D2A-BSP: Distilled Data Association Belief Space Planning with Performance Guarantees Under Budget Constraints
Supplementary Material

Moshe Shienman and Vadim Indelman

This document provides supplementary material to the paper [1]. Therefore, it should not be considered a self-contained document, but instead regarded as an appendix of [1]. Throughout this report, all notations and definitions are with compliance to the ones presented in [1].

1 Incrementally adapting $\mathcal{LB}\left[\eta\right], \mathcal{UB}\left[\eta\right]$

We denote the bounds presented in Theorem 4 in [1] as $\mathcal{LB}\left[\eta|b_k^s\right], \mathcal{UB}\left[\eta|b_k^s\right]$, i.e. with respect to a simplified belief b_k^s with M_k^s components. Given a belief component $r_k \notin M_k^s$ with associated weight w_k^r, we denote $M_k^{s+1} \triangleq M_k^s \cup r_k$. By definition (see eq. (11) in [1]) the simplified belief at time k for M_k^{s+1} components is given by

$$b_k^{s+1} \triangleq \sum_{j=1}^{M_k^{s+1}} w_k^{s+1,j} b_k^j \quad , \quad w_k^{s+1,j} \triangleq \frac{w_k^j}{w_k^{m,s+1}}, \tag{1}$$

where w_k^j corresponds to the original belief component weight (see eq. (3) in [1]) and $w_k^{m,s+1} = w_k^{m,s} + w_k^r$. As such, $\mathcal{LB}\left[\eta|b_k^{s+1}\right], \mathcal{UB}\left[\eta|b_k^{s+1}\right]$ represent the bounds for the measurement likelihood η given a simplified belief b_k^{s+1} with M_k^{s+1} components. Using eq. (28) in [1] and (1) we define

$$\eta^{s+1} \triangleq \sum_{i}^{|L|} \sum_{j}^{M_k^{s+1}} \tilde{\zeta}_{k+1}^{i,j} w_k^{s+1,j}. \tag{2}$$

We now present how to incrementally adapt the lower and upper bounds. We begin by writing the lower bound with respect to the simplified belief b_k^{s+1} using (2) and get the recursive update rule

$$\begin{aligned} \mathcal{LB}\left[\eta|b_k^{s+1}\right] =& \eta^{s+1} w_k^{m,s+1} = \sum_{i}^{|L|} \sum_{j}^{M_k^{s+1}} \tilde{\zeta}_{k+1}^{i,j} w_k^j = \sum_{i}^{|L|} \sum_{j}^{M_k^s} \tilde{\zeta}_{k+1}^{i,j} w_k^j + \sum_{i}^{|L|} \tilde{\zeta}_{k+1}^{i,r} w_k^r = \\ & \eta^s w_k^{m,s} + \sum_{i}^{|L|} \tilde{\zeta}_{k+1}^{i,r} w_k^r = \mathcal{LB}\left[\eta|b_k^s\right] + \sum_{i}^{|L|} \tilde{\zeta}_{k+1}^{i,r} w_k^r. \end{aligned} \tag{3}$$

Using similar derivations the recursive update rule for the upper bound is given by

$$\begin{aligned} \mathcal{UB}\left[\eta|b_k^{s+1}\right] =& \eta^{s+1} w_k^{m,s+1} + (1 - w_k^{m,s+1}) \sigma \sum_{i}^{|L|} \alpha^i = \eta^s w_k^{m,s} + \sum_{i}^{|L|} \tilde{\zeta}_{k+1}^{i,r} w_k^r + (1 - w_k^{m,s} - w_k^r) \sigma \sum_{i}^{|L|} \alpha^i = \\ & \eta^s w_k^{m,s} + (1 - w_k^{m,s}) \sigma \sum_{i}^{|L|} \alpha^i + \sum_{i}^{|L|} \tilde{\zeta}_{k+1}^{i,r} w_k^r - w_k^r \sigma \sum_{i}^{|L|} \alpha^i = \mathcal{UB}\left[\eta|b_k^s\right] + w_k^r \sum_{i}^{|L|} \left[\tilde{\zeta}_{k+1}^{i,r} - \sigma \alpha^i\right]. \end{aligned} \tag{4}$$

Moshe Shienman is with the Technion Autonomous Systems Program (TASP), Technion - Israel Institute of Technology, Haifa 32000, Israel, smoshe@campus.technion.ac.il. Vadim Indelman is with the Department of Aerospace Engineering, Technion - Israel Institute of Technology, Haifa 32000, Israel. vadim.indelman@technion.ac.il.

