Incorporating Data Association Within Belief Space Planning For Robust Autonomous Navigation

Antony Thomas

Collaborators: Asst. Prof. Vadim Indelman (Advisor)
Dr. Shashank Pathak

Master's Thesis Seminar (*January 9, 2017*)

TECHNION
Israel Institute of Technology

ANPL | Autonomous Navigation and Perception Lab

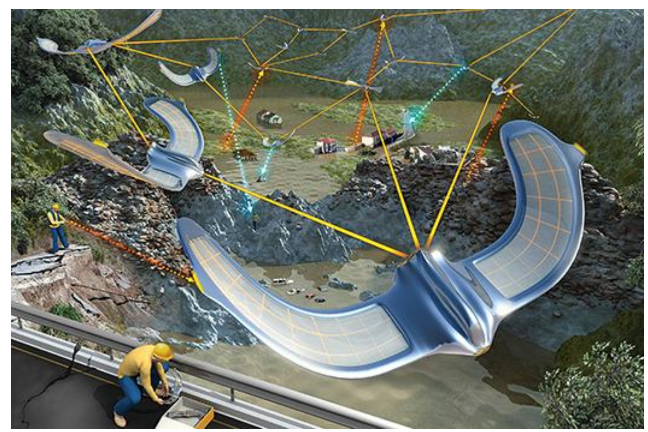
Introduction

- Why autonomous navigation ?

Autonomous Micro UAVs (Upenn)

Autonomous cars -
DARPA Urban Challenge 2007 winner 'Boss' (CMU)

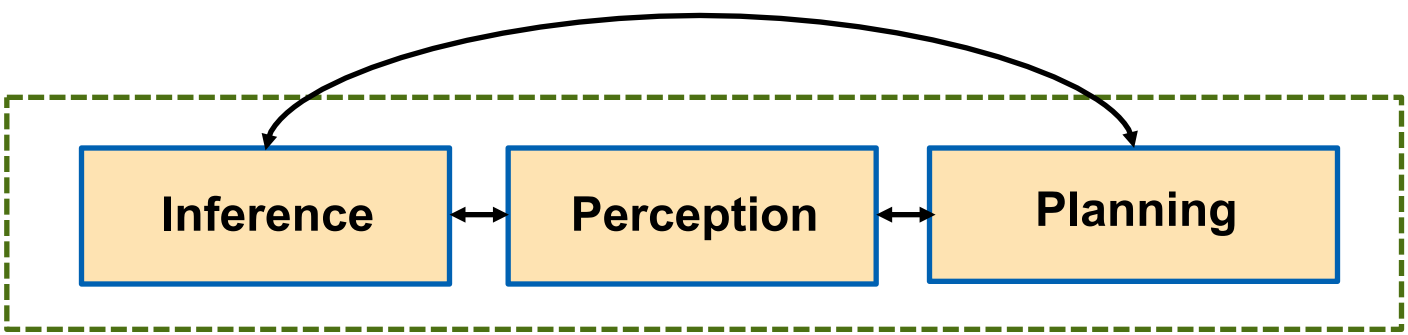
Autonomous robot janitor
(Fuji Heavy Industries)



Aerial sensor network (EPFL)

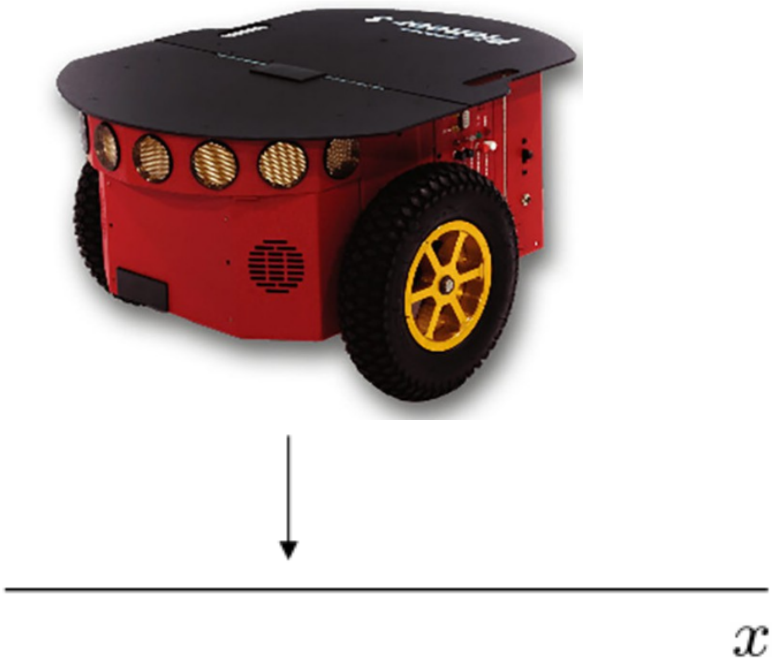
Introduction

- **Autonomous navigation involves:**
  - **Inference (estimation): Where am I ?**
  - **Perception: What is the environment perceived by sensors ?**
    e.g.: What am I looking at? Is that the same scene as before?
  - **Planning: What is the next best action(s) to realize a task ?**
    e.g.: where to look or navigate next?



Inference (Estimation)

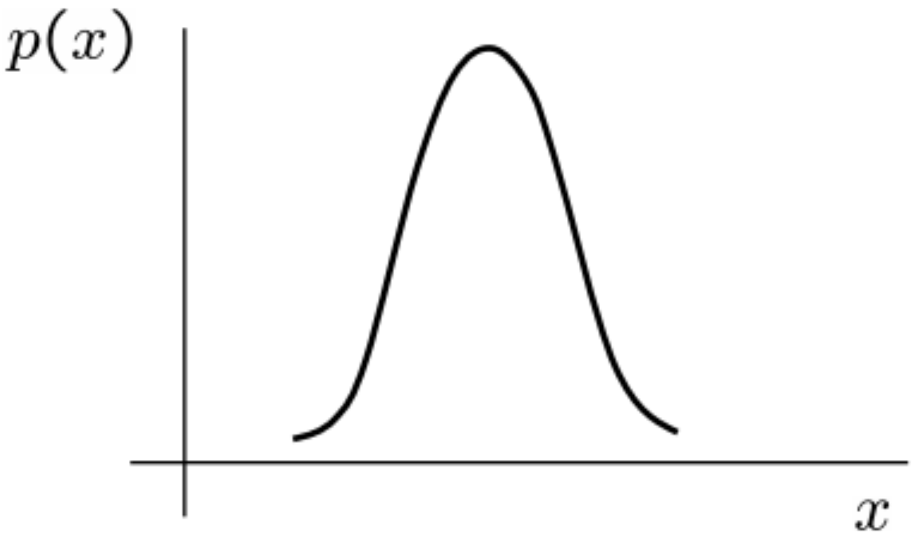
- Estimate the state $\boldsymbol{x}$ of the robot, given observations $\boldsymbol{z}$ and controls $\boldsymbol{u}$



Can we say that the robot is precisely at a particular location ?

Inference (Estimation)

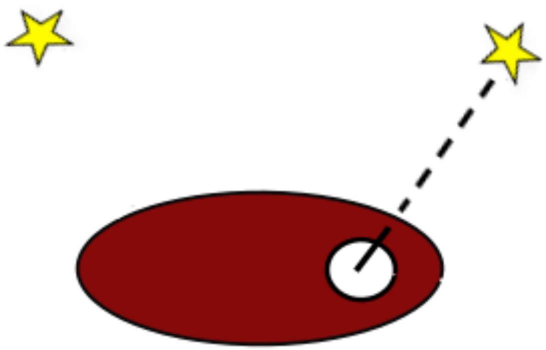
- Uncertainty in the robot's motions and observations
- Probability theory used to account for the uncertainty

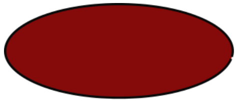


The robot is somewhere here

Perception

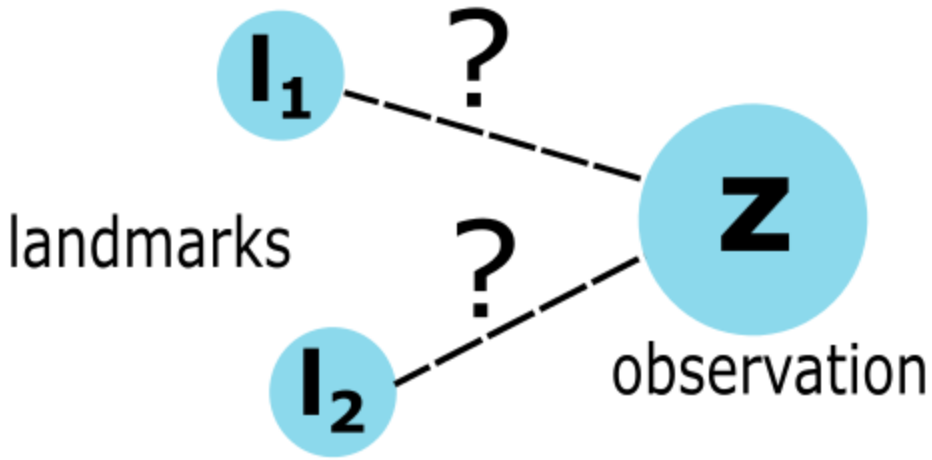
- Which is the landmark that the robot is looking at ?



- Robot 
- Landmark 
- Robot pose uncertainty 

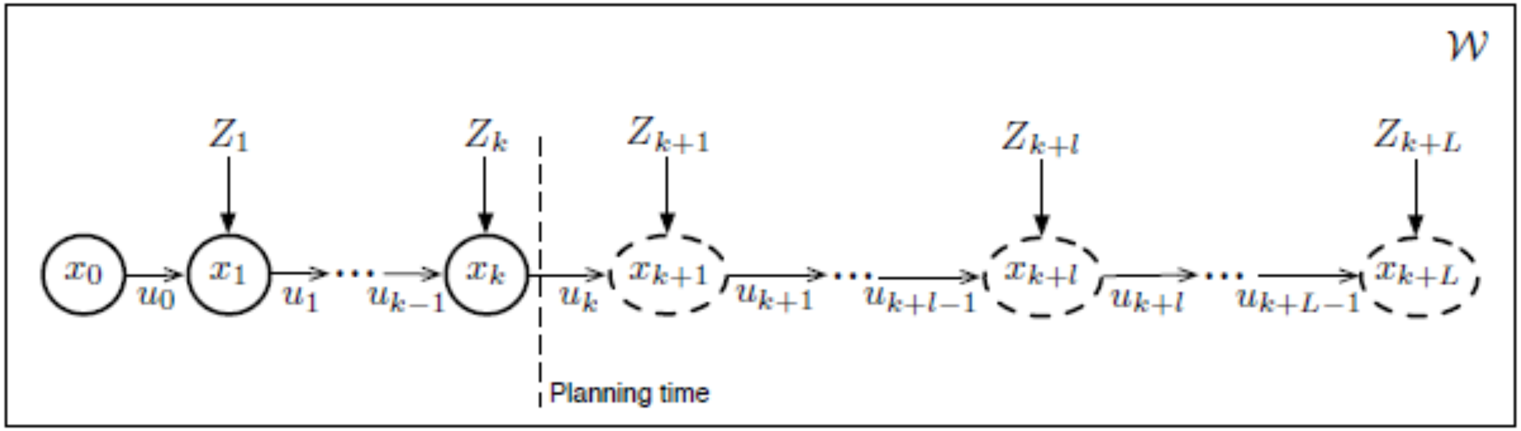
Data Association

- The problem of finding the correct correspondences between observations and landmarks



Planning

- **Belief space planning (BSP) and decision making under uncertainty**
  - Determine best *future* action(s) while accounting for different sources of uncertainty (stochastic control, imperfect sensing, uncertain environment)
  - Fundamental problem in robotics and AI



BSP for *L* look-ahead steps, Indelman et al., IJRR'15

Motivation

- What happens if the environment is ambiguous, perceptually aliased ?
  - Identical objects or scenes
  - Objects or scenes that appear similar for some viewpoints
- Examples:
  - Two corridors that look alike
  - Similar in appearance buildings, windows, …
- What if additionally, we have localization (or orientation) uncertainty ?

