finite borders, we take S samples from $b^{i-}[x_k]$.

$$\mathbb{P}(z_k|\gamma_{k-1}=i,H_k^-)\approx\cdot\sum_{n=1}^{S}\sum_{g=1}^{L}\frac{1}{|\Omega_{l_g}|}\mathbb{P}(z_k\mid l_{\beta_k=g},x_k^{n,i})\cdot\mathbb{1}_{\Omega_{l_g}}(x_k^{n,i})$$

  - In order to avoid calculating the denominator I denote, $\tilde{w}^i_{k-1|k}\doteq\mathbb{P}(z_k|\gamma_{k-1}=i,H_k^-)\cdot w^i_{k-1}$.
  - We received an un-normalized weight, $\tilde{w}^i_{k-1|k}$ is un-normalize, therefore we Normelize via,

$$w^i_{k-1|k}=\frac{\tilde{w}^i_{k-1|k}}{\sum_{j=1}^{M_{k-1}}\tilde{w}^j_{k-1|k}}$$

- **Multiple steps,** $0 \leqslant p < k$



- Let us try to **re-evaluate** a hypothesis from p **steps** prior to time k, via $m \doteq k - p$.

- **Multiple steps,** $0 \leqslant p < k$
  - Applying Bayes rule and performing marginalization yields:

$$\boxed{w^i_{m|k} = \underbrace{\left[\frac{\prod_{j=1}^{p} \mathbb{P}(z_{m+j} \mid \gamma_m, H^-_{m+j})}{\prod_{j=1}^{p} \mathbb{P}(z_{m+j} \mid H^-_{m+j})}\right]}_{\Psi_{m|k}} \cdot \underbrace{w^i_{m|m}}_{\mathrm{B}}}$$

  - A naive approach would be to calculate $\mathbb{P}(z_{m+j} \mid \gamma_m, H^-_{m+j})$ from scratch up for any j.

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- **Multiple steps, Calculation for any j**
  - $\mathbb{P}(z_{m+j} \mid \gamma_m = i, H^-_{m+j})$ for a given value of j,

$$\sum_{g=1}^{|L|} \frac{1}{|\Omega_{l_g}|} \int_{x_{m+j} \in \Omega_{l_g}} \mathbb{P}(z_{m+j} \mid l_g, x_{m+j}) b^{i-}_m[x_{m+j}] dx_{m+j}$$

  - $b^{i-}_m[x_{m+j}]$ is a GMM with S components,

$$b^{i-}_m[x_{m+j}] \doteq \sum_{n=1}^{S} \underbrace{\frac{f(x^{n,i}_{m+j-1}, z_{m+j-1})}{\hat{\eta}^i_{m+j-1}}}_{\zeta^{n,i}_{m+j-1}} \cdot \mathbb{P}(x_{m+j} \mid x^{n,i}_{m+j-1}, u_{m+j-1})$$

- **Multiple steps, Calculation for any j**
  - $\zeta_{m+j-1}^{n,i}$ (GMM weights) are calculated from the **previous step** $m+j-1$, where,
    - $f(x,z) \doteq \frac{1}{S} \cdot \sum_{g=1}^{L} \frac{1}{|\Omega_{l_g}|} \mathbb{P}(z \mid l_g, x) \cdot \mathbb{P}(l_g \mid x)$
    - $\hat{\eta} \triangleq \sum_{n=1}^{S} f(x^n, z)$
  - Since $\mathbb{P}(z_{m+j} \mid \gamma_m = i, H_{m+j}^{-})$ holds finite borders we need to sample $b_m^{i-}[x_{m+j}]$.

$$
\begin{aligned}
\mathbb{P}(z_{m+j} \mid \gamma_m, H_{m+j}^{-}) \quad &\approx \quad \frac{1}{S} \cdot \sum_{n=1}^{S} \sum_{g=1}^{L} \frac{1}{|\Omega_{l_g}|} \mathbb{P}(z_{m+j} \mid l_g, x_{m+j}^{n,i}) \cdot \mathbb{1}_{\Omega_{l_g}}(x_{m+j}^{n,i}) \\
&= \quad \sum_{n=1}^{S} f(x_{m+j}^{n,i}, z_{m+j})
\end{aligned}
$$

