Focused Topological Belief Space Planning

Moshe Shienman, Andrej Kitanov, and Vadim Indelman
Technion – Israel Institute of Technology, Israel

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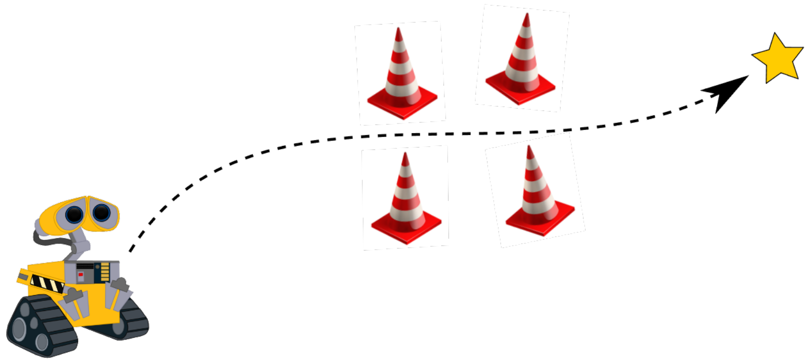
- Belief Space Planning (BSP) - determining optimal controls over the belief space with respect to a given objective (instantiation of POMDP)

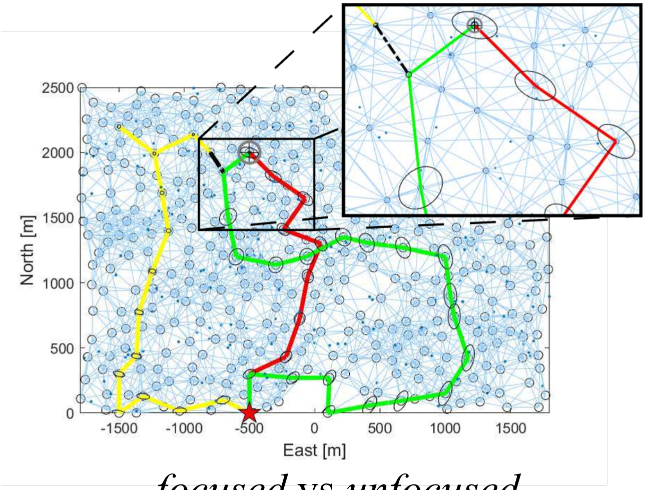
- *unfocused* BSP - the objective function considers all variables

- *focused* BSP - objective function prioritizes or only considers a predefined subset of *focused* variables



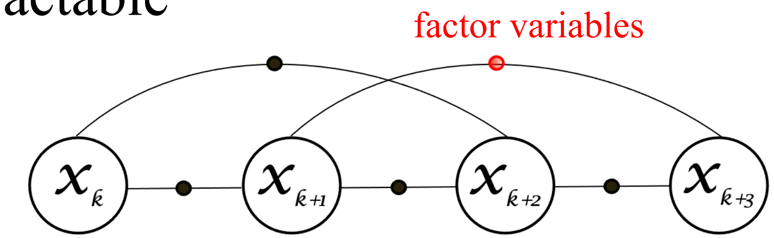
collision avoidance

focused reconstruction task



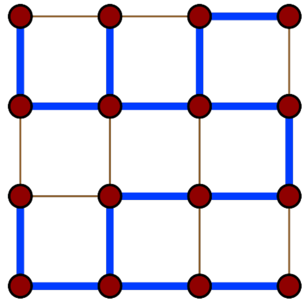
focused vs *unfocused*

- Finding a globally optimal solution to BSP is computationally intractable



Factor Graph

- An approximation should
  1. Discriminate between candidate actions
  2. Be less expensive to compute
  3. Returns the optimal solution of the original problem



A spanning tree (in blue) of a grid graph

- Recently, topological properties were used to approximate information theoretic cost functions

- Topological Aspects : **[Khosoussi et al. 2015 RSS, Khosoussi et al. 2019 IJRR]**

  ➡ do not address planning (only inference - measurement selection)