Wong et al., IJRR’15

Angeli et al.,TRO’08

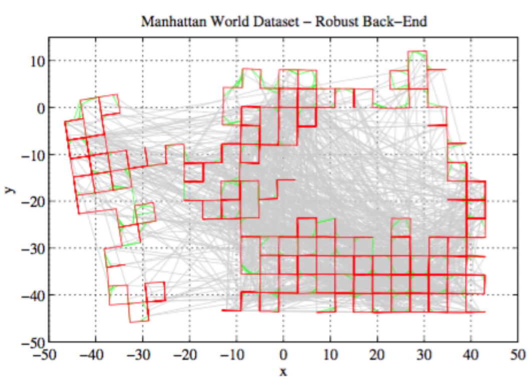
Motivation

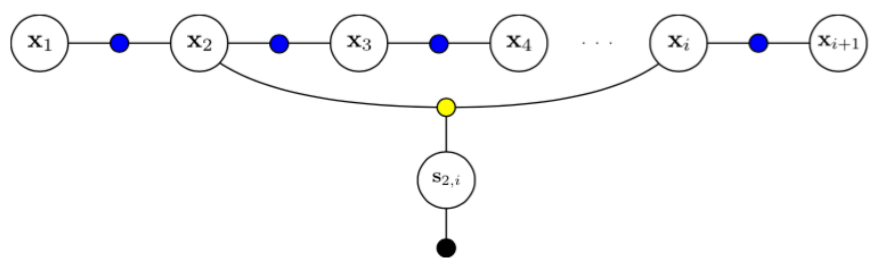
- What happens if the environment is ambiguous, perceptually aliased ?
  - Identical objects or scenes
  - Objects or scenes that appear similar for some viewpoints
- Examples:
  - Two corridors that look alike
  - Similar in appearance buildings, windows, …
- What if additionally, we have localization (or orientation) uncertainty ?

- *Identifying the true object from the aliasing object becomes particularly challenging (data association)*
- *Incorrect association (wrong scene) can be catastrophic*

Relation to Prior Work

- **Robust graph optimization approaches**:
  - Attempt to be resilient to incorrect data association (outliers overlooked by front-end algorithms, e.g. RANSAC)
  - Only consider the **passive** case whereas we consider the **active** case

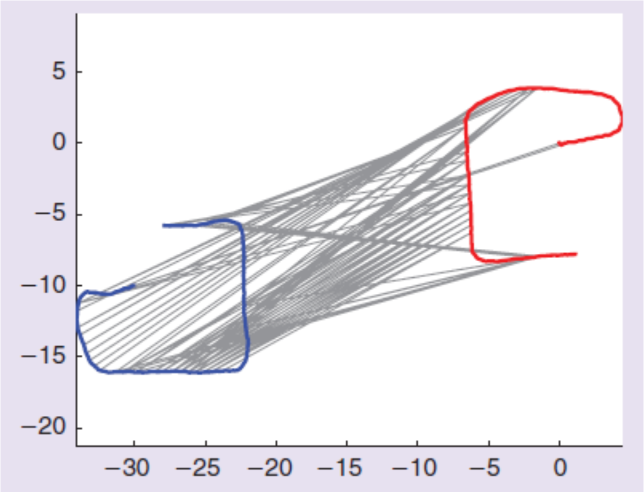




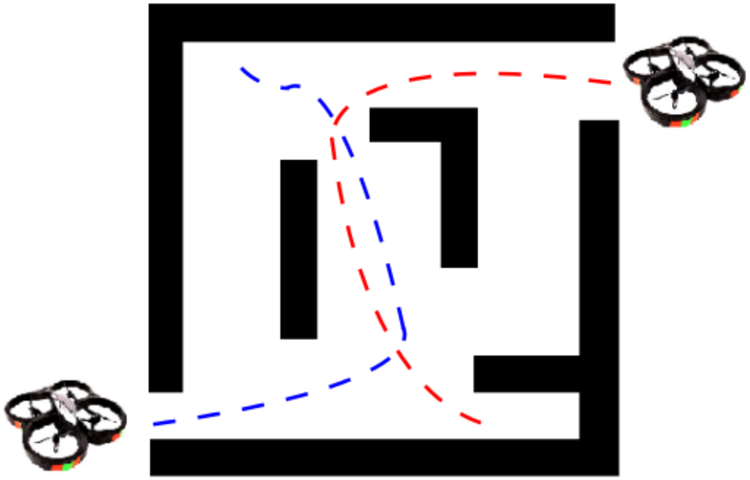
Sünderhauf et al., ICRA'12

Relation to Prior Work

- **Multi-robot pose graph localization from unknown initial relative poses and data association:**
  - Each possible data association modeled either as an inlier or an outlier
  - Only consider the **passive** case whereas we consider the **active** case

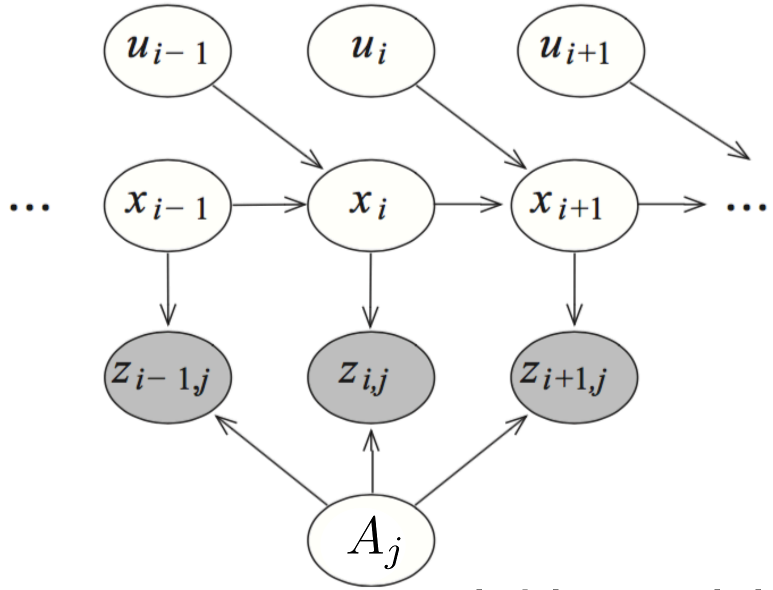


Indelman et al., CSM’16



Relation to Prior Work

- **Belief space planning (BSP) approaches**:
  - Typically assume data association (DA) to be **given** and **perfect**



Indelman et al., IJRR'15

Contribution

- **We develop a belief space planning (BSP) algorithm, considering both**
  - **Ambiguous data association** due to perceptual aliasing, and
  - **Localization uncertainty** due to stochastic control and imperfect sensing

- **Our approach - Data Association Aware Belief Space Planning (DA-BSP):**
  - Relaxes common assumption in BSP regarding **known** and **perfect** DA
  - To that end, we incorporate reasoning about DA within BSP

Formulation

- Consider a robot operating in a known environment (map given)
- The robot takes observations of different scenes or objects as it travels (e.g. images, laser scans)
- These observations are used to infer random variables of interest (e.g. robot pose)

Notations

- Belief at current time k:

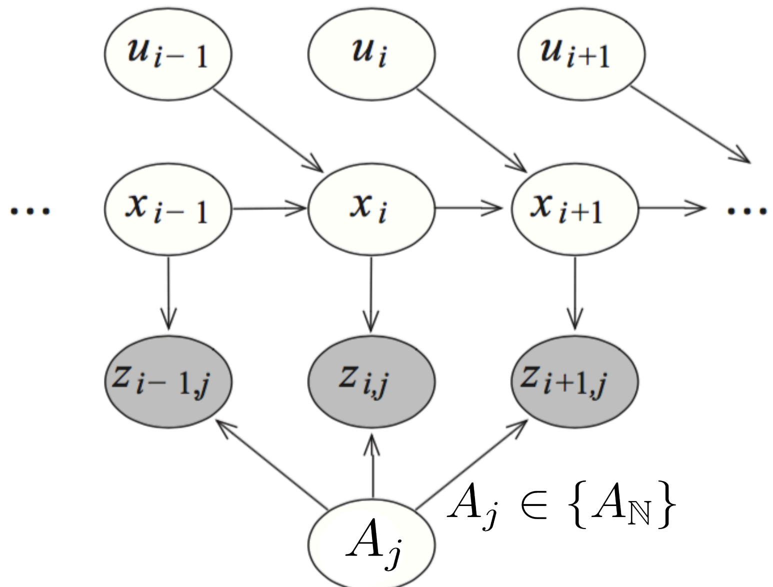
$$b[X_k] \doteq p(X_k | u_{0:k-1}, z_{0:k}) \text{ or } b[X_k] \doteq p(X_k | \mathcal{H}_k)$$

$\{x_0, \ldots, x_k\}$ ***robot poses***

$\{u_0, \ldots, u_{k-1}\}$ ***controls***

$\{z_0, \ldots, z_k\}$ ***observations***

$\{u_{0:k-1}, z_{0:k}\}$ ***history***



Indelman et al., IJRR'15

- Set of known objects/scenes: $\{A_\mathbb{N}\}$

Probabilistic Formulation

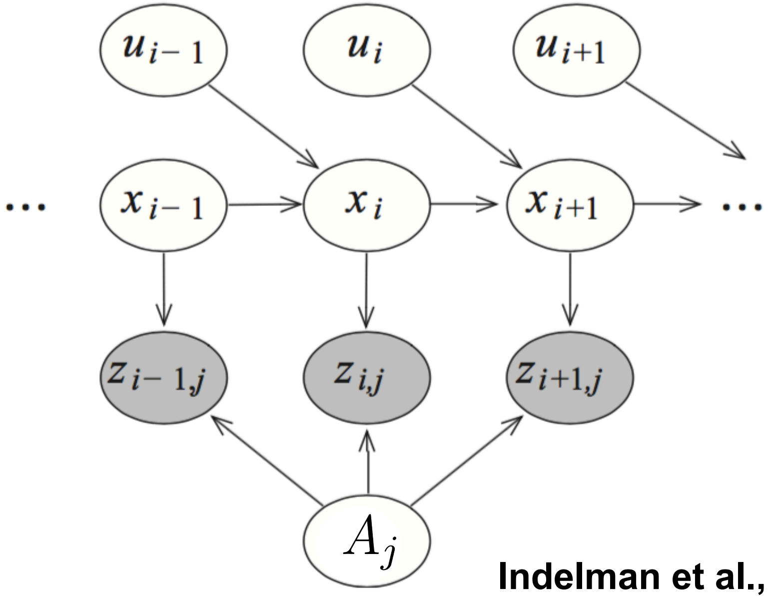
- Motion model:

$$p(x_{i+1}|x_i, u_i), \qquad x_{i+1} = f(x_i, u_i) + w_i, \quad w_i \sim \mathcal{N}(0, \Sigma_w)$$

- Observation model:

$$p(z_{i,j}|x_i, A_j), \qquad z_{i,j} = h(x_i, A_j) + v_i, \quad v_i \sim \mathcal{N}(0, \Sigma_v)$$

j-th scene or object



Indelman et al., IJRR'15

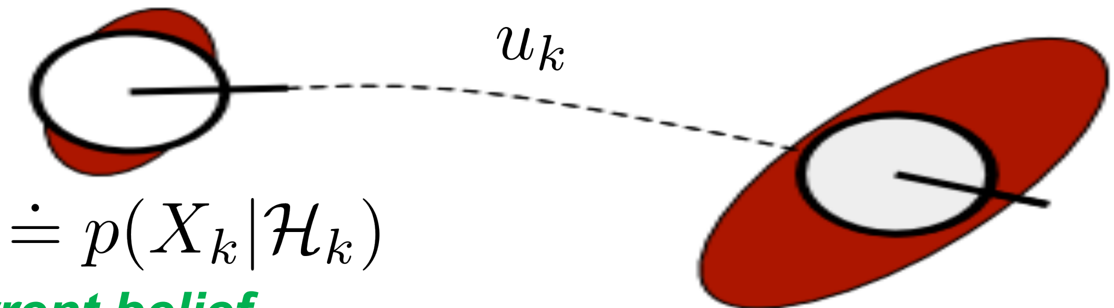
Propagated belief

- Given an action $\boldsymbol{u_k}$ we can propagate the belief using the motion model

$$b[X_{k+1}^{-}] \doteq p(X_{k+1}|\mathcal{H}_{k+1}^{-}) = p(X_k|\mathcal{H}_k)p(x_{k+1}|x_k, u_k)$$

$$\mathcal{H}_{k+1}^{-} = \mathcal{H}_k \cup u_k \qquad \mathcal{H}_k = \{u_{0:k-1}, z_{0:k}\}$$

history

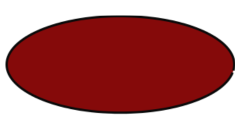


$$b[X_k] \doteq p(X_k|\mathcal{H}_k)$$

Current belief

$$b[X_{k+1}^{-}]$$

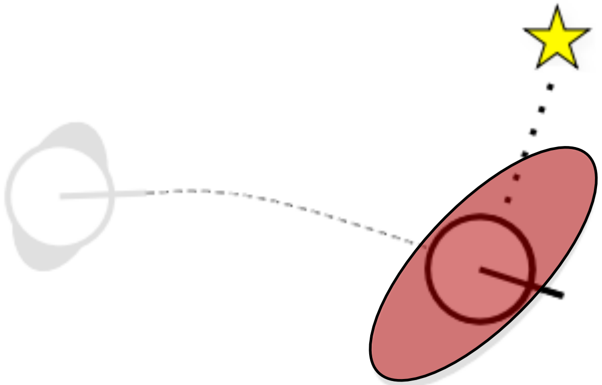
Propagated belief

- Robot 
- Robot pose uncertainty 

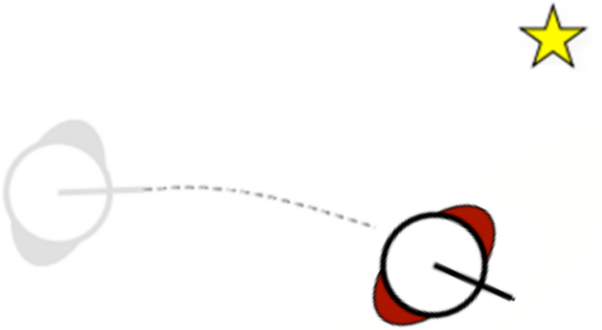
Posterior

- Observation model used to calculate the posterior belief at k+1

$$b[X_{k+1}] = \eta p(X_k|\mathcal{H}_k)p(x_{k+1}|x_k, u_k)p(z_{k+1}|x_{k+1}, A_j)$$