- **Multiple steps,** $0 \leqslant p < k$
  - Therefore, $w^i_{m|k}$ yields into,

$$w^i_{m|k} \approx \frac{\left[\sum_{j=1}^{p} \hat{\eta}^{n,i}_{m+j}\right] \cdot w^i_{m|m}}{\prod_{j=0}^{p-1} \mathbb{P}(z_{k-j} \mid H^-_{k-j})} = \frac{\tilde{w}^i_{m|m}}{\prod_{j=0}^{p-1} \mathbb{P}(z_{k-j} \mid H^-_{k-j})}$$

  - We normalize in the same form as in single step,

$$w^i_{m|k} = \frac{\tilde{w}^i_{m|k}}{\sum_{j=1}^{M_m} \tilde{w}^j_{m|k}}$$

Computational Complexity

- **Naïve approach** - Calculation performed from scratch for each step.

$$\underbrace{S}_{m+1} + \underbrace{2 \cdot S}_{m+2} + \ldots + \underbrace{p \cdot S}_{m+p} = \frac{(1+p) \cdot p}{2} \cdot S \in \mathcal{O}(p^2 S).$$

- **Incremental approach** - Calculation reuse from previous step.

$$\underbrace{S}_{m+1} + \underbrace{S}_{m+2} + \ldots + \underbrace{S}_{m+p} = p \cdot S \in \mathcal{O}(p \cdot S),$$

- Our approach reduces the complexity in one order of magnitude in p.

Approach

Specific DA re-evaluation

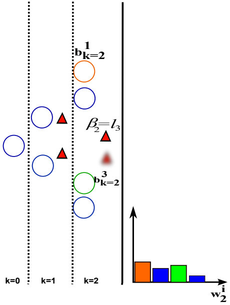
- A specific DA probebilty can be described as, $\mathbb{P}(\beta_{k-p} \doteq c \mid H_{k-p})$.
- Recall each hypothesis i at time $k-p$ equals,

$$\gamma_{k-p} = \{\underbrace{\beta_1, .., \beta_{k-p-1}}_{l}, \underbrace{\beta_{k-p}}_{r}\} \doteq i$$

- Therefore we suggest,

$$\mathbb{P}(\beta_{k-p} \doteq c \mid H_{k-p}) \doteq \sum_{i=1}^{M_{k-p}} w_{k-p}^{i} \mathbb{1}_{\{c\}}(i.r)$$

$$\mathbb{P}(\beta_2 = l_3 \mid H_2) = w_2^1 + w_2^3$$



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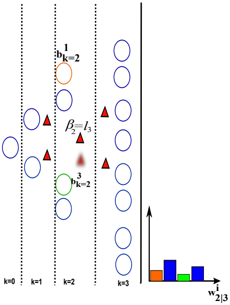
- **Specific DA re-evaluation**
  - My framework allows re-evaluation given the updated weights, $w^i_{k-p|k}$. Therefore,

$$\mathbb{P}(\beta_{k-p} \doteq c \mid H_k) \doteq \sum_{i=1}^{M_{k-p}} w^i_{k-p|k} \mathbb{1}_{\{c\}}(i.r)$$

$$\mathbb{P}(\beta_2 = l_3 \mid H_3) = w^1_{2|3} + w^3_{2|3}$$



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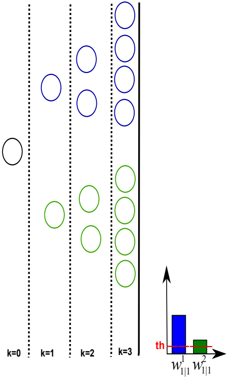
- **Enhanced Pruning**
  - In the common case pruning is performed at current time.