2 Incrementally adapting $\mathcal{LB}\left[\mathcal{H}\right], \mathcal{UB}\left[\mathcal{H}\right]$

We follow similar derivations as in Section 1 and denote the bounds presented in Theorem 2 in [1] as $\mathcal{LB}\left[\mathcal{H}|b_k^s\right], \mathcal{UB}\left[\mathcal{H}|b_k^s\right]$, i.e. with respect to a simplified belief b_k^s with M_k^s components. Given a belief component $r_k \notin M_k^s$ with associated weight w_k^r, we denote $M_k^{s+1} \triangleq M_k^s \cup r_k$. Using (1) we also denote the bounds over the cost term, given a simplified belief b_k^{s+1} with M_k^{s+1} components, as $\mathcal{LB}\left[\mathcal{H}|b_k^{s+1}\right], \mathcal{UB}\left[\mathcal{H}|b_k^{s+1}\right]$. Deriving a direct recursive update rule for these bounds is not trivial. Instead, we show how each term in $\mathcal{LB}\left[\mathcal{H}|b_k^{s+1}\right], \mathcal{UB}\left[\mathcal{H}|b_k^{s+1}\right]$ can be incrementally updated individually. Using (2) we begin with a recursive update rule for η^{s+1} given by

$$\eta^{s+1} = \sum_{i}^{|L|} \sum_{j}^{M_k^{s+1}} \tilde{\zeta}_{k+1}^{i,j} w_k^{s+1,j} = \frac{1}{w_k^{m,s+1}} \left[\eta^s w_k^{m,s} + \sum_{i}^{|L|} \tilde{\zeta}_{k+1}^{i,r} w_k^r \right]. \tag{5}$$

Using equations (24) and (28) in [1] we write the recursive update rule for $\mathcal{H}^{s+1}$, i.e. the cost given a simplified belief b_k^{s+1}

$$\begin{aligned}
&\mathcal{H}^{s+1} = -\frac{1}{\eta^{s+1}} \sum_{i}^{|L|} \sum_{j}^{M_k^{s+1}} \left[\tilde{\zeta}_{k+1}^{i,j} w_k^{s+1,j} log\left(\tilde{\zeta}_{k+1}^{i,j} w_k^{s+1,j} \right) \right] + log\left(\eta^{s+1}\right) = \\
&-\frac{1}{\eta^{s+1}} \left[\sum_{i}^{|L|} \sum_{j}^{M_k^s} \left[\frac{\tilde{\zeta}_{k+1}^{i,j} w_k^j}{w_k^{m,s+1}} log\left(\frac{\tilde{\zeta}_{k+1}^{i,j} w_k^j}{w_k^{m,s+1}} \right) \right] + \sum_{i}^{|L|} \left[\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} log\left(\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} \right) \right] \right] + log\left(\eta^{s+1}\right) = \\
&-\frac{1}{\eta^{s+1}} \left[\frac{w_k^{m,s}}{w_k^{m,s+1}} \sum_{i}^{|L|} \sum_{j}^{M_k^s} \left[\tilde{\zeta}_{k+1}^{i,j} w_k^{s,j} log\left(\frac{\tilde{\zeta}_{k+1}^{i,j} w_k^{s,j} w_k^{m,s}}{w_k^{m,s+1}} \right) \right] + \sum_{i}^{|L|} \left[\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} log\left(\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} \right) \right] \right] + log\left(\eta^{s+1}\right) = \\
&-\frac{1}{\eta^{s+1}} \left[\frac{w_k^{m,s}}{w_k^{m,s+1}} \left[\sum_{i}^{|L|} \sum_{j}^{M_k^s} \left[\tilde{\zeta}_{k+1}^{i,j} w_k^{s,j} log\left(\tilde{\zeta}_{k+1}^{i,j} w_k^{s,j} \right) \right] + \sum_{i}^{|L|} \sum_{j}^{M_k^s} \left[\tilde{\zeta}_{k+1}^{i,j} w_k^{s,j} log\left(\frac{w_k^{m,s}}{w_k^{m,s+1}} \right) \right] \right] + \sum_{i}^{|L|} \left[\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} log\left(\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} \right) \right] \right] + log\left(\eta^{s+1}\right) = \\
&-\frac{1}{\eta^{s+1}} \left[\frac{w_k^{m,s}}{w_k^{m,s+1}} \left[-\eta^s \left[\mathcal{H}^s - log\left(\eta^s\right) \right] + \eta^s log\left(\frac{w_k^{m,s}}{w_k^{m,s+1}} \right) \right] + \sum_{i}^{|L|} \left[\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} log\left(\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} \right) \right] \right] + log\left(\eta^{s+1}\right) = \\
&\frac{\eta^s}{\eta^{s+1}} \frac{w_k^{m,s}}{w_k^{m,s+1}} \left[\mathcal{H}^s - log\left(\eta^s\right) - log\left(\frac{w_k^{m,s}}{w_k^{m,s+1}} \right) \right] - \frac{1}{\eta^{s+1}} \sum_{i}^{|L|} \left[\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} log\left(\frac{\tilde{\zeta}_{k+1}^{i,r} w_k^r}{w_k^{m,s+1}} \right) \right] + log\left(\eta^{s+1}\right).
\end{aligned} \tag{6}$$