Observation



$b[X_{k+1}]$

Belief updated (posterior)

Objective Function

- Belief at time k+1, given control u_k and observation z_{k+1}:

$$b[X_{k+1}] \doteq p(X_{k+1} | u_{0:k}, z_{0:k+1})$$

- Objective function (single look ahead step):

$$J(u_k) \doteq \mathop{\mathbb{E}}_{z_{k+1}} \left\{ c\big(p(X_{k+1} | u_{0:k}, z_{0:k+1})\big) \right\} \equiv \mathop{\mathbb{E}}_{z_{k+1}} \left\{ c\big(b[X_{k+1}]\big) \right\}$$

  - $c(.)$ for example can be the trace of the covariance of X_{k+1}

- Why expectation ?
  - Observations are not given at planning time
  - Consider all possible realizations of a future observation z_{k+1}

- Optimal control:

$$u_k^\star \doteq \arg\min_{u_k} J(u_k)$$

Formulation – In Brief

- Given: a candidate action(s) and $b[X_k]$

- Calculate the posterior given u_k and particular future observation z_{k+1}

$$b[X_{k+1}] \doteq p(X_{k+1}|u_{0:k}, z_{0:k+1})$$

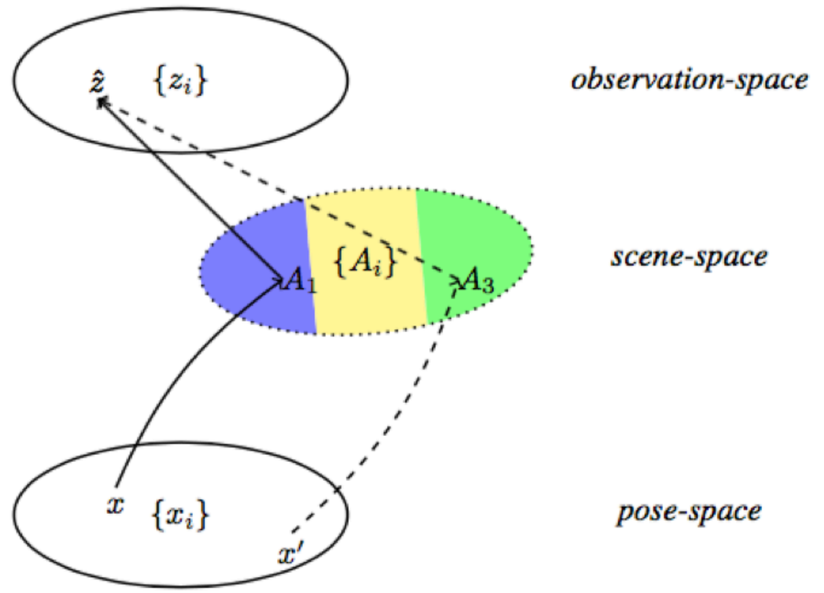
- Evaluate the cost function

- *Consider all possible values such an observation can assume* (expectation)

$$J(u_k) \doteq \underset{z_{k+1}}{\mathbb{E}} \left\{ c\big(p(X_{k+1}|u_{0:k}, z_{0:k+1})\big) \right\} \equiv \underset{z_{k+1}}{\mathbb{E}} \left\{ c\big(b[X_{k+1}]\big) \right\}$$

Concept

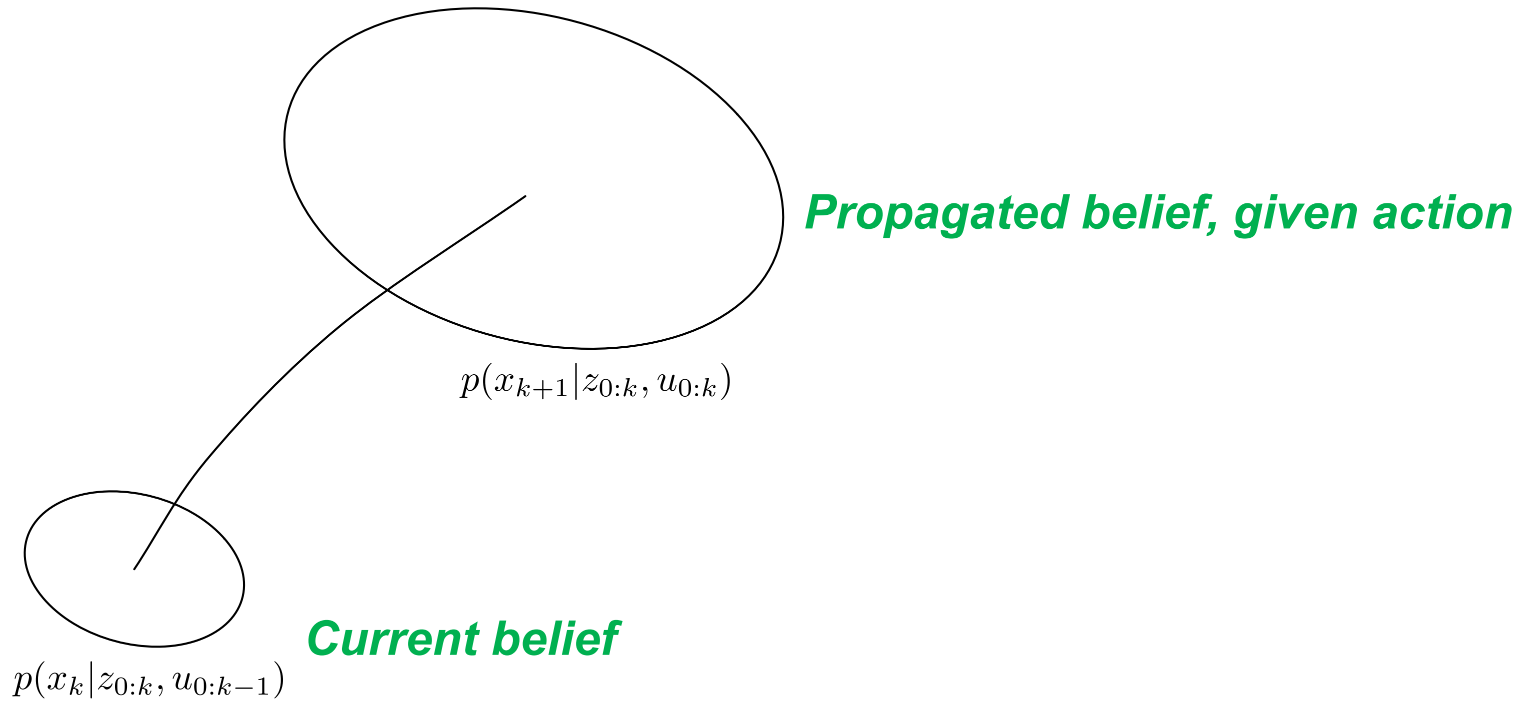
- In presence of perceptual aliasing, the **same observation** could be obtained from **different poses** viewing **different scenes**



How to capture this fact within belief space planning?

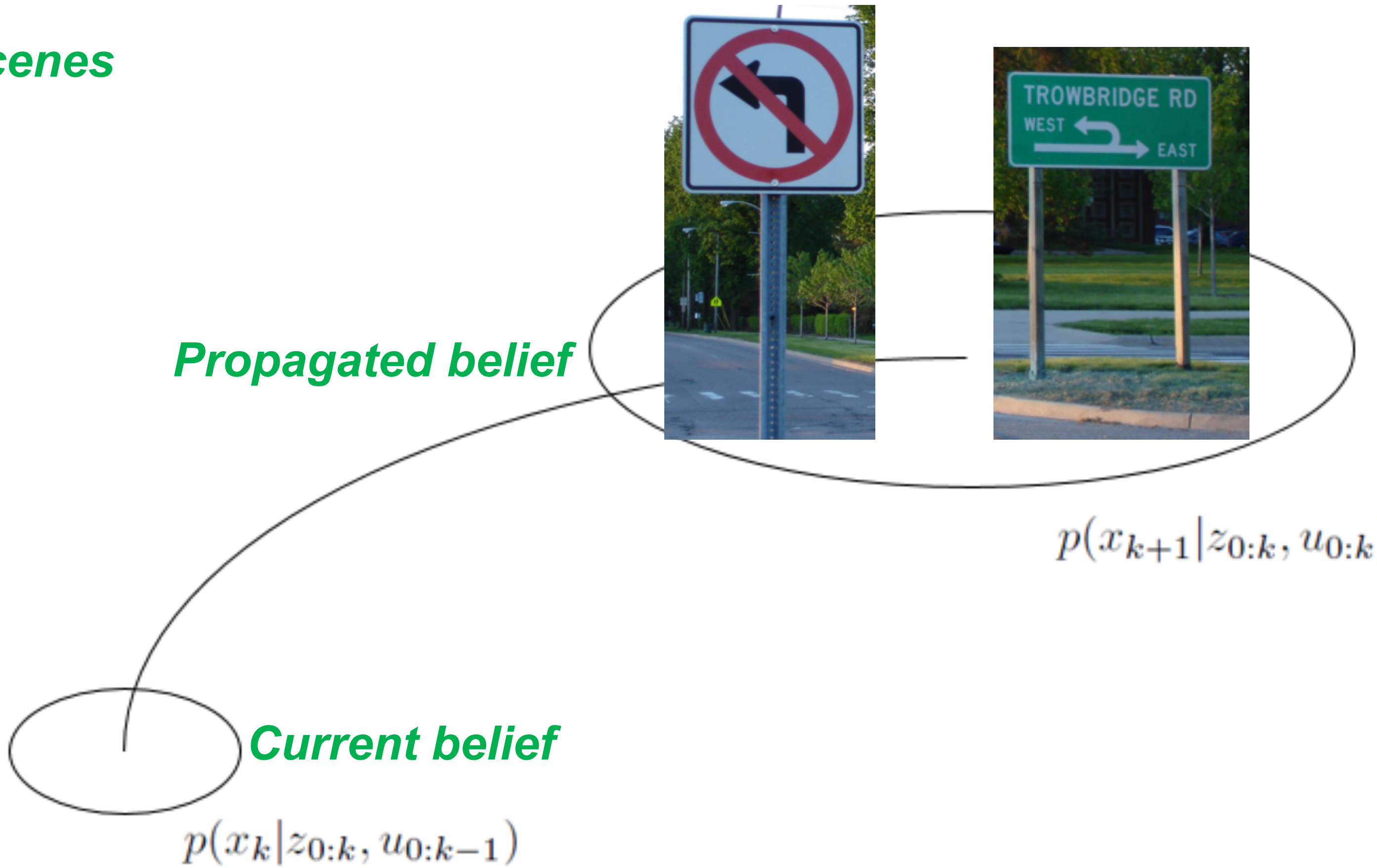
Concept – propagated belief

- It is unknown from what actual pose x_{k+1}, a future observation z_{k+1} will be acquired
- Robot pose x_{k+1} can be anywhere within $b[x_{k+1}^{-}] \doteq p(x_{k+1}|z_{0:k}, u_{0:k})$