- Active *unfocused* inference with topological aspects :
  [Kitanov and Indelman 2018 ICRA, Kitanov and Indelman 2019]

  ➡ do not consider the *focused* case

- Active *focused* inference :
  [Kopitkov and Indelman 2019 IJRR] , [Levine and How 2013 NIPS]

  ➡ do not incorporate any topological aspects in inference nor in planning

Our Contributions

1. FT-BSP - a novel approach for the *focused* information theoretic BSP problem
2. Derive two topological signatures to approximate the *focused* objective function
3. Prove asymptotic convergence and develop bounds (for one signature)
4. Provide empirical results
   - Correlation between topological signatures and a *focused* information theoretic objective function
   - Runtime comparison

focused BSP - Notations

- x_k : robot state at time k (position and orientation)
- X_k : the joint state up to and including time k
- $X_k^F \subseteq X_k$: a *focused* subset of states , $X_k^U = X_k / X_k^F$: the remaining *unfocused* states
- $z_{0:k}$, $u_{0:k-1}$: all observations and controls up to time k
- $b\left[X_k\right] \doteq \mathbb{P}\left(X_k | z_{0:k}, u_{0:k-1}\right)$: posterior probability density function over the joint state – the **belief**
- Λ_k : the information matrix of the Maximum Likelihood (ML) estimation

focused BSP – The Objective Function

- Main goal of BSP is $\mathcal{U}^* = \underset{\mathcal{U}}{\operatorname{argmin}}\, J(\mathcal{U})$
- We only consider the information theoretic term and minimize the differential entropy

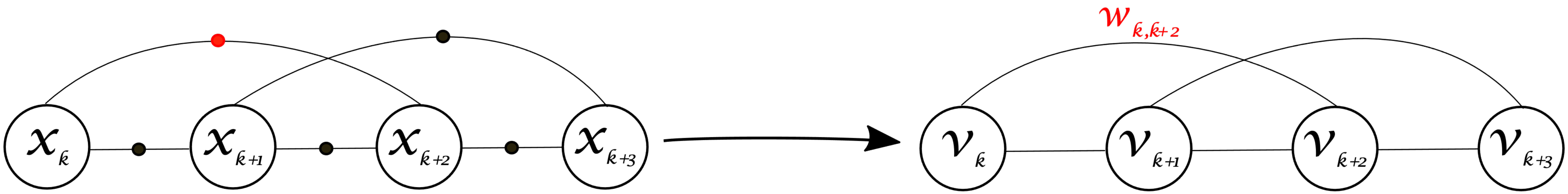
$$J_{\mathcal{H}}^{F}(\mathcal{U}) = \mathcal{H}\left(b\left[X_{k+L}^{F}\right]\right) = \frac{n^F}{2}\log(2\pi e) - \frac{1}{2}\log\left|\Lambda_{k+L}^{M,F}\right|$$

- The information matrix Λ_{k+L} can be partitioned as $\Lambda_{k+L} = \begin{bmatrix} \Lambda_{k+L}^{F} & \Lambda_{k+L}^{F,U} \\ (\Lambda_{k+L}^{F,U})^T & \Lambda_{k+L}^{U} \end{bmatrix}$. Using [Ouellette 1981]

$$J_{\mathcal{H}}^{F}(\mathcal{U}) = \frac{n^F}{2}\log(2\pi e) - \frac{1}{2}\log\left|\Lambda_{k+L}\right| + \frac{1}{2}\log\left|\Lambda_{k+L}^{U}\right|$$

Belief Topology

- The weighted topological graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, w)$



Factor Graph

Topological Graph

- The associated weighted Laplacian matrix to $\mathcal{G}$: $\mathcal{L}_w(i,j) = \begin{cases} W_{ii} & \text{if } i = j, \\ -w_{ij} & \text{if } v_i \text{ and } v_j \text{ are adjacent,} \\ 0 & \text{otherwise} \end{cases}$
- $t_w(\mathcal{G})$ - weighted number of spanning trees
- $\tau_w(\mathcal{G}) = \log t_w(\mathcal{G})$ - Weighted Tree Connectivity (WTC) [Khosoussi et al. 2019 IJRR]