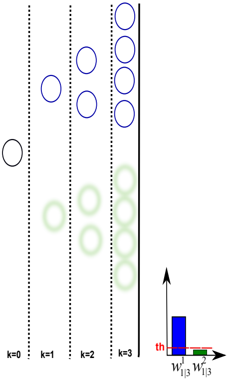
$$\tilde{w}^i_{k-p} \doteq w^i_{k-p} \cdot \mathbb{1}(w^i_{k-p} > th)$$

  - In the following example at $k = 1$ both hypotheses weights are above pruning threshold.
  - This resulted in a full hypothesis tree from both till current time, via $k = 3$.

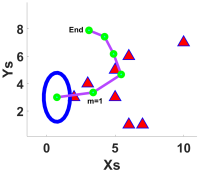


- **Enhanced Pruning**
  - In case the updated weight is below pruning threshold, we can prune also all of its descendants.
  - Specifically, the once at current time.
  - As before we can denote, $\gamma_k \doteq \{\underbrace{\gamma_{k-p}}_{\text{i.l}}, \underbrace{\beta_{k-p+1} \cdots, \beta_k}_{\text{i.r}}\} = i$
  - Therefore our weights at current time will be,

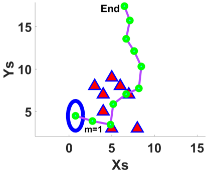
$$\tilde{w}_k^j \doteq w_k^j \cdot \mathbb{1}_{\{j.l\}}(w_{k-p|k}^{j.l} > th)$$



- **Weight update simulations**
  - A setting of eight **identical** landmarks.
  - First, calculate the GMM at the end of trajectory.
  - Second, GMM weight re-evaluation at point $m \doteq 1$.

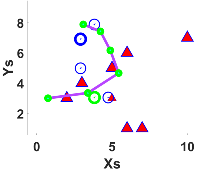


Five steps robot's trajectory

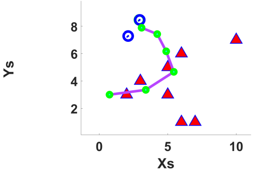


Ten steps robot's trajectory

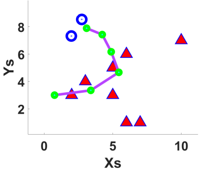
- **Weight update simulations - Five steps**



$b[X_1], M_1 = 5$



$b[X_5], M_5 = 4$

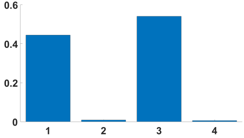


Merged belief, via $\tilde{b}[X_5], M_5 = 2$

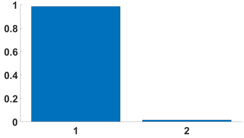
Evaluation

Weight update simulations - Five steps

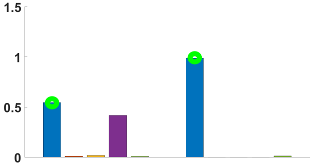
- Ambiguous hypotheses at $k = 5$ without the merging effect, and at $k = 1$.
- Our algorithm enables to disambiguate and extract the **correct hypothesis** at the re-evaluation point.



$w^i_{5|5}$ distribution of $b[X_5]$

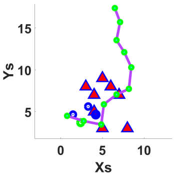


$\tilde{w}^i_5$ distribution of $\tilde{b}[X_5]$

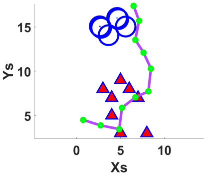


$w^i_{1|1}$ vs $w^i_{1|5}$

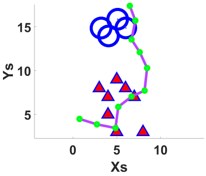
- **Weight update simulations - Ten steps**



$b[X_1], M_1 = 4$



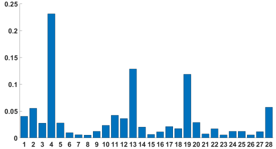
$b[X_{10}], M_{10} = 28$



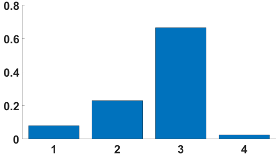
Merged belief, via $\tilde{b}[X_{10}], M_{10} = 4$