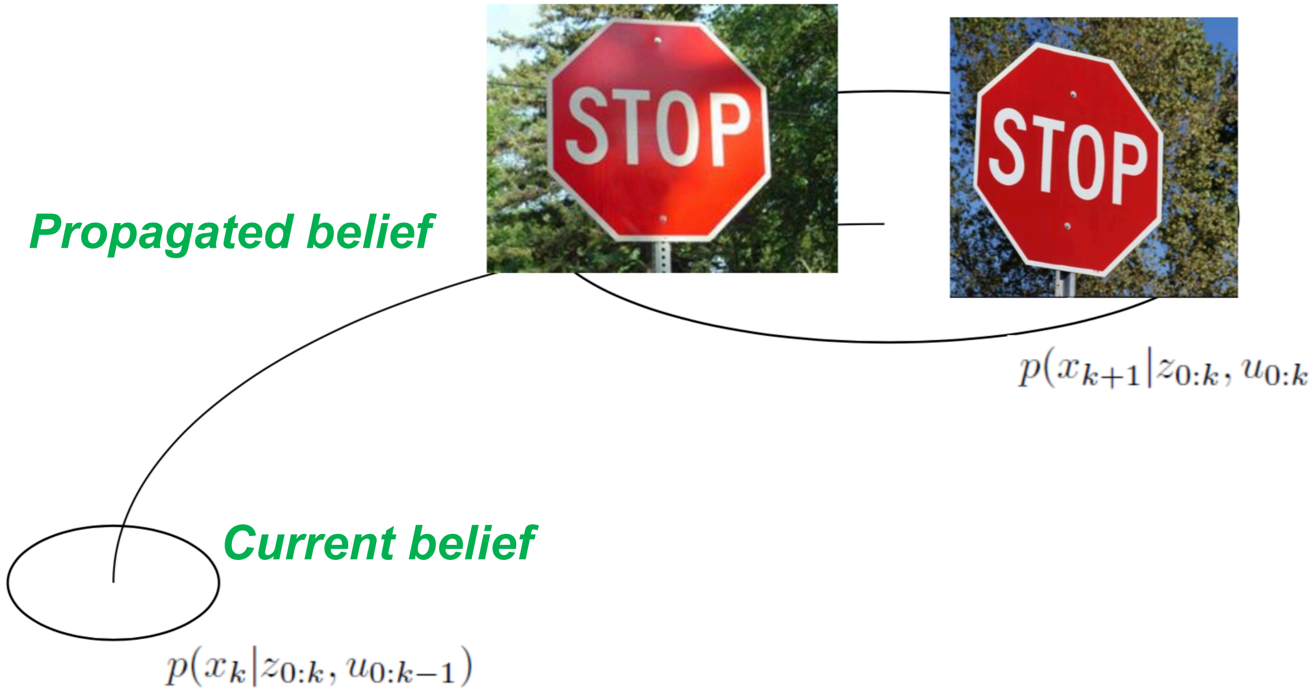
Concept - Intuition

Distinct scenes



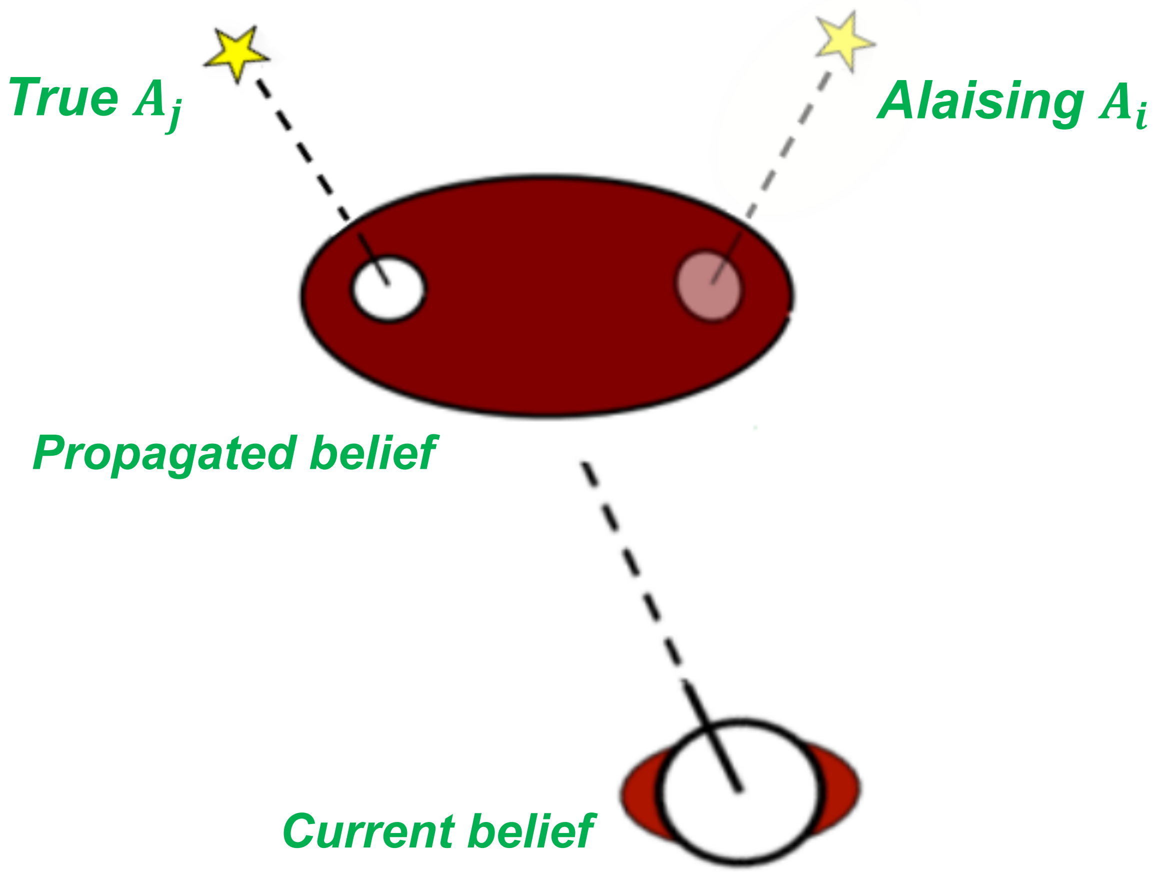
Concept - Intuition

Perceptually aliased scenes



Key Idea

- Reason about different scenes (or objects) that a specific future observation z_{k+1} could be generated from



Key Idea

- Reason about different scenes (or objects) that a specific future observation z_{k+1} could be generated from
- This means marginalizing over all the possible scenes/objects

$$b[X_{k+1}] = \sum_{j}^{\{A_{\mathbb{N}}\}} \underline{p(X_{k+1}, A_j | \mathcal{H}_{k+1}^{-}, z_{k+1})}$$

$$= \sum_{j}^{\{A_{\mathbb{N}}\}} \underline{p(X_{k+1} | \mathcal{H}_{k+1}^{-}, z_{k+1}, A_j) p(A_j | \mathcal{H}_{k+1}^{-}, z_{k+1})}$$

Key Idea

- Reason about different scenes (or objects) that a specific future observation z_{k+1} could be generated from
- This means marginalizing over all the possible scenes/objects

$$b[X_{k+1}] = \sum_{j}^{\{A_{\mathbb{N}}\}} \underline{p(X_{k+1}, A_j | \mathcal{H}_{k+1}^{-}, z_{k+1})}$$

$$= \sum_{j}^{\{A_{\mathbb{N}}\}} \underline{\boxed{p(X_{k+1} | \mathcal{H}_{k+1}^{-}, z_{k+1}, A_j)} p(A_j | \mathcal{H}_{k+1}^{-}, z_{k+1})}$$

- Posterior given that observation z_{k+1} was generated by scene A_j

Key Idea

- Reason about different scenes (or objects) that a specific future observation z_{k+1} could be generated from
- This means marginalizing over all the possible scenes/objects

$$b[X_{k+1}] = \sum_{j}^{\{A_{\mathbb{N}}\}} \underline{p(X_{k+1}, A_j | \mathcal{H}_{k+1}^{-}, z_{k+1})}$$

$$= \sum_{j}^{\{A_{\mathbb{N}}\}} \underline{p(X_{k+1} | \mathcal{H}_{k+1}^{-}, z_{k+1}, A_j) \boxed{p(A_j | \mathcal{H}_{k+1}^{-}, z_{k+1})}}$$

- Likelihood of scene A_j being actually the one which generated the observation z_{k+1}

Revisiting Objective Function

- Objective function (single look ahead step):

$$J(u_k) \doteq \underset{z_{k+1}}{\mathbb{E}} \left\{ c\Big(p(X_{k+1} | u_{0:k}, z_{0:k+1}) \Big) \right\} \equiv \underset{z_{k+1}}{\mathbb{E}} \left\{ c\Big(b[X_{k+1}] \Big) \right\}$$

- Write expectation explicitly:

$$J(u_k) \doteq \int_{z_{k+1}} p(z_{k+1} | \mathcal{H}_{k+1}^{-}) c \left(p(X_{k+1} | \mathcal{H}_{k+1}^{-}, z_{k+1}) \right)$$

- $c(.)$ for example can be the trace of the covariance of X_{k+1}

Posterior belief

$$J(u_k) \doteq \int_{z_{k+1}} p(z_{k+1}|\mathcal{H}_{k+1}^{-})c\left(\boxed{p(X_{k+1}|\mathcal{H}_{k+1}^{-}, z_{k+1})}\right)$$

- We already saw this term before

$$b[X_{k+1}] = \sum_{j}^{\{A_{\mathbb{N}}\}} \underline{p(X_{k+1}, A_j|\mathcal{H}_{k+1}^{-}, z_{k+1})}$$

$$= \sum_{j}^{\{A_{\mathbb{N}}\}} \underline{p(X_{k+1}|\mathcal{H}_{k+1}^{-}, z_{k+1}, A_j)p(A_j|\mathcal{H}_{k+1}^{-}, z_{k+1})}$$

- In other words
  - Observation is given, hence, **must** capture **one** (unknown) scene
  - Which one? Consider all possible scenes

Likelihood of an observation

$$J(u_k) \doteq \int_{z_{k+1}} \boxed{p(z_{k+1}|\mathcal{H}^-_{k+1})} c\left(p(X_{k+1}|\mathcal{H}^-_{k+1}, z_{k+1}) \right)$$

- Observation model

$$p(z_i|x_i, A_j)$$

  - Calculate corresponding likelihood for each A_j

$$p(z_{k+1}|\mathcal{H}^-_{k+1}) \equiv \sum_j p(z_{k+1}, A_j|\mathcal{H}^-_{k+1})$$

  - Accounting for all viewpoints x_{k+1}

$$p(z_{k+1}|\mathcal{H}^-_{k+1}) \equiv \sum_j \int_x p(z_{k+1}, x, A_j|\mathcal{H}^-_{k+1})$$

- Likelihood of a specific z_{k+1} to be captured

Likelihood of an observation

$$J(u_k) \doteq \int_{z_{k+1}} \boxed{p(z_{k+1}|\mathcal{H}_{k+1}^{-})} c\left(p(X_{k+1}|\mathcal{H}_{k+1}^{-}, z_{k+1}) \right)$$

- Corresponding to each A_j we get w_j

$$p(z_{k+1}|\mathcal{H}_{k+1}^{-}) \equiv \sum_j \int_x p(z_{k+1}, x, A_j|\mathcal{H}_{k+1}^{-}) \doteq \sum_j w_j$$

Summarizing

$$J(u_k) \doteq \int_{z_{k+1}} \boxed{p(z_{k+1}|\mathcal{H}_{k+1}^{-})} c\left(\boxed{p(X_{k+1}|\mathcal{H}_{k+1}^{-}, z_{k+1})}\right)$$

- Likelihood of a specific z_{k+1} to be captured

$$\underline{p(z_{k+1}|\mathcal{H}_{k+1}^{-})} \equiv \sum_j \int_x p(z_{k+1}, x, A_j|\mathcal{H}_{k+1}^{-}) \doteq \sum_j w_j$$