Weighted Tree Connectivity (WTC)

Reminder: *focused* objective function

$$J^F_{\mathcal{H}}(\mathcal{U}) = \frac{n^F}{2}\log(2\pi e) - \frac{1}{2}\log\left|\Lambda_{k+L}\right| + \frac{1}{2}\log\left|\Lambda^U_{k+L}\right|$$

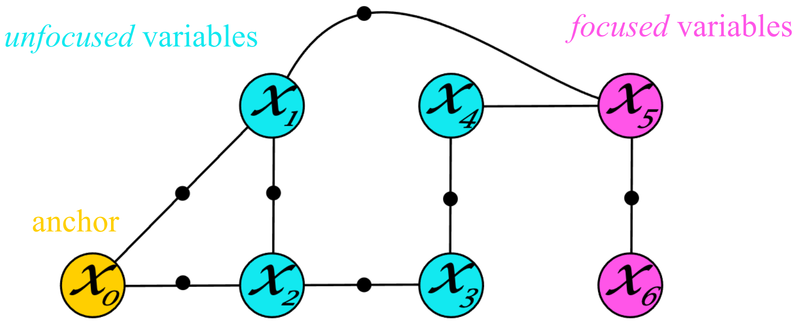
- [Khosoussi et al. 2019 IJRR] identified Laplacian structures in $\Lambda = \begin{bmatrix} L_{w_p} \otimes \cdots & \cdots \\ \vdots & L_{w_\theta} + \cdots \end{bmatrix}$
- Introduced a topological signature $\tau_w(\mathcal{G}) = 2\tau_{w_p}(\mathcal{G}) + \tau_{w_\theta}(\mathcal{G})$
- Used to bound the D-criterion of the SLAM graph $\tau_w(\mathcal{G}) \leq \log|\Lambda| \leq \tau_w(\mathcal{G}) + n \cdot \log(1 + \delta/\lambda_1)$
- [Kitanov and Indelman 2018 ICRA] extended to BSP problems
- We can use this signature to approximate $\log\left|\Lambda_{k+L}\right|$ but what about $\log\left|\Lambda^U_{k+L}\right|$?

The *unfocused* Augmented Graph

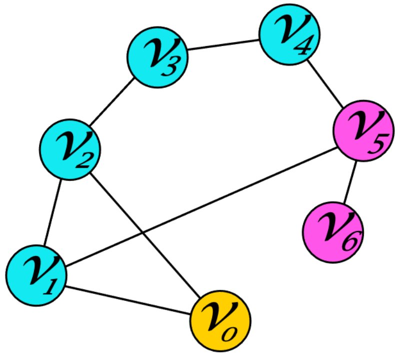
Reminder: *focused* objective function

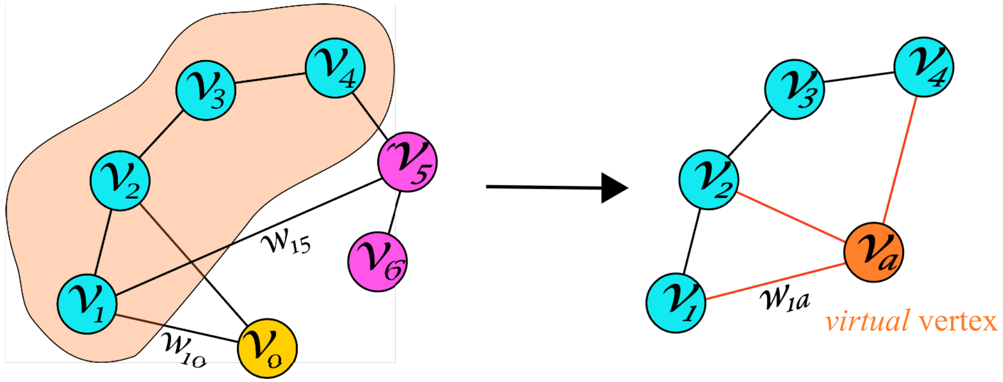
$$J^F_{\mathcal{H}}(\mathcal{U}) = \frac{n^F}{2}\log(2\pi e) - \frac{1}{2}\log\left|\Lambda_{k+L}\right| + \frac{1}{2}\log\left|\Lambda^U_{k+L}\right|$$