Using Theorem 2 in [1] we explicitly write the lower bound with respect to the simplified belief b_k^{s+1}

$$\begin{aligned}
\mathcal{LB}\left[\mathcal{H}|b_k^{s+1}\right] =& \frac{\eta^{s+1} w_k^{m,s+1}}{\mathcal{UB}\left[\eta|b_k^{s+1}\right]} \left[\mathcal{H}^{s+1} - log(\eta^{s+1}) \right] - \frac{w_k^{m,s+1}}{\mathcal{UB}\left[\eta|b_k^{s+1}\right]} \sum_{i}^{|L|} \sum_{j}^{M_k^{s+1}} \tilde{\zeta}_{k+1}^{i,j} w_k^{s+1,j} log\left(\frac{w_k^{m,s+1}}{\mathcal{LB}\left[\eta|b_k^{s+1}\right]} \right) = \\
& \frac{\eta^{s+1} w_k^{m,s+1}}{\mathcal{UB}\left[\eta|b_k^{s+1}\right]} \left[\mathcal{H}^{s+1} - log(\eta^{s+1}) \right] - \frac{w_k^{m,s+1} \eta^{s+1}}{\mathcal{UB}\left[\eta|b_k^{s+1}\right]} log\left(\frac{w_k^{m,s+1}}{\mathcal{LB}\left[\eta|b_k^{s+1}\right]} \right),
\end{aligned} \tag{7}$$

and observe that each term can be incrementally updated individually using (5), (6) and Section 1. Similarly, using Theorem 2 in [1], we explicitly write the upper bound with respect to the simplified belief b_k^{s+1}

$$\begin{aligned}
\mathcal{UB}\left[\mathcal{H}|b_k^{s+1}\right] =& \frac{\eta^{s+1} w_k^{m,s+1}}{\mathcal{LB}\left[\eta|b_k^{s+1}\right]} \left[\mathcal{H}^{s+1} - log(\eta^{s+1}) \right] - \frac{w_k^{m,s+1}}{\mathcal{LB}\left[\eta|b_k^{s+1}\right]} \sum_{i}^{|L|} \sum_{j}^{M_k^{s+1}} \tilde{\zeta}_{k+1}^{i,j} w_k^{s+1,j} log\left(\frac{w_k^{m,s+1}}{\mathcal{UB}\left[\eta|b_k^{s+1}\right]} \right) - \gamma log\left(\frac{\gamma}{|L| \left|\neg M_k^{s+1}\right|} \right) = \\
& \frac{\eta^{s+1} w_k^{m,s+1}}{\mathcal{LB}\left[\eta|b_k^{s+1}\right]} \left[\mathcal{H}^{s+1} - log(\eta^{s+1}) \right] - \frac{w_k^{m,s+1} \eta^{s+1}}{\mathcal{LB}\left[\eta|b_k^{s+1}\right]} log\left(\frac{w_k^{m,s+1}}{\mathcal{UB}\left[\eta|b_k^{s+1}\right]} \right) - \gamma log\left(\frac{\gamma}{|L| \left|\neg M_k^{s+1}\right|} \right),
\end{aligned} \tag{8}$$

where $\gamma \triangleq 1 - \frac{\eta^{s+1} w_k^{m,s}}{\mathcal{UB}\left[\eta|b_k^{s+1}\right]}$. Since $0 \leq \gamma \leq 1$ by definition, the upper bound (8) holds when $|L| \left|\neg M_k^{s+1}\right| > 2$. We observe that each term can be incrementally updated individually using (5), (6) and Section 1.

References

[1] M. Shienman and V. Indelman. D2a-bsp: Distilled data association belief space planning with performance guarantees under budget constraints. In *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2022.