- Posterior *given* a specific observation z_{k+1}

$$\underline{b[X_{k+1}]} = \sum_j p(X_{k+1}|\mathcal{H}_{k+1}^{-}, z_{k+1}, A_j)p(A_j|\mathcal{H}_{k+1}^{-}, z_{k+1})$$

$$= \sum_j \tilde{w}_j b[X_{k+1}^{j+}]$$

Summarizing

$$J(u_k) \doteq \int_{z_{k+1}} \underline{p(z_{k+1}|\mathcal{H}^-_{k+1})} c \left(\underline{p(X_{k+1}|\mathcal{H}^-_{k+1}, z_{k+1})} \right)$$

$$\Downarrow$$

$$J(u_k) \doteq \int_{z_{k+1}} \underline{(\sum_j w_j)} c \left(\underline{\sum_j \tilde{w}_j b[X^{j+}_{k+1}]} \right)$$

- $\tilde{w}_j = \eta w_j$
- $b[X^{j+}_{k+1}] = p(X_{k+1}|H^-_{k+1}, z_{k+1}, A_j)$
- In short we get a GMM with weights $\tilde{w}_j$ corresponding to each $b[X^{j+}_{k+1}]$
- Do this for all possible realizations of a future observation z_{k+1}

Perceptual Aliasing Aspects

$$J(u_k) \doteq \int_{z_{k+1}} \underbrace{(\sum_j w_j)}\, c\Big(\underbrace{\sum_j \tilde{w}_j b[X_{k+1}^{j+}]}\Big)$$

- No perceptual aliasing:
  - Only **one** non-negligible weight $\tilde{w}_j$
  - Corresponds to the true scene A_j
  - Reduces to state of the art belief space planning
- With perceptual aliasing:
  - Multiple non-negligible weights $\tilde{w}_j$
  - Correspond to aliased scenes, given z_{k+1}
  - Posterior **becomes a mixture of pdfs (GMM)**

Summary

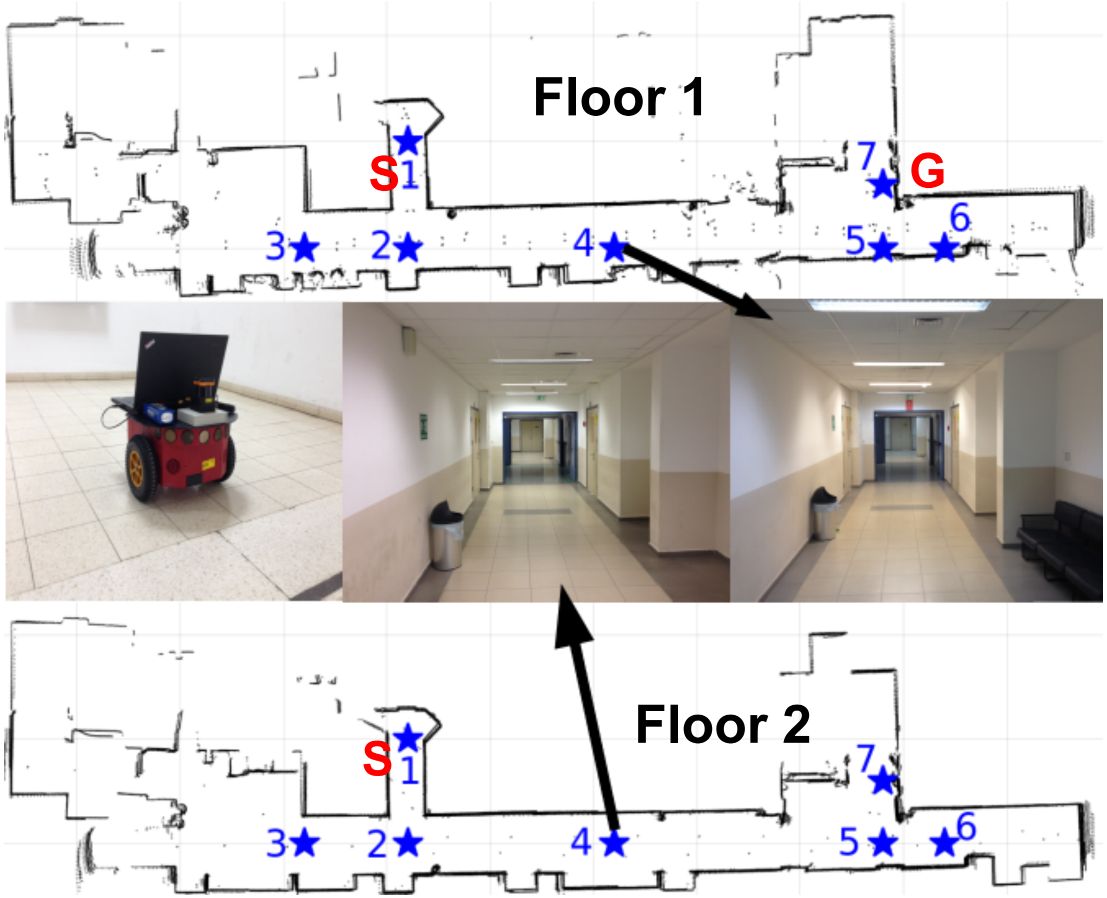
- Given belief at time k
  - $b[X_k]$
- Reason about possible scenes that can generate a future observation
  - GMM posterior
- Reason this belief evolution for different candidate actions
  - select best action
- Repeat

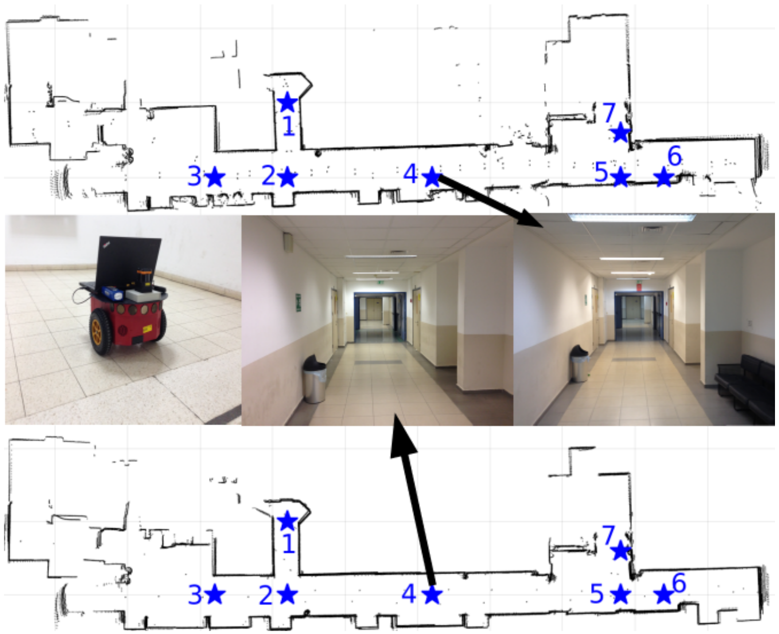
Results - Considered scenarios

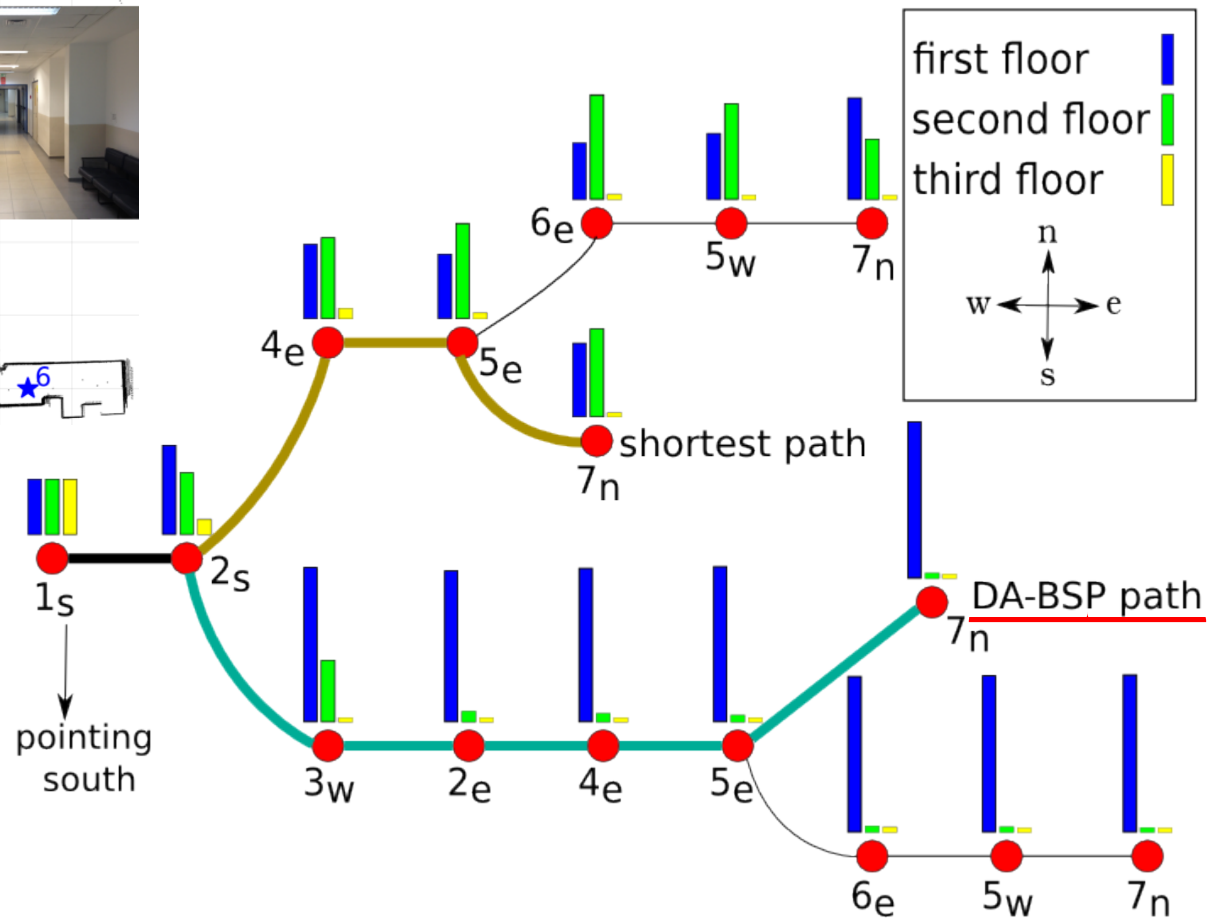
- **Scenario 1:** Real experiment in Ullman building using laser scanner
- **Scenario 2:** Simulation in Gazebo environment
- **Scenario 3:** Real experiment in Industrial Engg. building with April tags

Scenario 1: Real experiment in Ullman building using laser scanner

- DA-BSP in a 3-floor aliased environment with Pioneer robot
- Floor and position disambiguation considered