- Observation: Λ^U_{k+L} contains Laplacian structures $\Lambda^U = \begin{bmatrix} L^U_{w_p} \otimes \cdots & \cdots \\ \vdots & L^U_{w_\theta} + \cdots \end{bmatrix}$
- Construct a topological graph $\mathcal{G}^{U,A}$ with a WTC signature $\tau^{U,A}_w = 2\tau_{w_p}(\mathcal{G}^{U,A}) + \tau_{w_\theta}(\mathcal{G}^{U,A})$



Factor Graph


Topological Graph



unfocused Augmented Topological Graph

Weighted Tree Connectivity Difference (WTCD) signature

Reminder: *focused* objective function

$$J_{\mathcal{H}}^{F}(\mathcal{U}) = \frac{n^F}{2}\log(2\pi e) - \frac{1}{2}\log\left|\Lambda_{k+L}\right| + \frac{1}{2}\log\left|\Lambda_{k+L}^{U}\right|$$

- **Theorem 1** $\tau_w^{U,A}$ asymptotically bounds $log\left|\Lambda^U\right|$
- Following **[Kitanov and Indelman 2018]** and the WTCs for both terms we define

$$S_{WTCD} = \frac{n^F}{2}\log(2\pi e) - \frac{1}{2}\left[\tau_w - \tau_w^{U,A}\right]$$

- **Theorem 2** the approximation error $\epsilon(J_{\mathcal{H}}^F) \doteq J_{\mathcal{H}}^F - S_{WTCD}$ is bounded

Can we do better (computationally)?

Von Neumann Difference signature

Reminder: *focused* objective function

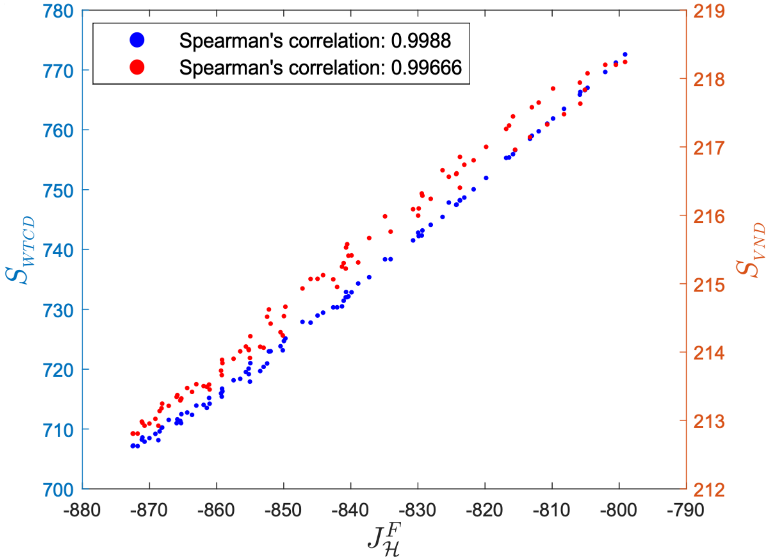
$$J^F_{\mathcal{H}}(\mathcal{U}) = \frac{n^F}{2}\log(2\pi e) - \frac{1}{2}\log\left|\Lambda_{k+L}\right| + \frac{1}{2}\log\left|\Lambda^U_{k+L}\right|$$