Weight update simulations - Ten steps

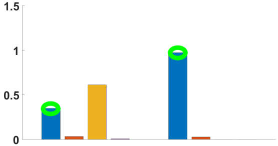
- Ambiguous hypotheses at $k = 10$ with or without merging.
- Ambiguous setting as well at $k = 1$, the re-evaluation point.
- Our algorithm enables to disambiguate and extract the **correct hypothesis** at the re-evaluation point.



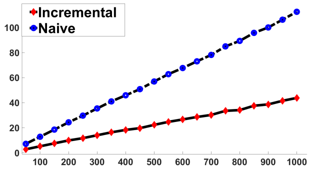
$w^i_{10}, i \in [1..28]$



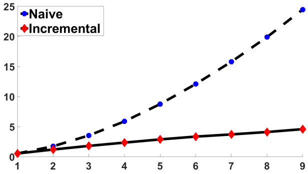
$\tilde{w}^i_{10}, i \in [1..4]$



- **Run-Time Calculations**



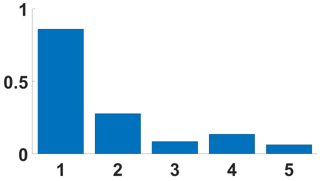
Run-Time as a function of samples



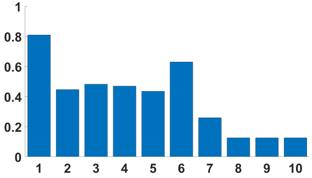
Runtime as a function of p

- Both analyses show an **advantage to calculation re-use** approach.
- **Run-Time as a function of** p - Linear rate to the incremental approach, while the Naive increments in a square rate, as expected.

- **Histogram Calculation**



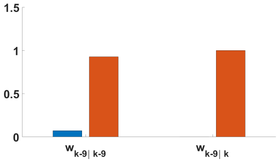
$H(w_{1|j}), j \in [1..5]$



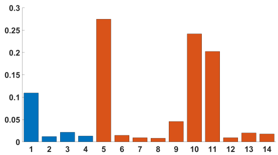
$H(w_{1|j}), j \in [1..10]$

- In certain cases $H(w_{p|j})$ where $j \in [p..k]$ can reduce as j increases.

Example for Enhanced pruning



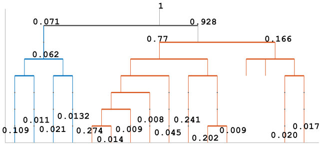
$w^i_{1|1}$ vs $w^i_{1|10}$



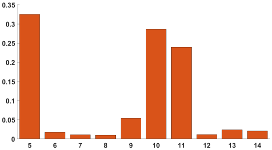
w^i_{10}, $i \in [1..14]$

- A ten steps trajectory, where the re-evaluation point at $p = 1$.
- $w_{1|1}$ holds two hypotheses, while $w^1_{1|10}$ is disambiguated.
- Current time holds four descendants of $w^1_{1|10}$, via $w^i_{10|10}$ where $i \in [1..4]$.

- **Example for Enhanced pruning**



GMM weights hypothesis tree.



$\tilde{w}^i_{10}$, $i \in [5..14]$

- In general the entire belief hypothesis tree of $w^1_{1|10}$ is obsolete. Specifically, descendants at current time, via $w^i_{10} i \in [1..4]$.
- Therefore our current time GMM reduces to ten components.

Conclusions

- New information can have an impact on the weight distribution, and our ability to disambiguate between past hypotheses.
- Incremental approach to calculations.
- Enhanced pruning.

Future Work

- Examine our approach in real world settings, in the aspects of run-time and performance.
- Add uncertainty to the landmarks, and add them to our state.
- Examine if there is an effect of two hypotheses from different time points.
- Decision making under uncertainty - Take this approach into planning.

Thank you for listening!