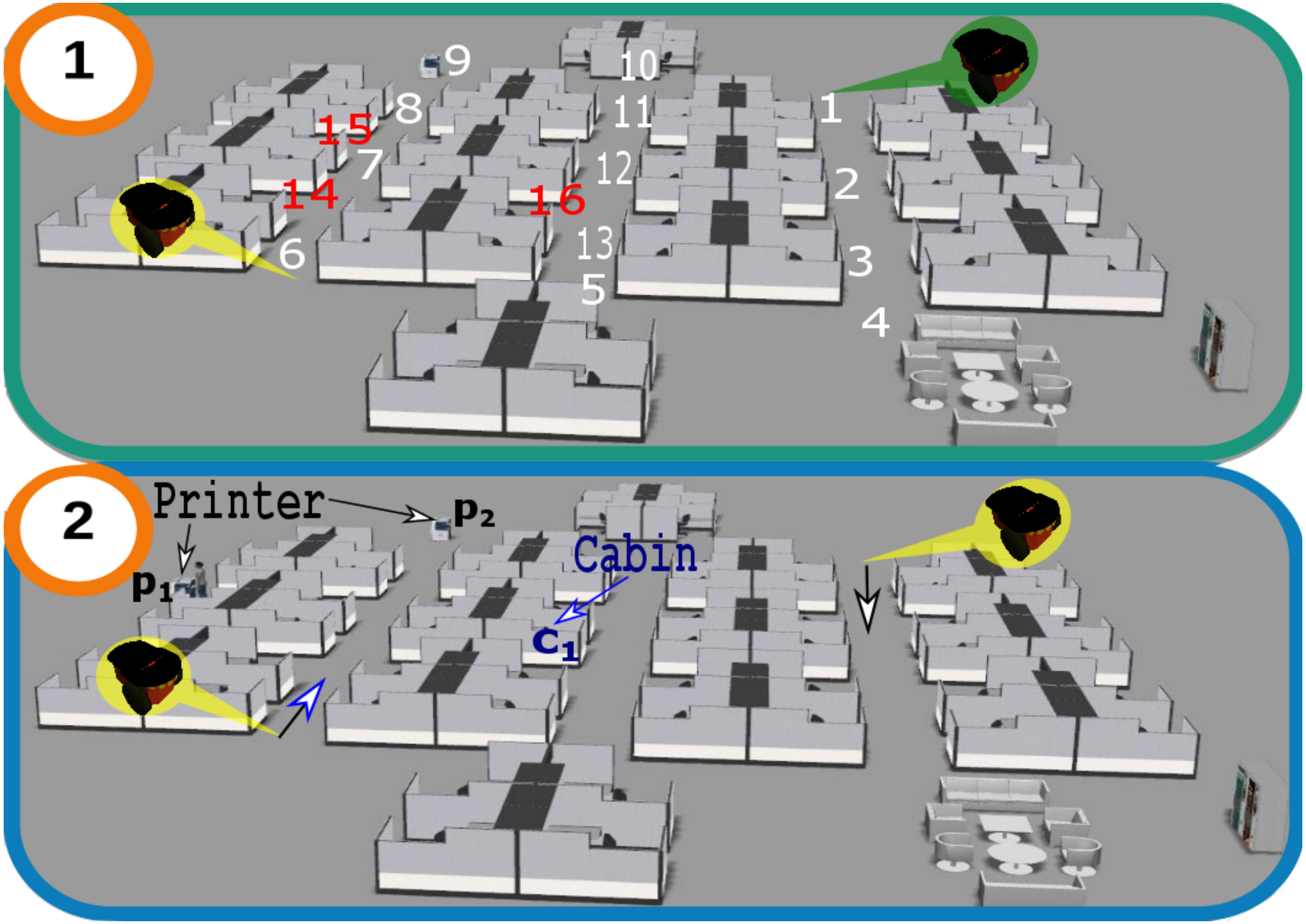



1
2
3
4
5
6
7


first floor
second floor
third floor
n
w
e
s
1s
2s
4e
5e
6e
5w
7n
shortest path
7n
DA-BSP path
3w
2e
4e
5e
6e
5w
7n
pointing south

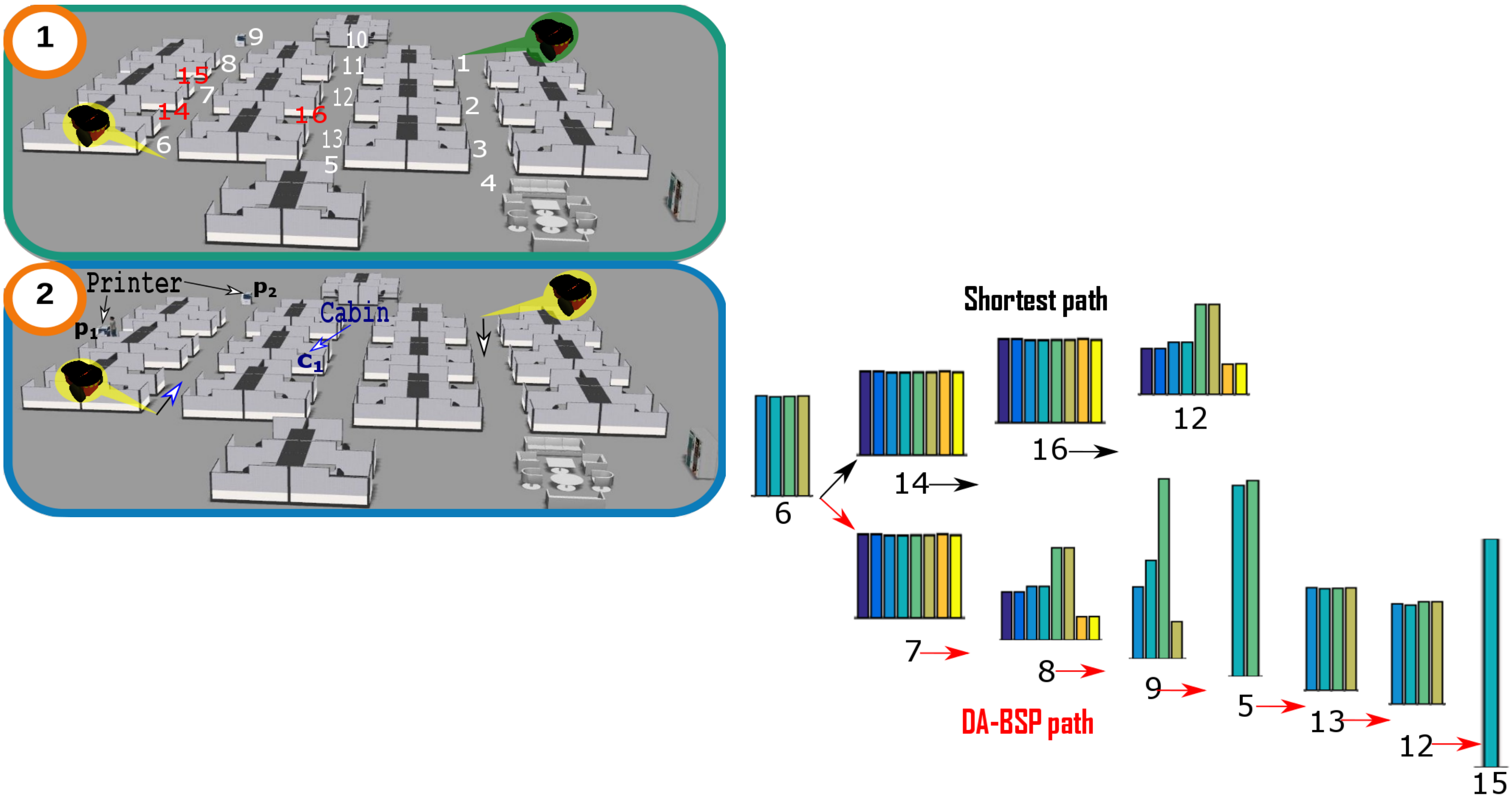
Scenario 2: Simulation in Gazebo environment

- Two floor aliased office floor environment



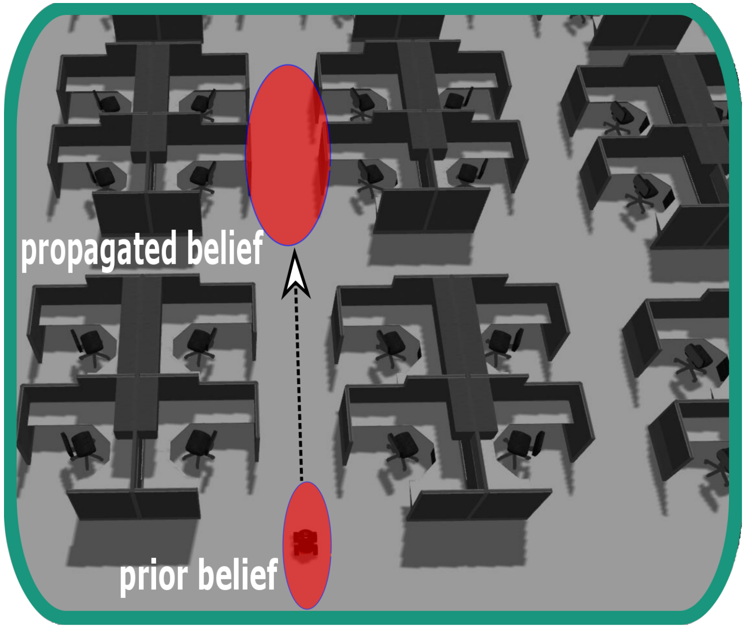
Scenario 2: Simulation in Gazebo environment

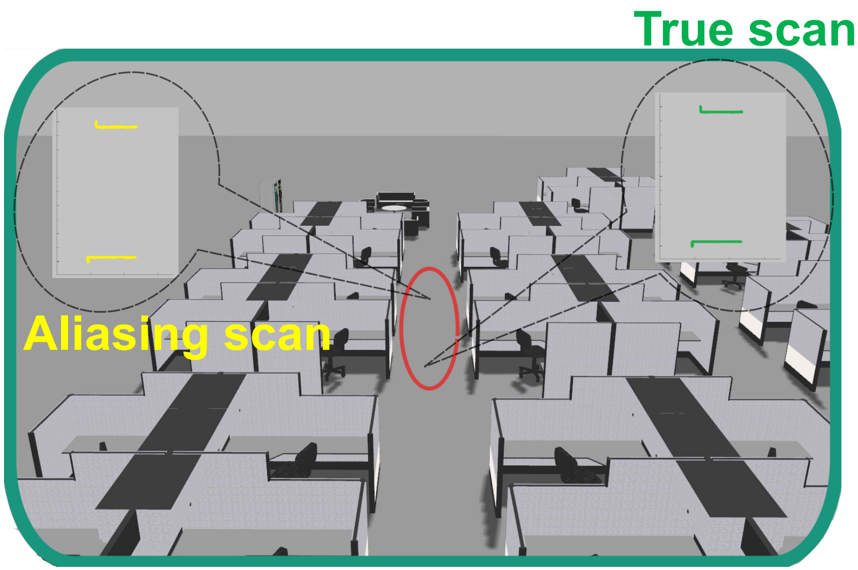
- Two floor aliased office floor environment