- [Passerini and Severini 2009 IJTAS] Von Neumann graph signature $\mathcal{H}_{VN}\left(\hat{\mathcal{L}}_w\right) = -\sum_{i=1}^{n}\frac{\hat{\lambda}_i}{2}\log\frac{\hat{\lambda}_i}{2}$
- [Kitanov and Indelman 2019] used $\mathcal{H}_{VN} \approx \tilde{\mathcal{H}}_{VN} = \frac{n\log 2}{2} - \frac{1}{2}\left(\mathrm{Tr}\left[\hat{\mathcal{L}}^2_w\right] - n\right)$ for BSP in the *unfocused* case
- $\tilde{\mathcal{H}}_{VN}$ is more efficient to compute and supports incremental update to allow online operation
- Following similar derivations, and utilizing the Laplacian structures of Λ_{k+L} and Λ^U_{k+L} we define

$$S_{VND} = \frac{n^F}{2}\log(2\pi e) - \frac{1}{2}\left[h_w - h^{U,A}_w\right]$$

Measurement Selection

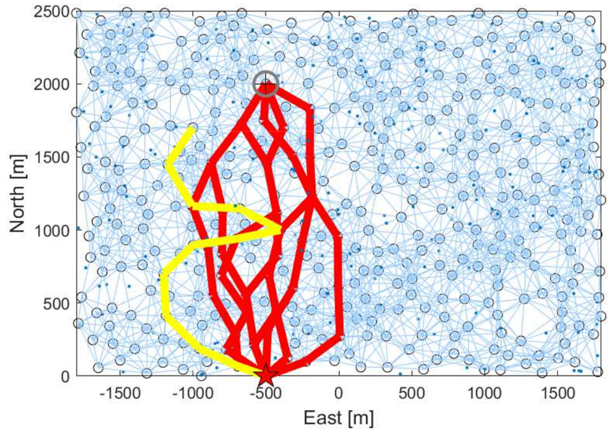
- 2D pose SLAM Intel dataset with $n = 1228$ robot poses and 278 loop closure observations
- Generated $n_L = 278$ subgraphs with a random set of $n^F < n/2$ *focused* variables
- Goal: find the most informative subset of observations with respect to a random *focused* set of variables



Topological signatures vs marginal entropy for randomly sampled loop closures

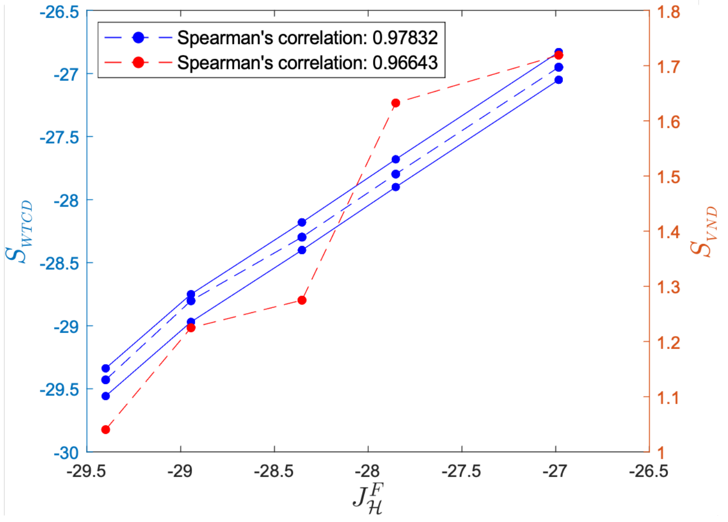
Active 2D Pose SLAM

- 25 different candidate paths generated on top of a PRM
- Yellow path represents the initial belief
- *focused* set of variables is defined as the last robot pose (the goal) and is shared among all candidate paths
- Goal: reduce uncertainty at the goal position



signature	measurement selection	active SLAM
S_{WTCD}	18.88	1.21
S_{VND}	**12.02**	**0.14**
$J^F_{\mathcal{H}}$	146.24	6.34

average running time over 20 experiments in *ms*



Topological signatures vs $J^F_{\mathcal{H}}$ in a *focused* BSP problem. Solid lines represent the bounds from **Theorem 2**

Conclusions

- FT-BSP – a novel concept for belief space planning with respect to a *focused* set of variables
- Introduced two topological signatures to approximate the *focused* information theoretic cost function
- The approximation error is bounded
- Topological signatures can be used to discriminate between candidate actions

Thank you!