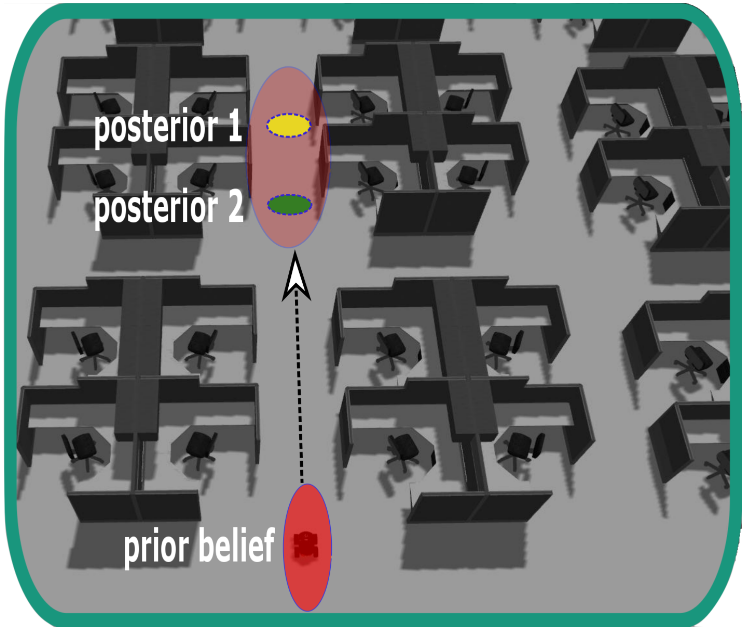


A. Thomas, Incorporating Data Association Within Belief Space Planning For Robust Autonomous Navigation

Incorporating aliasing

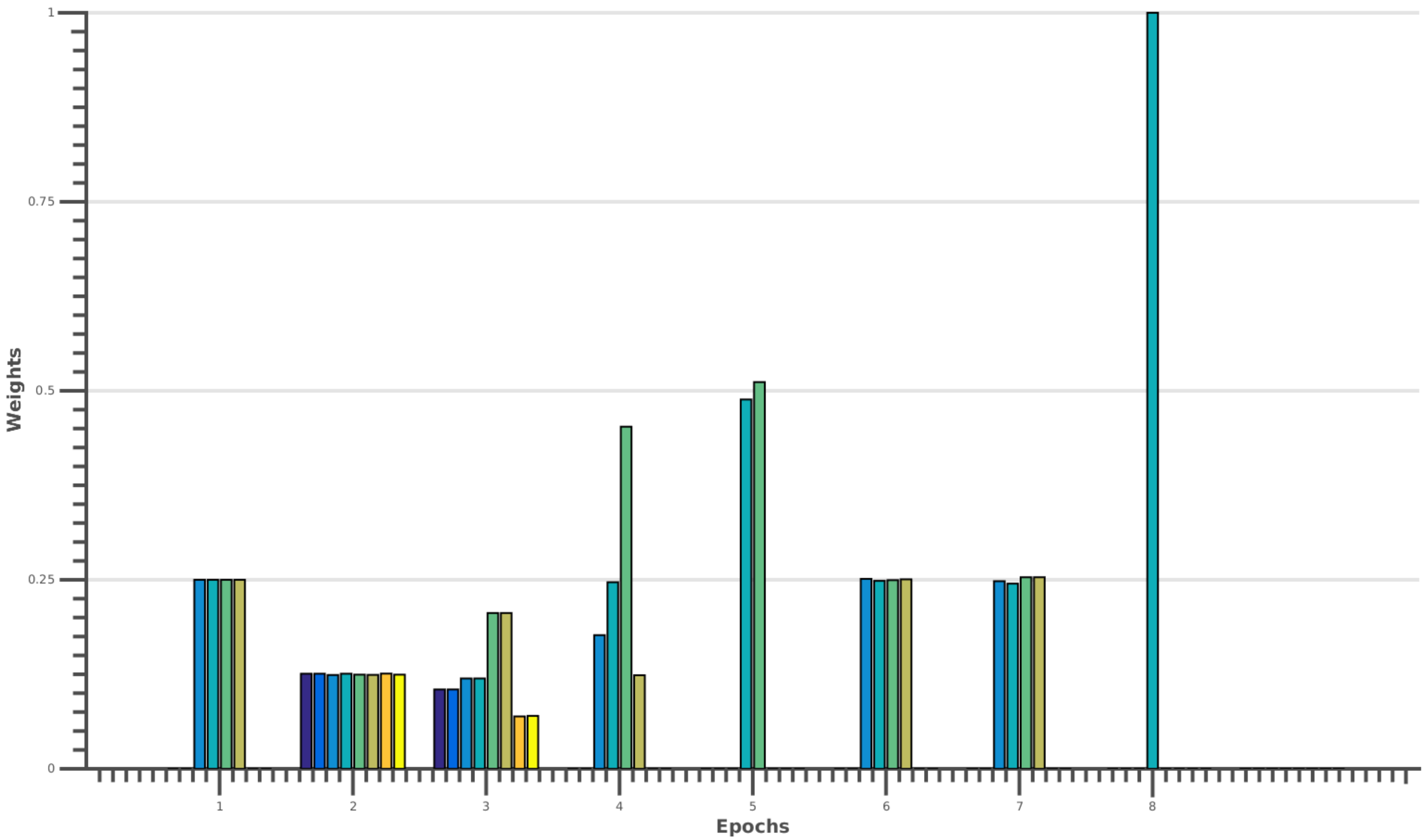






Weights and modes for DA-BSP path

- Evolution of weights for L = 2



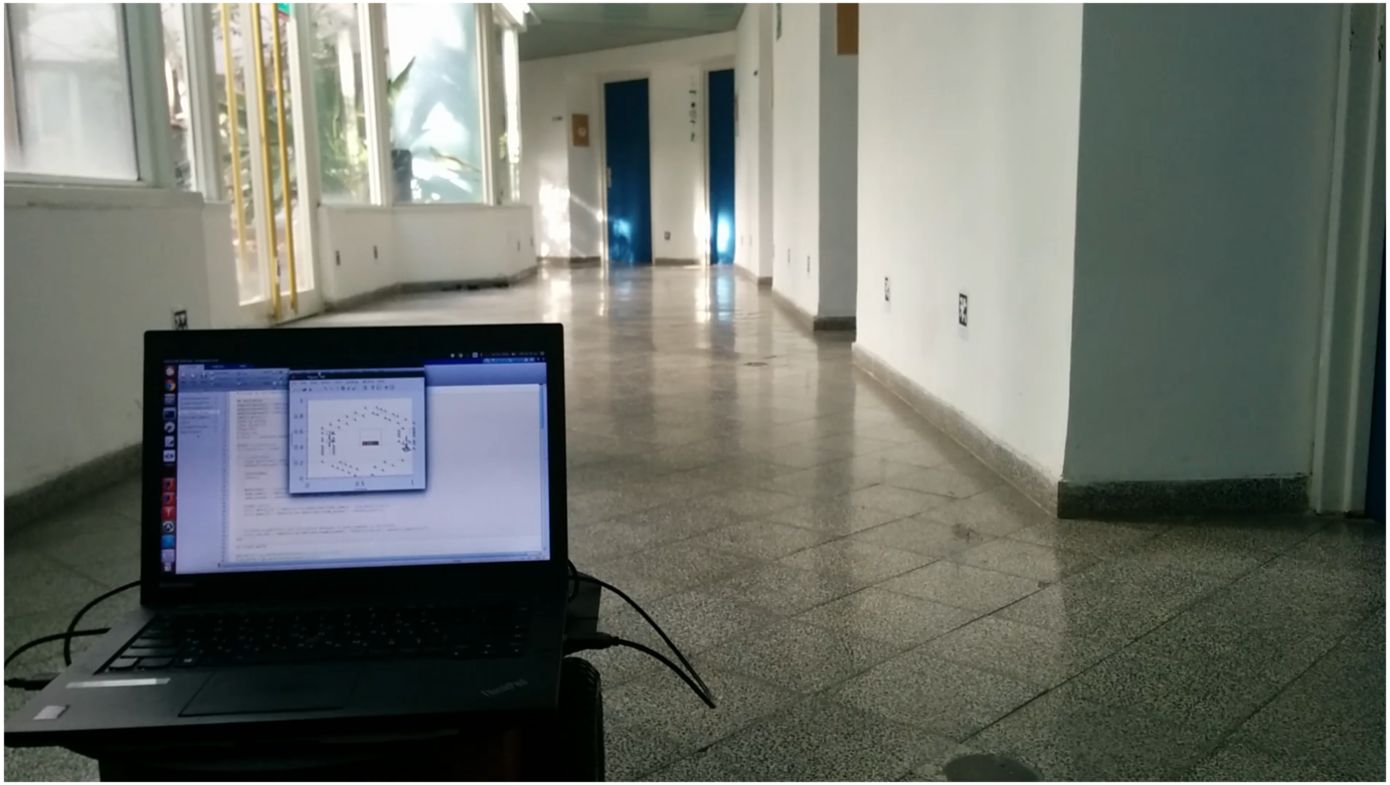
Quantitative evaluation

- η_{da} - weight of the true component
- BSP-uni - an association randomly chosen as the correct one(unimodal belief)
- η - measures whether association chosen in BSP-uni is correct or not (1/0)

Algorithm	Epoch	$L=2$		$L=4$		Inference	
		t(s)	$(\eta_{da}, \tilde{m})$	t(s)	$(\eta_{da}, \tilde{m})$	t(s)	$(\eta_{da}, \tilde{m})$
DA-BSP	2	293.45	(0.13,8)	733.67	(0.49,2)	29.40	(0.12,8)
	3	262.37	(0.25,4)	557.57	(0.25,4)	26.80	(0.12,8)
	5	10.05	(0.25,4)	115.95	(1,1)	2.40	(0.26,4)
	7	2.47	(1,1)	2.57	(1,1)	1.46	(1,1)
		t(s)	η	t(s)	η	t(s)	η
BSP-uni	2	7.04	1	18.96	1	4.17	1
	3	1.23	1	2.20	0	0.77	0
	5	1.04	0	1.90	0	0.56	1
	7	0.47	0	0.50	0	0.46	0

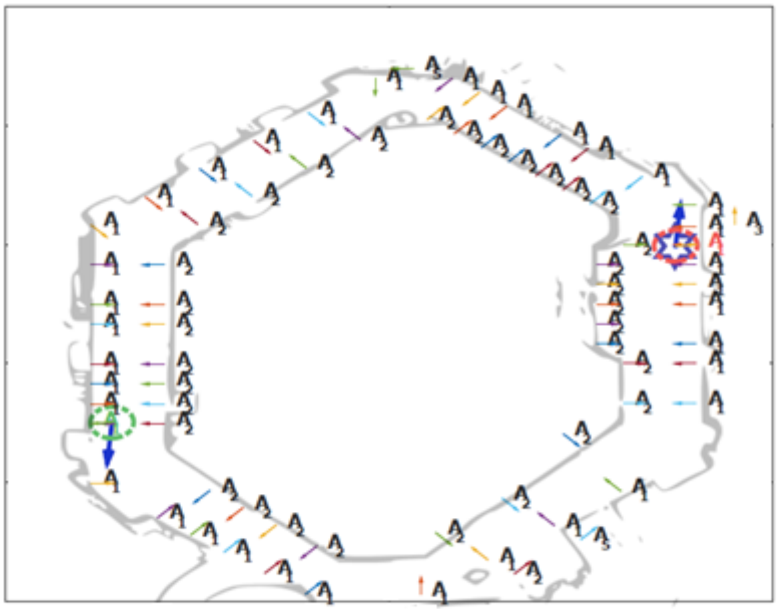
Scenario 3: Real experiment in Industrial Engg. building with April tags

- Octagonal world with a known map
- April Tags used to simulate aliasing environment and for localization

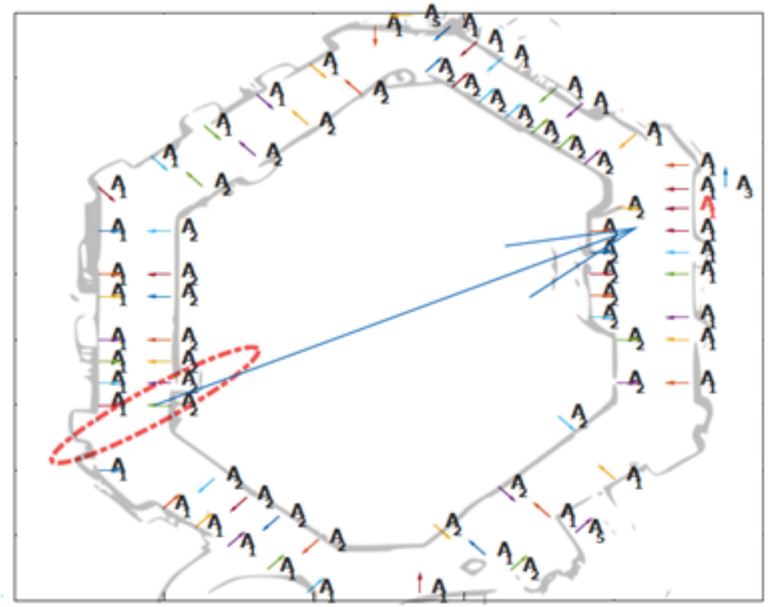


Starting configuration

- After 4 steps

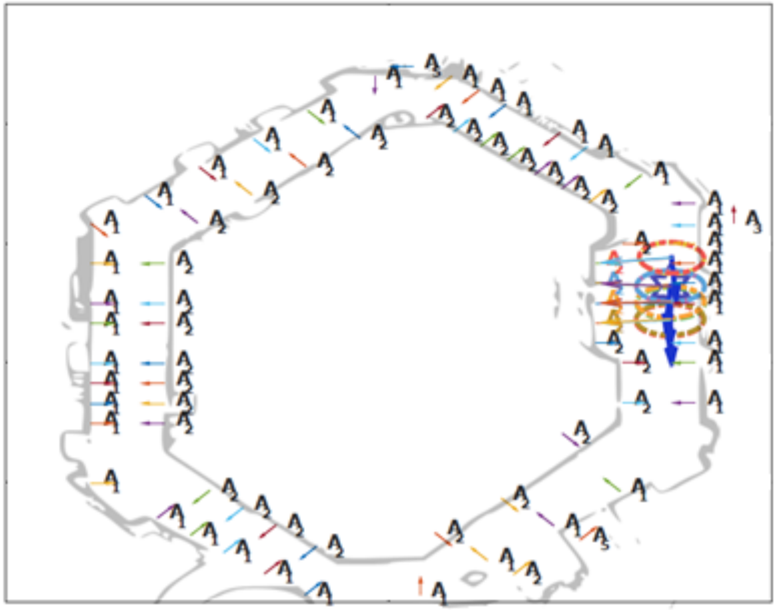


DA-BSP

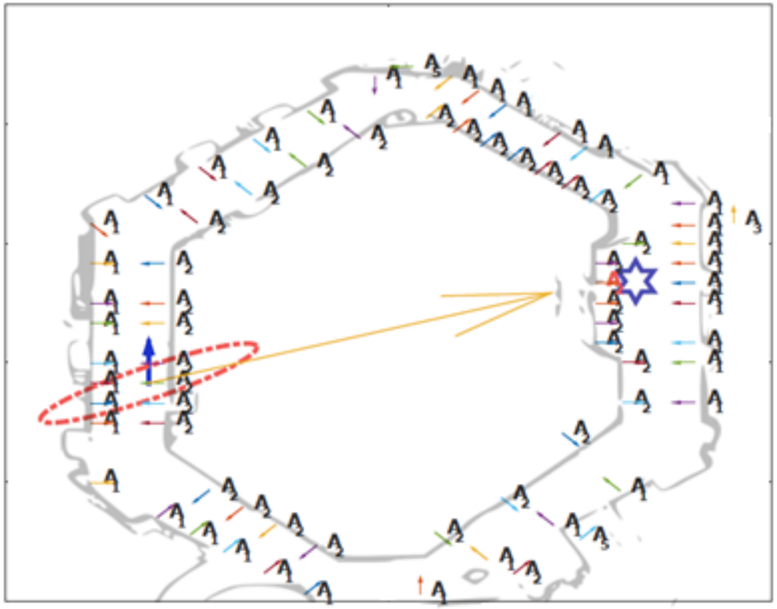


BSP- *uni*

- After 18 steps



DA-BSP



BSP- ***uni***

Quantitative evaluation

- ξ_{ca} - averaged η for 5 random runs

Algorithm	Epoch	$L=1$				$L=3$				Inference			
		t(s)	η_{da}	$\tilde{m}$	DA	t(s)	η_{da}	$\tilde{m}$	DA	t(s)	η_{da}	$\tilde{m}$	DA
DA−BSP	1	2.60	0.11	4.00	✓	95.57	0.08	5.95	✓	0.80	0.22	4.00	✓
	2	1.21	0.29	2.00	✓	5.75	0.13	1.37	✓	0.05	-	4.00	-
	4	1.00	0.35	2.00	✓	4.29	-	1.00	-	0.61	0.50	2.00	✓
	8	0.11	-	1.00	-	0.35	-	1.00	-	0.02	-	1.00	-
	12	3.90	0.11	4.80	✓	191.48	0.08	6.79	✓	1.16	0.28	4.20	✓
	16	2.62	0.12	3.03	✓	3.58	-	3.02	-	0.60	0.11	4.60	✓
	19	3.14	0.09	2.60	✓	82.16	0.04	6.10	✓	0.94	0.14	6.60	✓
		t(s)	ξ_{ca}		DA	t(s)	ξ_{ca}		DA	t(s)	ξ_{ca}		DA
BSP−uni	1	0.43	0.90		×	2.19	-		-	0.20	1.00		✓
	2	0.15	-		-	1.43	0.86		×	0.03	-		-
	4	0.25	1.00		✓	4.51	0.98		×	0.17	1.00		✓
	8	0.15	-		-	1.10	-		-	0.05	-		-
	12	0.26	1.00		✓	3.90	-		-	0.17	1.00		✓
	16	0.16	-		-	1.11	-		-	0.08	-		-
	19	0.30	1.00		✓	1.24	-		-	0.17	-		-

Conclusions

- **Data association aware belief space planning (DA-BSP)**
  - Considers **data association** within BSP
  - Relaxes typical assumption in BSP that DA is **given** and **correct**
  - Approach in particular suitable to handle scenarios with **perceptual aliasing** and **localization uncertainty**
  - Unified framework for **robust active** and **passive perception**

Thank you

Asst. Prof. Vadim Indelman

Dr. Shashank Pathak

Asaf Feniger

Term a)

$$J(u_k) \doteq \int_{z_{k+1}} \boxed{p(z_{k+1}|H_{k+1}^{-})} c\left(p(X_{k+1}|\mathcal{H}_{k+1}^{-}, z_{k+1}) \right)$$

- Likelihood of a specific z_{k+1} to be captured

$$p(z_{k+1}|\mathcal{H}_{k+1}^{-}) \equiv \sum_j \int_x p(z_{k+1}, x, A_j|\mathcal{H}_{k+1}^{-}) \doteq \sum_j w_j$$

$$\equiv \sum_j \int_x p(z_{k+1}|x, A_j, \mathcal{H}_{k+1}^{-}) p(A_j|x, \mathcal{H}_{k+1}^{-}) b[x_{k+1}^{-} = x]$$

- $b[x_{k+1}^{-}] = \int_{\neg x_{k+1}} b[X_{k+1}^{-}]$

Term b)

$$J(u_k) \doteq \int_{z_{k+1}} p(z_{k+1}|H^-_{k+1})c\left(\boxed{p(X_{k+1}|\mathcal{H}^-_{k+1}, z_{k+1})}\right)$$

- $$b[X_{k+1}] = \sum_j^{\{A_{\mathbb{N}}\}} p(X_{k+1}, A_j|\mathcal{H}^-_{k+1}, z_{k+1})$$
  $$= \sum_j^{\{A_{\mathbb{N}}\}} p(X_{k+1}|\mathcal{H}^-_{k+1}, z_{k+1}, A_j)\underline{p(A_j|\mathcal{H}^-_{k+1}, z_{k+1})}$$

- $$p(A_j|\mathcal{H}^-_{k+1}, z_{k+1}) = \int_x p(A_j, x|\mathcal{H}^-_{k+1}, z_{k+1})$$
  $$\doteq \eta \int_x p(z_{k+1}|A_j, x, \mathcal{H}^-_{k+1})p(A_j, x|\mathcal{H}_{k+1})$$
  $$\doteq \eta \int_x p(z_{k+1}|x, A_j, \mathcal{H}^-_{k+1})p(A_j|x, \mathcal{H}^-_{k+1})b[x^-_{k+1} = x]$$

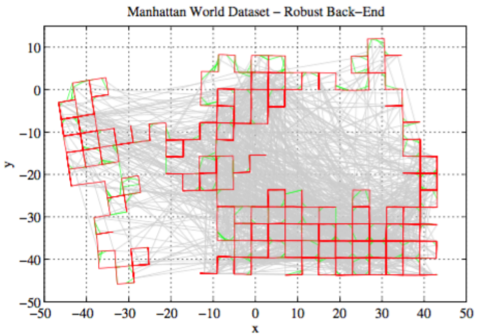
Weights

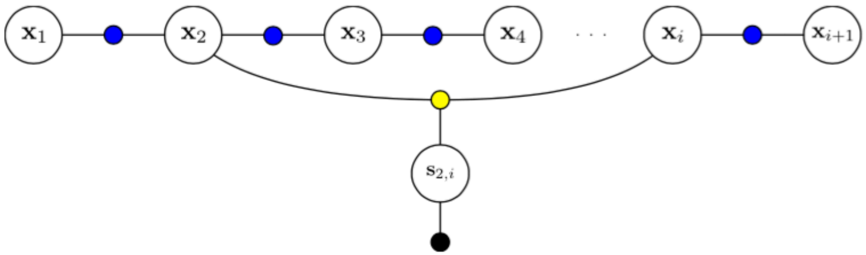
- $$p(z_{k+1}|\mathcal{H}^-_{k+1}) \equiv \sum_j \int_x p(z_{k+1}, x, A_j|\mathcal{H}^-_{k+1}) \doteq \sum_j w_j$$
$$\equiv \sum_j \int_x p(z_{k+1}|x, A_j, \mathcal{H}^-_{k+1})p(A_j|x, \mathcal{H}^-_{k+1})b[x^-_{k+1} = x]$$
$$\doteq \sum_j w_j$$

- $$p(A_j|\mathcal{H}^-_{k+1}, z_{k+1}) = \int_x p(A_j, x|\mathcal{H}^-_{k+1}, z_{k+1})$$
$$\doteq \eta \int_x p(z_{k+1}|A_j, x, \mathcal{H}^-_{k+1})p(A_j, x|\mathcal{H}_{k+1})$$
$$\doteq \eta \int_x p(z_{k+1}|x, A_j, \mathcal{H}^-_{k+1})p(A_j|x, \mathcal{H}^-_{k+1})b[x^-_{k+1} = x]$$
$$= \eta w_j \doteq \tilde{w}_j$$

Relation to Prior Work

- **Robust graph optimization approaches**:
  - Attempt to be resilient to incorrect data association (outliers overlooked by front-end algorithms, e.g. RANSAC)
  - Only consider the **passive** case (actions/controls are given)
  - In contrast, we consider the **active** case (belief space planning)





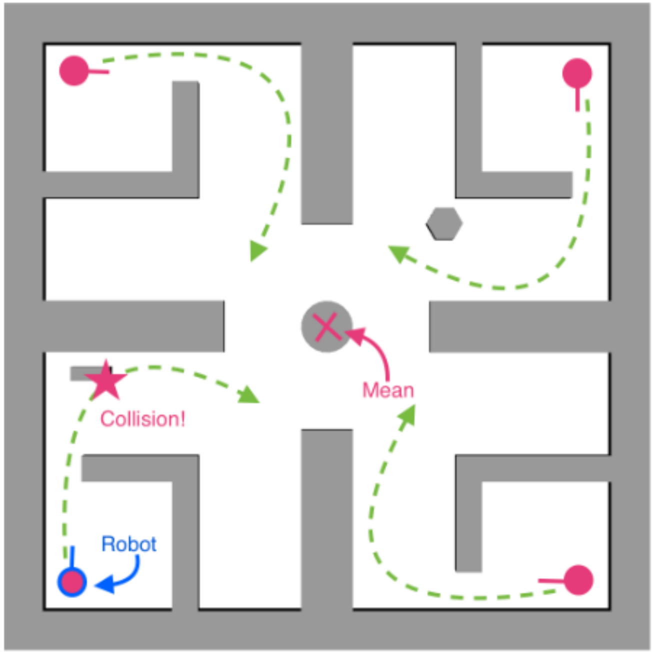
Images from Sünderhauf et al., ICRA'12

Relation to Prior Work

- Probably the closest work to our approach is by Agarwal et al., arXiv 2015
- Hypotheses due to ambiguous data association considered and method developed for active disambiguation
- Consider ambiguous data association only within the prior belief
- Assume there indeed exists an action that can yield complete disambiguation.

Intuition regarding GMM prior

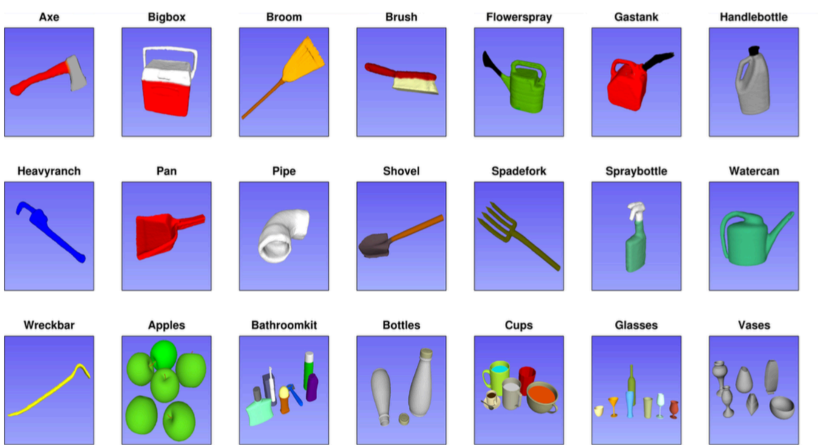
- Kidnapped robot scenrio
- Robot can be in either of the 4 rooms initially

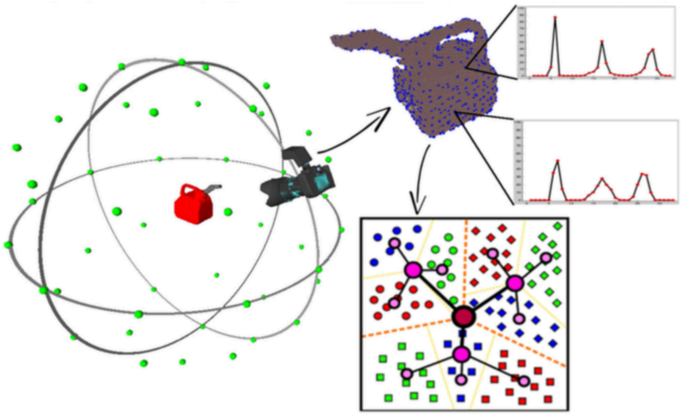


Agarwal et al., arXiv 2015

Relation to Prior Work

- **Active hypothesis disambiguation, active object classification**
  - Finding the correct hypothesis (associations) from a sequence of viewpoints
  - Assumes sensor to be localized
  - We consider both **localization uncertainty** and **data association** aspects within the belief





Atanasov et al., TRO'14

Key Idea

Bimodal posterior belief (GMM)